DISCUSSION PAPER SERIES

No. 10862

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INDUSTRIAL ORGANIZATION and INTERNATIONAL TRADE AND REGIONAL ECONOMICS

ISSN 0265-8003

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Discussion Paper No. 10862
October 2015
Submitted 30 September 2015

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PRICE COMPETITION IN PRODUCT VARIETY NETWORKS†

Abstract

We develop a product-differentiated model where the product space is a network defined as a set of varieties (nodes) linked by their degree of substituabilities (edges). In this network, we also locate consumers so that the location of each consumer (node) corresponds to her "ideal" variety. We show that there exists a unique Nash equilibrium in the price game among firms. Equilibrium prices are determined by firms' weighted Bonacich centralities and the average willingness to pay across consumers. They both hinge on the network structure of the firm-product space. We also investigate how local product differentiation and the spatial discount factor affect the equilibrium prices. We show that these effects non-trivially depend on the network structure. In particular, we find that, in a star-shaped network, the firm located in the star node does not always enjoy higher monopoly power than the peripheral firms.

JEL Classification: D43, L11 and L13
Keywords: monopolistic competition, networks, product variety and spatial competition

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† We owe special thanks to J.-F. Thisse for a detailed and useful discussion. We are also grateful to K. Behrens, S. Dhingra, Y. Murata, M. Parenti and M. Sandomirskaya for comments and suggestions. Philip Ushchev acknowledges financial support from the Government of the Russian Federation under the grant 11.G34.31.0059.

1 Introduction

In industrial organization, there are two dominating approaches to modeling product differentiation: (*i*) *spatial competition*, also known as the *address* approach, which was first suggested by Hotelling (1929) and further developed by Lancaster (1966), and (*ii*) *monopolistic competition*, introduced by Chamberlin (1933) and formalized by Spence (1976) and Dixit and Stiglitz (1977).[1]

Each of these approaches has generated a large flow of contributions, which have been applied to a wide range of economic issues (see, e.g. Helpman and Krugman, 1989; Fujita and Thisse, 2013). However, these approaches have limitations. On the one hand, spatial competition a la Hotelling relies on the principle of mutually exclusive choices, meaning that each consumer purchases only one variety. On the other hand, all varieties are assumed to be equally good substitutes in models of monopolistic competition and due to consumers' love for variety, all varieties are consumed in equal volumes. As a consequence, the two frameworks feature strong dissimilarities in patterns of consumers' behavior, which, in turn, imply different properties of market outcomes.

The aim of our paper is to provide a unifying framework for studying imperfect competition in a "firm-product"space, which would capture both features of Hotelling's and Chamberlin's models. In the 1980s and early 1990s, a number of attempts (Sattinger, 1984; Perloff and Salop, 1985; Hart, 1985; Deneckere and Rotschild, 1992) have been made to bridge these seemingly orthogonal views of the world. Having improved substantially our understanding of the relationships between various models of imperfect competition, these contributions eventually failed to produce a workable model for studying imperfect competition under product differentiation. This is why the various intermediate possibilities remain almost ignored in the literature, a feature of the modern state of the art that we seek to change.

Our point of departure is that understanding the nature of market competition requires a flexible tool that maps the fundamental features of the product space into substitutability patterns across varieties available in the market. In order to design such a tool, we use a *network approach*.[2] This allows us to develop a new model of price competition, which combines features of both spatial and monopolistic competition, while retaining the tractability of linear-quadratic representative consumer models, which are also widely used in the industrial organization literature. To be precise, the salient feature of our setting is that we model the product space as a *network*, which captures simultaneously the following two features of the demand side: (i) *proximity* of each variety to a consumer's ideal variety, and (ii) the binary relationship of *direct substitutability* between varieties. In our model, there is a link between two varieties/nodes if and only if they are direct substitutes, while consumer's willingness to pay decays with the distance between a specific variety and her ideal variety. Thus, consumers exhibit love for variety, as in monopolistic competition, but are willing to pay less for more distant varieties, like in the address approach. This way of representing the product space has two main benefits.

[1]For overviews, see Beath and Katsoulacos (1991), Anderson et al. (1992) and Matsuyama (1995).

[2]The economics of networks is a growing field. For overviews, see Jackson (2008, 2014), Ioannides (2012), Jackson and Zenou (2015) and Jackson et al. (2015).

First, we depart from Chamberlin's assumption of symmetry, i.e. firms no longer face identical demand curves. Instead, each firm demand's behavior depends on the firm's position in the product-variety network. In particular, Chamberlin-type and Hotelling-type product spaces are obtained as special cases, when the substitutability network is a complete graph or a chain, respectively. Second, we capture the idea of Hart (1979), who considered the space of varieties as a compact metric space, in a way which leads to intuitive and testable predictions while working with a general network.[3] As a consequence, our model allows for a rich set of regimes of imperfect competition.[4]

Our main findings can be summarized as follows. First, we propose a new way of modeling firms' heterogeneity with respect to, e.g., Melitz (2003), Melitz and Ottaviano (2008), who put firms with asymmetric marginal costs into a symmetric consumers' taste space. Instead, in our model, asymmetries solely stem from the different positions of firms in the product-variety space, which we model as a product-variety network. More precisely, the structure of the network begets two effects: (i) the *market access effect*, i.e. a firm that enjoys locational advantage reaches more consumers, and (ii) the *localized competition effect*, which means that toughness of competition varies from one location to another, depending on the number of firm's potential competitors. Observe that, unlike standard differentiated oligopoly theory (Vives, 1999), here, Bertrand competition among firms *never exhibits strategic complementarity* (except for the Chamberlinian type of competition, i.e. when the network is a complete graph). This is because the prices of firms producing direct substitutes of a specific firm's variety are strategic complements for the firm's price, while prices of second-order substitutes are strategic substitutes.

Second, we totally characterize the Nash equilibrium in terms of prices for any possible network and show that the equilibrium price of each firm is a function of her *weighted Bonacich centrality*,[5] where the discount factor is an inverse measure of the degree of global product differentiation. It is also a function of the *average willingness to pay across consumers*, which depends on the structure of the network, in particular, the diameter of the network. Interestingly, what matters for the characterization of the equilibrium is a "signed"modification of the Bonacich centrality (where the discount factor changes sign with the distance in the network), rather than the standard Bonacich centrality with positive discount factor. Furthermore, we find that the Bonacich centrality matrix has a similar role as the Slutsky matrix of the demand system generated by the underlying product-variety network.

Third, we investigate how *local product differentiation* γ affects the equilibrium prices. We find that, when products are highly differentiated, a small reduction in the degree of differentiation makes competition tougher and reduces all prices. However, the magnitude of price reduction

[3] Indeed, each network is endowed with the geodesic distance, hence it can be viewed as a compact metric space. The purpose of Hart (1979) was very different from ours: his highly abstract model was aimed at studying the foundations of Chamberlinian monopolistic competition, whereas both existence of equilibrium and the ability of his model to generate intuitive and testable predictions about the market outcome were out of focus.

[4] In this paper, our understanding of what a "regime of imperfect competition"means is different from that in d'Aspremont et al. (2009), who develop a unifying approach of oligopolistic competition allowing for varying competitive toughness. We, instead, focus on Bertrand competition with a differentiated good, but make no restrictive assumptions on the structure of the product space. As a consequence, our model allows for versatile behavior of markups and industrial concentration.

[5] which is due to Bonacich (1987) and discounts further away paths less than closer ones.

depends on both the network structure and the distance decay factor. If, for example, we consider a *star-shaped network*, we find that the firm located in the star node does not always enjoy higher monopoly power than the other firms. This is because the firm located at the star node has better access to the market than the periphery firms, but it also faces tougher competition (all peripheral firms instead of one). Which of the two effects prevails depends on the value of the product substitutability parameter γ.

Fourth, we study how ϕ, the *spatial discount factor*, which captures the fact that there is an "exponential decay" of the attractiveness of varieties with distance, affects equilibrium prices. We show that, for sufficiently low values of γ, an increase in the spatial discount factor ϕ leads to higher prices. The intuition behind this result is as follows: when ϕ increases, firm i will have less monopoly power over consumer i, but better access to all the other consumers. The latter effect clearly dominates the former when γ is not too large. When γ is sufficiently large, the opposite may occur, for which we have examples obtained via simulations. Furthermore, we provide a reformulation of our model in terms of transportation cost t instead of spatial discount factor ϕ and show that the comparative statics results obtained for t are similar to the ones obtained for ϕ. In this version of the model, the key force that shapes the market outcome is the interaction between two types of firms' centrality: the Bonacich centrality and the inverse of the closeness centrality. The latter plays the same role as the average willingness-to-pay in the baseline model.

We also investigate the case of symmetric equilibrium, in which all firms charge the same price. We show that, if the product-variety network is regular (i.e. all nodes have the same number of links) and γ is small enough, then if we add links in the network keeping the number of nodes constant (so that competition becomes less localized), there exist two threshold values of the spatial discount factor, $\underline{\phi}$ and $\overline{\phi}$, such that the *competition effect* (i.e. competition becomes tougher when new links are added) dominates the *market access effect* (i.e. adding new links skews the distribution of distances in the network toward zero, thus bringing all consumers closer to each firm) if and only if $\underline{\phi} < \phi < \overline{\phi}$. Otherwise, the market access effect dominates the competition effect. This is because, when adding new links, the market access effect drives prices upwards while the competition effect leads to a reduction in prices.

Finally, we investigate other implications of our model. In particular, we calculate the Herfindahl index to measure market competitiveness. We show that it depends on the network structure and that denser networks do not always have more market competitiveness.

The rest of the paper unfolds as follows. In the next section, we review the related literature and highlight our contribution. In Section 3, we describe our model and determine the Nash equilibrium. In particular, we illustrate our results with two extreme cases of competition: the *Chamberlinian* competition for which the network is complete and the *Chen-Riordan* competition for which the network has a star shaped. We perform the comparative-statics exercises of our model in Section 4. Symmetric equilibria are analyzed in Section 5. The implications of our model are investigated in Section 6. Finally, Section 7 concludes. All proofs can be found in the Appendix.

2 Related literature

Our model can be viewed as a further development of monopolistic competition models (Matsuyama, 1995), as well as a radical generalization of the "spokes" model proposed by Chen and Riordan (2007), where the network represents the variety space. From the technical viewpoint, our approach is related to literature on games on networks (Jackson and Zenou, 2015) where the network is explicitly modeled as a graph and the payoff functions are linear-quadratic (see, in particular, Ballester et al., 2006; Bramoullé et al., 2014; Calvó-Armengol et al., 2009). The focus and results are, however, very different to ours. There is also a growing literature that models price competition between firms with an explicit network. Two important papers in this literature are that of Bloch and Quérou (2013) and Candogan et al. (2012).[6] Bloch and Quérou (2013) study optimal monopoly pricing in the presence of network externalities across consumers. The setting proposed by these authors involves a homogeneous good produced by a monopolist, and many consumers whose probability to purchase the good. Candogan et al. (2012) develop a similar approach, but with a divisible good.[7] In contrast to these papers, we account for product differentiation and consider a price-setting game among several firms. There is also an interesting literature on more general aspect of industrial organization and networks. However, most papers in this literature (Goyal and Moraga-Gonzalez, 2001; Goyal and Joshi, 2003; Westbrock, 2010; König, 2013; König et al., 2014) introduce the network through R&D collaborations. We believe we are the first to apply the toolkit of games on networks to modeling competition in product-variety space within an address approach.[8]

To be precise, the main novelty of our modeling strategy compared to the previous literature may be described as follows. First, the ideal variety of each consumer, or, equivalently, consumer's location in a "firm-product" space, is a node in the network. Second, the geodesic distance between nodes measures the degree of taste heterogeneity. In addition, the degree of pairwise substitutability between product varieties is high (low) - or, equivalently, firms are (are not) involved into head-to-head competition - when there is a link (there is no link) between the corresponding nodes. In other words, the principal role of a network in our model is that it captures the *substitutability relationship* between differentiated products. This is where our work departs from the modern "non-spatial"paradigm of modeling imperfect competition in international trade (Ottaviano et al., 2002; Melitz, 2003; Melitz and Ottaviano, 2008; Dhingra, 2013; Mayer et al., 2014), where the substitution term is the same across varieties and where heterogeneities are mostly on the supply side.[9] The relationship of our work to this strand of literature is best described as follows. Recent

[6] See also Shi (2003), Deroian and Gannon (2006), Banerji and Dutta (2009), Billand et al. (2014), Carroni and Righi (2015), Currarini and Feri (2015) and Chen et al. (2015).

[7] See also Bimpikis et al. (2015) who develop a model with a bipartite graph where nodes are either firms or markets and a link between firm i and market j exists if firm i operates in market j.

[8] Note that Gabszewicz and Thisse (1986) have developed within spatial competition theory a graph-theoretic setup to determine rigorously the set of firms forming an industry in the spatial economy.

[9] In the modern trade literature, network perspective is typically used for studying free-trade agreements (see, e.g., Furusawa and Konishi, 2007). A notable exception is Behrens et al. (2007), who consider trading countries as nodes of a spatial network and stress the role of Bonacich centrality for understanding the equilibrium trade patterns. Note also that Osharin et al. (2014) and Tarasov (2014) study income-taste heterogeneities across consumers within

studies of monopolistic competition under variable elasticity of substitution (Behrens and Murata, 2007; Zhelobodko et al., 2012; Parenti et al., 2014) go in the direction of dealing with more and more general classes of symmetric consumers' utilities, remaining within the non-spatial paradigm. We, instead, choose to study the consequences of a *non-specified network structure of the product space*. This is done at the cost of working with a relatively specific family of utilities (namely, linear-quadratic), which are well known to be best suited for studying games on networks.

Finally, our paper echoes the logit model of product differentiation (Anderson et al., 1992; Anderson et al., 1995), in which combining the ideal variety approach and the love for variety approach is achieved by introducing a probabilistic choice on the consumers' side. Furthermore, our comparative statics results, while being generically different from those obtained within the standard spatial competition approach a la Hotelling (see Section 4.2), parallel recent findings by Chen and Riordan (2008) on price-increasing competition. We differ from all these authors by stressing the role of the *topology* of the product space, which we model by means of a network.

3 The model

3.1 Notations and definitions

There are N firms that produce N different varieties, each firm $i = 1, ..., N$ producing one variety i. Each firm/variety is embedded into a *network* $(\mathcal{N}, \mathbf{G})$, where each variety $i \in \mathcal{N} \equiv \{1, 2, ..., N\}$ is a *node*, while $\mathbf{G} = (g_{ij})_{i,j=1...N}$ is the *adjacency matrix* that keeps track of the *substitutability* between varieties in the network. To be more precise, there is a link (i.e. $g_{ij} = 1$) between varieties i and j if and only if these two varieties are *direct* substitutes. Otherwise, a link does not exist, i.e. $g_{ij} = 0$. By convention, $g_{ii} = 0$. Quite naturally, $g_{ij} = g_{ji}$ so that the network is *undirected*, which implies that $\mathbf{G}$ is a square $(0, 1)$ *symmetric matrix* with zeros on its diagonal.

We have the following standard network-related definitions. A *walk* in a network $(\mathcal{N}, \mathbf{G})$ refers to a sequence of nodes, $i_1, i_2, i_3, \ldots, i_{L-1}, i_L$ such that $g_{i_l i_{l+1}} = 1$ for each l from 1 to $L-1$. The *length* of the walk is the number $L-1$ of links in it. A *path* in a network $(\mathcal{N}, \mathbf{G})$ is a walk in $(\mathcal{N}, \mathbf{G})$, $i_1, i_2, i_3, \ldots, i_{K-1}, i_K$, such that all the nodes are distinct. The (geodesic) *distance* d_{ij} between two nodes i and j in a network is the length of a shortest path between them. The sth power $\mathbf{G}^s = \mathbf{G} \times \overset{(s\ times)}{\cdots} \mathbf{G}$ of the adjacency matrix $\mathbf{G}$ keeps track of indirect connections in $\mathbf{G}$. More precisely, the coefficient $g_{ij}^{[s]}$ in the (i, j) cell of $\mathbf{G}^s$ gives the number of walks of length s in $\mathbf{G}$ between i and j. The set of neighbors (here *direct substitutes*) of node (here variety) i in network $(\mathcal{N}, \mathbf{G})$ are denoted by $\mathcal{N}_i = \{\text{all } j | g_{ij} = 1\}$.

In our model, the distance between two products in the network measures the degree of substitutability between these two products so that the higher is the distance, the poorer substitutes

non-spatial settings. Their approaches, however, are substantially different from ours, for the demand side in their model is described by the standard CES utility, hence, any two varieties are equally substitutable. A setting closer to ours is used by Di Comite et al. (2014), who work with asymmetric linear-quadratic preferences to study the empirical implications of country-specific taste mismatch patterns on trade flows.

are these products. In other words, the network $(\mathcal{N}, \mathbf{G})$ plays the role of a "firm-product"space in the model and captures the degree of substitution between N varieties supplied in the economy. Because we do not impose any specific assumptions about the network structure (except in Section 5, which mainly serves for illustrative purposes), we find that our approach is flexible enough to encompass different types of spatial structures, commonly studied in the industrial-organization literature. Figure 1 illustrates some of these networks. For example, *Chamberlinian* competition (due to Chamberlin, 1933) corresponds to the *complete network* (Figure 1a), Hotelling competition (due to Hotelling, 1929) to the *line* or *chain network* (Figure 1b), Salop competition (due to Salop, 1979) to the *circle network* (Figure 1c) and the *Chen-Riordan* competition (due to Chen and Riordan, 2008) to the *star network* (Figure 1d).

[*Insert Figure* 1 *here*]

3.2 Consumers

We have seen that a network is composed of varieties (*nodes*) linked by their degrees of substitutability (*edges*). In the network, we can also locate consumers so that the location of each consumer $k = 1, ..., N$ corresponds to her "ideal" variety. As a result, there are as many consumers as varieties.

3.2.1 Preferences

A consumer located at location/node $k \in \mathcal{N}$, i.e. whose ideal variety is k, has the following linear-quadratic utility function:

$$U(k, \mathbf{G}) = x_{0k} + \sum_{i \in \mathcal{N}} \alpha_{ik} x_{ik} - \frac{1}{2} \left(\sum_{i \in \mathcal{N}} x_{ik}^2 + \sum_{i,j \in \mathcal{N}} \gamma_{ij} x_{ik} x_{jk} \right) \tag{1}$$

where x_{0k} is the level of consumption of an outside good, x_{ik} is the volume of consumer ks purchases of variety produced by firm i (i.e. located at node i) whereas α_{ik} is the *consumer-specific willingness to pay* for variety i.

Quite naturally, we assume that α_{ik} is a decreasing function of the (geodesic) distance d_{ik} in the network between variety i and the ideal variety of consumer k. Indeed, as in Jackson and Wolinsky (1996), we assume that

$$\alpha_{ik} = \alpha \phi^{d_{ik}}, \tag{2}$$

where $0 < \phi < 1$ is a *spatial discount factor*. This captures the fact that there is an "exponential decay" of the attractiveness of varieties with distance. Observe that the term $\sum_{i \in \mathcal{N}} \alpha_{ik} x_{ik}$ captures the *proximity of other varieties to the ideal variety* of consumer k and depends on the location of k in the network (i.e. her ideal product). It is referred to as the *proximity network effect.*

Furthermore, the coefficients γ_{ij} depend on the structure of the network $(\mathcal{N}, \mathbf{G})$. More specifically, we assume that $\gamma_{ij} \equiv \gamma g_{ij}$ (where $\gamma > 0$ is a positive substitution parameter) so that only direct substitutes for which $g_{ij} = 1$ have an impact on the utility function. In other words, γ captures the *global* substitution effect while g_{ij} accounts for *local* substitution effect. Observe that γ_{ij} is network specific but not consumer specific. A higher γ means that varieties are less differentiated and thus the consumption of close substitutes reduces the utility of consuming variety i. The term $\frac{1}{2}\left(\sum_{i\in\mathcal{N}} x_{ik}^2 + \sum_{i,j\in\mathcal{N}} \gamma_{ij} x_{ik} x_{jk}\right)$ accounts for the consumer's *love for variety* but does not depend on her position on the network. This is referred to as the *love-for-variety effect*.

At this stage, we find useful to compare our setting to the standard approaches to product differentiation (Belleflamme and Peitz, 2010, Chap. 5). On the one hand, in love-for-variety models of market competition (monopolistic competition) with linear-quadratic utility, γ serves as the sole (inverse) measure of product differentiation while α is not individual-specific. On the other hand, in spatial competition models, the transportation cost is a measure of product differentiation. Our setting differs from both these frameworks in at least two respects. First, instead of α, we have here α_{ik}, defined by (2), which depends on the *location* of the consumer k in the network, while the spatial discount factor ϕ is a counterpart of the shopping cost in models a la Hotelling.[10] Second, instead of γ, we have here $\gamma_{ij} \equiv \gamma g_{ij}$, which depends on the *structure* of the network. To sum up, we have an essentially multidimensional description of how differentiated varieties are, given by ϕ, γ, and the network $\mathbf{G}$. Indeed, ϕ keeps track of the degree to which a consumer's valuation of the ideal variety exceeds that of any other variety, γ measures the degree of love for variety and $\mathbf{G}$ captures the topology of the product space, which affects the interplay between ϕ and γ.

To illustrate the nature of the terms ϕ and γ in the utility function (1), consider the star network of Figure 1d (Chen-Riordan competition with $N = 4$). In that case, for the consumer whose ideal variety is the star (node $k = 1$), the proximity effect amounts to

$$\sum_{i=1}^{i=4} \alpha_{i1} x_{i1} = \alpha x_{11} + \alpha\phi \sum_{i=2}^{i=4} x_{i1},$$

while the love-for-variety effect is given by

$$\sum_{i=1,j=1,i\neq j}^{i=4,j=4} \gamma_{ij} x_{i1} x_{j1} = \gamma \sum_{i=1,j=1,i\neq j}^{i=4,j=4} g_{ij} x_{i1} x_{j1} = \gamma x_{11}\left(x_{21} + x_{31} + x_{41}\right).$$

For any peripheral agent, for example the one whose ideal variety is $k = 2$, we have:

$$\sum_{i=1}^{i=4} \alpha_{i2} x_{i2} = \alpha x_{22} + \alpha\phi x_{12} + \alpha\phi^2 \sum_{i=3}^{i=4} x_{i2}$$

[10] In Section 4.2.3, we explicitly reformulate our model in terms of transportation cost instead of spatial discount factor to investigate if the properties of the model remain the same under the two formulations.

and

$$\sum_{i=1,j=1,i\neq j}^{i=4,j=4} \gamma_{ij} x_{i2} x_{j2} = \gamma \sum_{i=1,j=1,i\neq j}^{i=4,j=4} g_{ij} x_{i2} x_{j2} = \gamma x_{12} \left(x_{22} + x_{32} + x_{42} \right)$$

Observe that the substitution effect is global and network specific, while the willingness-to-pay effect is consumer specific.

For the Salop-type product space, the network is regular of order 2, which means that each variety has two direct substitutes. In that case, for example, for the consumer whose ideal variety is $k = 1$, we have:

$$\sum_{i=1}^{4} \alpha_{i1} x_{i1} = \alpha x_{11} + \alpha\phi \left(x_{21} + x_{41} \right) + \alpha\phi^2 x_{31}$$

and

$$\sum_{i=1,j=1,i\neq j}^{i=4,j=4} \gamma_{ij} x_{i1} x_{j1} = \gamma \sum_{i=1,j=1,i\neq j}^{i=4,j=4} g_{ij} x_{i1} x_{j1} = \gamma \left(x_{11} x_{21} + x_{11} x_{41} + x_{21} x_{31} + x_{31} x_{41} \right)$$

Let us compare these two terms for these two different networks. Consider first the *proximity network effect*. In the star network, all varieties are relatively close to each other so consumers do not lose too much from consuming other varieties than their ideal one while, the reverse occurs for the Salop competition (circle network) where some variety can be as far as 2 edges away and, in the case of N varieties, one can be as far as $N/2$ edges away. As a result, consuming more varieties different from the ideal variety has a higher impact on the proximity term of utility in the star network than in the circle network. If we now consider the *love-for-variety effect*, then we have the opposite. Indeed, the Chen-Riordan competition implies that many varieties (the ones located at the periphery of the network) are not direct substitutes (because they have a good match with local consumers' tastes, while consumers' tastes are very heterogeneous across locations) and compete with only one variety (the one located in the center of the network), which is a direct substitute for any "local" variety. On the contrary, in the circular model, each variety competes only with its two nearest neighbors. Therefore, consumers obtain more utility in terms of the love-for-variety effect in the circle network than in the star network. As a result, there is a trade off between different networks (or competition regimes) that we want to study in equilibrium.

Observe that when $\phi = 1$, then no consumer has ideal variety. If, in addition, the network is complete (Chamberlin network, Figure 1a), we are back to the standard representative consumer's approach, where only the love for-variety effect matters, while the proximity network effect does not.

3.2.2 Individual demand

For the sake of the exposition, we rewrite the utility function (1) in vector-matrix form:

$$U\left(k,\mathbf{G}\right)=x_{0k}+\boldsymbol{\alpha}_k^T\left(\phi,\boldsymbol{G}\right)\mathbf{x}_k-\frac{1}{2}\mathbf{x}_k^T(\mathbf{I}+\gamma\boldsymbol{G})\mathbf{x}_k, \tag{3}$$

where $\boldsymbol{\alpha}_k\left(\phi,\boldsymbol{G}\right)=\left(\alpha_{ik}\right)_{i=1\ldots N}$ and $\mathbf{x}_k\equiv\left(x_{ik}\right)_{i=1\ldots N}$ are $N\times 1$ vectors, $\mathbf{I}$ is the identity matrix of order N and $\mathbf{x}^T$ is the transpose of vector $\mathbf{x}$. Denote by $\lambda_1(\mathbf{G}),\ldots,\lambda_N(\mathbf{G})$, the eigenvalues of $\boldsymbol{G}$ where, without loss of generality, $\lambda_1(\mathbf{G})\geq\lambda_2(\mathbf{G})\geq\ldots\geq\lambda_N(\mathbf{G})$, so that $\lambda_1(\mathbf{G})$ is the largest eigenvalue of $\mathbf{G}$ while $\lambda_N(\mathbf{G})$ is the lowest eigenvalue of $\mathbf{G}$.

Lemma 1. *The utility function (3) is strictly concave in* $\mathbf{x}_k$ *if and only if:*

$$\gamma<-\frac{1}{\lambda_N(\mathbf{G})} \tag{4}$$

Consumer k seeks to maximize her utility (3) with respect to $(x_0,\mathbf{x})$ subject to the budget constraint:

$$x_{0k}+\mathbf{p}^T\mathbf{x}_k=Y_k,$$

where Y_k is consumer k's income whereas $\mathbf{p}=(p_i)_{i\in\mathcal{N}}$ is the *price vector*. Plugging the value of x_{0k} from this budget constraint into (3) yields:

$$U\left(k,\mathbf{G}\right)=Y_k-\mathbf{p}^T\mathbf{x}_k+\boldsymbol{\alpha}_k^T\left(\phi,\boldsymbol{G}\right)\mathbf{x}_k-\frac{1}{2}\mathbf{x}_k^T(\beta\mathbf{I}+\gamma\boldsymbol{G})\mathbf{x}_k$$

The inverse demand of consumer k for variety $i\in\mathcal{N}$ is given by:

$$p_i=\alpha_{ik}\left(\phi,\boldsymbol{G}\right)-x_{ik}-\gamma\sum_{j\neq i}g_{ij}x_{jk},\ i=1,\ldots,N \tag{5}$$

or, in vector-matrix form,

$$\mathbf{p}=\boldsymbol{\alpha}_k\left(\phi,\boldsymbol{G}\right)-(\mathbf{I}+\gamma\boldsymbol{G})\mathbf{x}_k. \tag{6}$$

By solving (6) for $\mathbf{x}_k$, we obtain consumer k's individual demands for all varieties:

$$\mathbf{x}_k^*=\mathbf{B}\left(-\gamma,\boldsymbol{G}\right)\left[\boldsymbol{\alpha}_k\left(\phi,\boldsymbol{G}\right)-\mathbf{p}\right], \tag{7}$$

where $\mathbf{B}\left(-\gamma,\boldsymbol{G}\right)\equiv\left(\mathbf{I}+\gamma\boldsymbol{G}\right)^{-1}$. Note that (4) implies that $\mathbf{B}$ is well-defined. Moreover, if

$$\gamma<\frac{1}{\lambda_1(\mathbf{G})} \tag{8}$$

then $\mathbf{B}$ can be expanded in a power series as follows:[11]

[11] For example, under *Hotelling competition* (Figure 1b) with N varieties, we have N distinct eigenvalues and the largest eigenvalue $\lambda_1(\mathbf{G})$ of this matrix is less than 2 (but converges to 2 when $N\to\infty$). Moreover, if λ is an eigenvalue of $\mathbf{G}$, then $-\lambda$ is also an eigenvalue of $\mathbf{G}$. Hence, a sufficient condition for (4) and (8) to hold is: $\gamma\leq 1/2$.

$$\mathbf{B}(-\gamma, \boldsymbol{G}) = \mathbf{I} - \gamma \boldsymbol{G} + \gamma^2 \boldsymbol{G}^2 - \gamma^3 \boldsymbol{G}^3 + \ldots \tag{9}$$

Following Bonacich (1987) and using (9), we may define *the vector of weighted Bonacich centrality measures of varieties* as:

$$\mathbf{b}(-\gamma, \boldsymbol{G}, \mathbf{u}) \equiv \mathbf{B}(-\gamma, \boldsymbol{G})\,\mathbf{u}, \tag{10}$$

where $\mathbf{u}$ is any $N \times 1$ vector. In the coordinate form, we have: $b_{ij} = \sum_{s=0}^{\infty} (-\gamma)^s \, g_{ij}^{[s]}$, or equivalently

$$b_{ij} = \begin{cases} 1 + \gamma^2 g_{ii}^{[2]} - \gamma^3 g_{ii}^{[3]} + ... & \text{for } i = j \\ -\gamma g_{ij} + \gamma^2 g_{ij}^{[2]} - \gamma^3 g_{ij}^{[3]} + ... & \text{for } i \neq j \end{cases} \tag{11}$$

Observe that $g_{ij}^{[s]}$ is the number of walks of length s in the network, which starts at variety i and ends at variety j (see Section 3.1). In particular, $g_{ii}^{[2]}$ is just the number of neighbors (i.e. direct substitutes) of variety i while $g_{ii}^{[3]}$ is the number of triangles involving i, i.e. the number of couples of direct substitutes which are also good substitutes for each other. As can be seen from (11), b_{ii} can be viewed as a firm-specific measure for *toughness of competition* faced by a firm producing variety i. When firm i's closest competitors also compete with each other, this relaxes the burden of competition borne by firm i. That is why $g_{ii}^{[3]}$ enters (11) with the negative coefficient $(-\gamma)^3$. The cycles of higher orders are also accounted for, but their weight decays exponentially with s.

The intuition behind the matrix $\mathbf{B}(-\gamma, \boldsymbol{G})$ and the Bonacich centrality measure (10) in our context can be further clarified using the demand system (7). Indeed, $\mathbf{B}(-\gamma, \boldsymbol{G})$ is the Slutsky matrix of the consumers' demand, while the Bonacich centrality measure can serve to directly compute the responses of quantities purchased to changes in prices. More precisely, consider a vector $\mathrm{d}\mathbf{p}$ of changes in prices. Then, a vector of corresponding changes in the quantities purchased by each consumer k will be given by

$$\mathrm{d}\mathbf{x}_k^* = -\mathbf{B}(-\gamma, \boldsymbol{G})\,\mathrm{d}\mathbf{p} = -\mathbf{b}(-\gamma, \boldsymbol{G}, \mathrm{d}\mathbf{p}). \tag{12}$$

To sum up, *a variety that has a higher (lower) Bonacich centrality in the product-variety network means that the demand for this variety is more (less) sensitive to changes in prices.*

Applying (11) to (12) yields the following decomposition of price effects:

$$\mathrm{d}x_{ik}^* = -\mathrm{d}p_i + \gamma \sum_{j \in \mathcal{N}} g_{ij} \mathrm{d}p_j - \gamma^2 \sum_{j \in \mathcal{N}} g_{ij}^{[2]} \mathrm{d}p_j + \gamma^3 \sum_{j \in \mathcal{N}} g_{ij}^{[3]} \mathrm{d}p_j + \ldots \tag{13}$$

The intuition behind equation (13) may be described as follows. Assume that prices change by $\mathrm{d}\mathbf{p}$. What happens to the consumption level of variety i? The immediate (and myopic) response of consumer k, captured by the first term in the right-hand side of (13), is just to reduce consumption by $\mathrm{d}p_i$, without paying any attention to changes in prices for other varieties. This is not the

Under *Salop* competition (Figure 1c) with N varieties, the largest eigenvalue $\lambda_1(\mathbf{G})$ of this matrix is equal to 2. Hence, (4) and (8) boil down to $\gamma < 1/2$.

end of the story, however. Individual k realizes that the initial decision has been fairly myopic, and seeks to adjust better her consumption bundle to the new circumstances. She does so by accounting for changes in prices of the closest substitutes of variety i, the resulting change in x_{ik}^* being captured by $\gamma \sum_{j \in \mathcal{N}} g_{ij} \mathrm{d}p_j$, the second term in the right-hand side of (13). Quite naturally, this term is positive (negative) if prices for varieties neighboring to i have increased (decreased). However, our individual refines even better her decision by taking into account also the prices for the substitutes of the substitutes. In other words, she now looks at varieties whose geodesic distance from i is at most 2. The magnitude of the corresponding change in x_{ik}^* is shown by the third term, $-\gamma^2 \sum_{j \in \mathcal{N}} g_{ij}^{[2]} \mathrm{d}p_j$. Observe that, unlike the second-order price effect $\gamma \sum_{j \in \mathcal{N}} g_{ij} \mathrm{d}p_j$, the third-order price effect $-\gamma^2 \sum_{j \in \mathcal{N}} g_{ij}^{[2]} \mathrm{d}p_j$ differs from zero even when $\mathrm{d}p_j = 0$ for all $j \neq i$, for $g_{ii}^{[2]} = N_i \neq 0$, the degree of node i. Thus, the decision-making process described above involves several steps even when prices for all varieties except i remain unchanged, the reason being that the substitution effect raised by the price shocks $\mathrm{d}p_i$ propagates through the whole network and impacts consumption levels of other varieties, which, in turn, leads to a feedback effect on the choice of how much of variety i to purchase. To sum up, (13) illustrates the "iterative" nature of consumers' responses to price shocks.[12]

Observe also that, in the literature on games on networks (Jackson and Zenou, 2015), the (weighted) Bonacich centrality is usually defined as $\mathbf{B}(\gamma, \boldsymbol{G})\, \mathbf{u}$ where

$$\mathbf{B}(\gamma, \boldsymbol{G}) = \mathbf{I} + \gamma \boldsymbol{G} + \gamma^2 \boldsymbol{G}^2 + \gamma^3 \boldsymbol{G}^3 + \dots$$

so that there are *no negative terms*. Here, we have a different definition that allows for negative values of the decay factor, which in fact, is also considered in the original article of Bonacich (1987). Indeed, Bonacich (1987) discusses the interpretation of his centrality measure when the decay factor alternates between negative and positive values. In our case, this means that even powers of $\mathbf{G}$ are weighted positively and odd powers negatively. This implies that having many direct ties (degree) contributes negatively to centrality, but, if one's connections themselves have many connections, so that there are many paths of length two, centrality is augmented.

To gain more intuition about the nature of individual demands, we can write (7) as follows:

$$x_{ik}^* = \sum_{j \in \mathcal{N}} b_{ij} \alpha_{jk} (\phi, \boldsymbol{G}) - b_{ii} p_i - \sum_{j \in \mathcal{N}, i \neq j} b_{ij} p_j \tag{14}$$

The individual demand (14) of consumer k for variety i is made of three terms. First, the intercept of the individual demand, $\sum\limits_{j \in \mathcal{N}} b_{ij} \alpha_{jk} (\phi, \boldsymbol{G})$, shows the maximum demand for this variety when prices are equal to zero. It is, in fact, the *weighted Bonacich centrality* of variety i for consumer k,

[12] This way of thinking about consumers' decisions bears some resemblance with recent studies in neuroeconomics (see Camerer et al., 2005, and Glimcher and Fehr, 2013).

where the weights are the exponential decay factors $\phi^{d_{jk}}$:

$$\sum_{j \in \mathcal{N}} b_{ij} \alpha_{jk} (\phi, \boldsymbol{G}) = \alpha \left[b_{ik} + \phi \sum_{j \in \mathcal{N}_1(k)} b_{ij} + \phi^2 \sum_{j \in \mathcal{N}_2(k)} b_{ij} + ... \right]$$

where $\mathcal{N}_l(k)$ is the set of nodes (varieties) such that the geodesic distance between i and k equals l, where $i \in \mathcal{N}_l(k)$. This means that, if a consumer is very "central", i.e. she is close to all varieties in the network (like the star in the Chen-Riordan competition network in Figure 1d), then this consumer's willingness to pay is very high. Observe that the intercept of the individual demand is the only part of the demand function which is *individual* specific and depends of the consumer k's position in the network.

Second, the own price effect $b_{ii} p_i$ captures the effect of price of variety i on its own demand. The marginal impact of the price p_i on demand x_{ik}^* is equal to b_{ii}, which is, by (11), the discounted number of cycles involving i. Thus, b_{ii} is *network specific*, but *not individual specific*. Finally, the last term $\sum_{j \in \mathcal{N}, i \neq j} b_{ij} p_j$ comprises the cross-price effects and has a similar interpretation as $b_{ii} p_i$. This means that the price effect crucially depends on the structure of the network. For instance, a complete network will generate price effects very different from those in a star network. Let us now illustrate these differences.

Examples Consider the Chamberlin-type spatial structure (Figure 1a) with N varieties. In this case, $\mathbf{G}$ has only two distinct eigenvalues, $\lambda_1 = N - 1$, $\lambda_2 = \lambda_3 = ... = \lambda_N = -1$. Hence, (4) and (8) boil down to $\gamma < 1/(N-1)$. Consider the case of $N = 4$. Then, if $\gamma < 1/3$, we have:

$$\mathbf{B}(-\gamma, \boldsymbol{G}) \equiv (\mathbf{I} + \gamma \boldsymbol{G})^{-1} = \frac{1}{(1 + 2\gamma - 3\gamma^2)} \begin{pmatrix} 1+2\gamma & -\gamma & -\gamma & -\gamma \\ -\gamma & 1+2\gamma & -\gamma & -\gamma \\ -\gamma & -\gamma & 1+2\gamma & -\gamma \\ -\gamma & -\gamma & -\gamma & 1+2\gamma \end{pmatrix}, \tag{15}$$

and

$$\alpha_{ik}(\phi, \boldsymbol{G}) = \begin{cases} \alpha & \text{if } i = k, \\ \alpha\phi & \text{if } i \neq k. \end{cases}$$

As a result, the individual demand (say for individual 1) for all four varieties is given by:

$$\mathbf{x}_1^* = \frac{1}{(1 + 2\gamma - 3\gamma^2)} \begin{pmatrix} \alpha(1 + 2\gamma - 3\gamma\phi) - (1+2\gamma)p_1 + \gamma(p_2 + p_3 + p_4) \\ \alpha(\phi - \gamma) - (1+2\gamma)p_2 + \gamma(p_1 + p_3 + p_4) \\ \alpha(\phi - \gamma) - (1+2\gamma)p_3 + \gamma(p_1 + p_2 + p_4) \\ \alpha(\phi - \gamma) - (1+2\gamma)p_4 + \gamma(p_1 + p_2 + p_3) \end{pmatrix}. \tag{16}$$

As implied by (16), when $\phi \to 1$, the demand system becomes completely symmetric. On the contrary, under *Chen-Riordan* spatial structure (Figure 1d) with N varieties, the eigenvalues of are:

$\lambda_1 = \sqrt{N-1},\ \lambda_2 = \ldots = \lambda_{N-1} = 0,\ \lambda_N = -\sqrt{N-1}$. Hence, a necessary and sufficient condition for (4) and (8) to hold is $\gamma < 1/\sqrt{N-1}$. For $N = 4$ and $\gamma < 1/\sqrt{3}$, we have (1 is the star variety):

$$\mathbf{B}\left(-\gamma, \boldsymbol{G}\right) = \frac{1}{(1-3\gamma^2)} \begin{pmatrix} 1 & -\gamma & -\gamma & -\gamma \\ -\gamma & 1-2\gamma^2 & \gamma^2 & \gamma^2 \\ -\gamma & \gamma^2 & 1-2\gamma^2 & \gamma^2 \\ -\gamma & \gamma^2 & \gamma^2 & 1-2\gamma^2 \end{pmatrix}, \tag{17}$$

and

$$\alpha_{i1}\left(\phi, \boldsymbol{G}\right) = \begin{cases} \alpha & \text{if } i = 1, \\ \alpha\phi & \text{if } i \neq 1, \end{cases}$$

while for $k \neq 1$,

$$\alpha_{ik}\left(\phi, \boldsymbol{G}\right) = \begin{cases} \alpha & \text{if } i = k, \\ \alpha\phi & \text{if } i = 1, \\ \alpha\phi^2 & \text{otherwise.} \end{cases}$$

As a result, the individual demand for individual 1 (whose ideal variety is the star variety) for all four varieties is given by:

$$\mathbf{x}_1^* = \frac{1}{(1-3\gamma^2)} \begin{pmatrix} \alpha\left(1-3\gamma\phi\right) - p_1 + \gamma\left(p_2+p_3+p_4\right) \\ \alpha\left(\phi-\gamma\right) - \left(1-2\gamma^2\right)p_2 + \gamma p_1 - \gamma^2\left(p_3+p_4\right) \\ \alpha\left(\phi-\gamma\right) - \left(1-2\gamma^2\right)p_3 + \gamma p_1 - \gamma^2\left(p_2+p_4\right) \\ \alpha\left(\phi-\gamma\right) - \left(1-2\gamma^2\right)p_4 + \gamma p_1 - \gamma^2\left(p_2+p_3\right) \end{pmatrix}.$$

The individual demand for all four varieties of a peripheral consumer, say individual 2, whose ideal variety is variety 2, is given by:

$$\mathbf{x}_2^* = \frac{1}{(1-3\gamma^2)} \begin{pmatrix} \alpha\left(\phi-2\gamma\phi^2-\gamma\right) - p_1 + \gamma\left(p_2+p_3+p_4\right) \\ \alpha\left(1+2\gamma^2\phi^2-2\gamma^2-\gamma\phi\right) - \left(1-2\gamma^2\right)p_2 + \gamma p_1 - \gamma^2\left(p_3+p_4\right) \\ \alpha\left(\gamma^2-\gamma^2\phi^2+\phi^2-\gamma\phi\right) - \left(1-2\gamma^2\right)p_3 + \gamma p_1 - \gamma^2\left(p_2+p_4\right) \\ \alpha\left(\gamma^2-\gamma^2\phi^2+\phi^2-\gamma\phi\right) - \left(1-2\gamma^2\right)p_4 + \gamma p_1 - \gamma^2\left(p_2+p_3\right) \end{pmatrix}.$$

Two comments are in order. First, as stated above, the network-specific effect on the individual demand, that is the price effect, is the same for all consumers (star and peripheral) in the network as can be seen by the terms after the intercept. We see that (i) own price effect, $\partial x_{ik}^*/\partial p_i$, is always negative for the demand of each consumer and for each variety, (ii) the cross-price effects, $\partial x_{ik}^*/\partial p_j$, $j \neq i$, are positive (negative) if and only if either i or j is the star (both i and j are peripheral varieties). In other words, any peripheral variety is a gross substitute for the star (central) variety (positive effect) while peripheral varieties come to be gross complements to each other (negative effect). This is very different from what we find in the Chamberlinian type of product space (see (16)) where all varieties are direct substitutes to each other: $\partial x_{ik}^*/\partial p_j = \gamma > 0$, $j \neq i$.

Second, consider the intercept in the demand function, which is individual specific and depends on the position in the network. We can see that, for the star variety, which is at distance 1 to every other variety, the consumer's demand is much higher than for any peripheral variety. This does not imply, however, that the "star" firm $i = 1$ will always enjoy more monopoly power than any peripheral firm. We will consider this issue in more detail in Section 4.

3.2.3 Aggregate demand

We now turn to deriving the aggregate demand for variety i. Using (7), it is straightforward to see that the vector $\mathbf{X}^* \equiv \sum_{k \in \mathcal{N}} \mathbf{x}_k^*$ of aggregate demands faced by firms is given by:

$$\mathbf{X}^* = \mathbf{B}\left(-\gamma, \boldsymbol{G}\right) \left[\sum_{k \in \mathcal{N}} \left(\boldsymbol{\alpha}_k\left(\phi, \boldsymbol{G}\right) - \mathbf{p}\right)\right]. \tag{18}$$

Define the vector of *average willingness to pay across consumers* as

$$\overline{\boldsymbol{\alpha}}\left(\phi, \boldsymbol{G}\right) \equiv \frac{1}{N} \sum_{k \in \mathcal{N}} \boldsymbol{\alpha}_k\left(\phi, \boldsymbol{G}\right),$$

Since $\alpha_{ik} = \alpha \phi^{d_{ik}}$, we have

$$\overline{\alpha}_i\left(\phi, \boldsymbol{G}\right) = \alpha \left[\frac{1}{N} + \phi \frac{N_1(i)}{N} + \phi^2 \frac{N_2(i)}{N} + \ldots + \phi^d \frac{N_d(i)}{N}\right], \tag{19}$$

where $d \equiv \max_{i,k \in \mathcal{N}} d_{ik}$ is the diameter of the network while $N_l(i)$ is the number of nodes (varieties) whose geodesic distance from node i equals l. In particular, $N_1(i)$ is the degree of node i. Using (19) and (18), we can write aggregate demand for variety i as follows:

$$X_i^* = N \left(\sum_{j \in \mathcal{N}} b_{ij} \overline{\alpha}_j - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j\right), \tag{20}$$

where b_{ij} are the elements of $\mathbf{B} \equiv \mathbf{B}\left(-\gamma, \boldsymbol{G}\right)$ (see (11)).

In order to guarantee that expressions (20) make economic sense, we have to require that the choke-off price of aggregate demand for each variety is positive. In other words, the following inequalities must hold:

$$\sum_{j \in \mathcal{N}} b_{ij} \overline{\alpha}_j > 0 \quad \text{for all } i \in \mathcal{N}. \tag{21}$$

Observe that (21) always holds when γ is sufficiently close to zero, i.e. when the degree of local product differentiation is not very low. Indeed, as implied by (11), we have:

$$\lim_{\gamma \to 0} b_{ij} = \begin{cases} 1, & \text{for } i = j, \\ 0, & \text{for } i \neq j. \end{cases} \tag{22}$$

A sufficient condition for (21) to hold is that $\mathbf{B}$ is a *strictly diagonally dominant matrix* with weights $\bar{\alpha}_i$:

$$b_{ii}\overline{\alpha}_i > \sum_{j\neq i} |b_{ij}|\overline{\alpha}_j \quad \text{for all } i \in \mathcal{N}. \tag{23}$$

Because of (22), (23) also holds when γ is small enough.

3.3 Firms

Each firm $i \in \mathcal{N}$ faces the aggregate demand (20) for its variety, and seeks to maximize its profit given by $\pi_i \equiv p_i X_i$. Using (20) yields:[13]

$$\pi_i = Np_i \left(\sum_{j\in\mathcal{N}} b_{ij}\overline{\alpha}_j - b_{ii}p_i - \sum_{j\neq i} b_{ij}p_j \right). \tag{24}$$

where b_{ij} are the elements of $\mathbf{B}$ (see (11)). Firm i chooses p_i that maximizes (24). The first-order conditions are given by

$$2b_{ii}p_i + \sum_{j\neq i} b_{ij}p_j = \sum_{j\in\mathcal{N}} b_{ij}\overline{\alpha}_j, \tag{25}$$

where $i \in \mathcal{N}$.

Since π_i is strictly concave in p_i, second-order conditions hold automatically, given that the first-order condition holds.

Solving (25) for p_i, we obtain the best-reply function of firm i:

$$p_i^*\left(\mathbf{p}_{-i}, \boldsymbol{G}\right) = \frac{1}{2}\left(\overline{\alpha}_i + \sum_{j\in\mathcal{N}, j\neq i} \frac{b_{ij}}{b_{ii}}(\overline{\alpha}_j - p_j) \right). \tag{26}$$

An interesting result here is that, except when the network is of Chamberlinian type, the price game never exhibits strategic complements. This is to be contrasted with the literature on non-spatial Bertrand competition, where most games display strategic complementarities (see Vives, 1999). Indeed, it is easily verified that

$$\frac{\partial p_i^*\left(\mathbf{p}_{-i}, \boldsymbol{G}\right)}{\partial p_j} = -\frac{1}{2}\frac{b_{ij}}{b_{ii}}. \tag{27}$$

Observe that, when (8) holds, $b_{ii} > 0$. However, this is not necessarily true for b_{ij}. More precisely, as implied by (11), $b_{ij} > 0$ if and only if the geodesic distance between i and j is even

[13] For simplicity, we impose the standard assumption that there is no cost of production. Assuming a constant marginal cost $c > 0$ will not change any of our results.

while the opposite is true when geodesic distance between i and j is odd. As a result, it should be clear from (27) that, to have *strategic complementarities*, all b_{ij}s have to be negative. This is the case only under Chamberlinian type of network (Figure 1a). Indeed, the network is complete so that all firms are in direct competition with each other and we are back to the standard case with no network. In that case, if the direct competitors of firm i decrease their prices, they will attract more consumers and thus i's best-reply is to also decrease her price. Consider now the Chen-Riordan type of network (Figure 1d). In that case, $b_{12} = -\gamma + \mathcal{O}\left(\gamma^3\right) < 0$ while $b_{23} = \gamma^2 + \mathcal{O}\left(\gamma^4\right) > 0$, i.e. b_{ij} is negative if and only if either i or j is the star (node 1) while b_{ij} is positive if and only if both i and j are peripheral nodes. Here, the game does not exhibit strategic complementarities. The same reasoning applies to *strategic substitutes*, which requires that all b_{ij}s have to be positive. This is impossible here because each node needs to be connected to another node and thus some b_{ij}s have to be negative. As a result, in most cases, our price-competition game with networks exhibits neither strategic complements nor strategic substitutes. To sum up, equation (27) entails that the proverb "an enemy of my enemy is my friend" applies to our setting. This echoes the paper by Arie et al. (2015) who show that, if two small firms serve two separate markets and a large firm serves both these markets, then, under Bertrand competition, a merger is profitable for each of the small firms.

Another interesting implication of (27) is that, when $g_{ij} = 1$, i.e. when products i and j are neighbors in the substitutability network, then, as implied by (11), (27) boils down to

$$\frac{\partial p_i^*\left(\mathbf{p}_{-i}, \boldsymbol{G}\right)}{\partial p_j} = \frac{\gamma}{2} + \mathcal{O}\left(\gamma^2\right).$$

Hence, γ yields a first-order approximation for $\partial p_i^* / \partial p_j$ when firms i and j produce close varieties. Based on that, we refer to γ as the *toughness of local competition.*

3.4 Equilibrium

An equilibrium price vector is a non-negative solution of (25), or, equivalently, a fixed point of the best-reply mapping given by (26).

The following proposition shows that a unique equilibrium exists when the degree of product differentiation is sufficiently high.

Proposition 1. *Assume that the value of γ is low enough for both (8) and (23) to hold. Then, there exists a unique interior Nash equilibrium $(\mathbf{p}^*, \mathbf{X}^*)$ where $\mathbf{p}^*$ is given by*

$$\mathbf{p}^* = \left(\mathbf{I} + \gamma \widetilde{\boldsymbol{G}}\right)^{-1} \widetilde{\boldsymbol{\alpha}}\left(\phi, \boldsymbol{G}\right), \tag{28}$$

with

$$\widetilde{\alpha}_i\left(\phi, \boldsymbol{G}\right) \equiv \frac{\overline{\alpha}_i\left(\phi, \boldsymbol{G}\right)}{1 + b_{ii}}, \tag{29}$$

and

$$\widetilde{g}_{ij} \equiv \frac{b_{jj}}{1+b_{ii}} g_{ij}, \tag{30}$$

and where $\mathbf{X}^*$ *is given by (20).*

To better understand these results, let us calculate the equilibrium for the two extreme cases of competition: Chamberlinian competition (complete network in Figure 1a) and Chen-Riordan competition (star network in Figure 1d).

Chamberlinian competition Consider the *complete* network in Figure 1a with $N = 4$ varieties. If $\gamma < 1/3$, then the unique (symmetric) Nash equilibrium is given by:

$$p_1^* = p_2^* = p_3^* = p_4^* = \frac{\alpha\,(1-\gamma)\,(3\phi+4)}{4\,(\gamma+2)}$$

$$X_1^* = X_2^* = X_3^* = X_4^* = \frac{\alpha\,(2\gamma+1)\,(3\phi+4)}{3\gamma^2+7\gamma+2}$$

Chen-Riordan competition Consider the complete network in Figure 1d with $N = 4$ varieties. If $\gamma < 1/\sqrt{3}$, then the unique Nash equilibrium is:

$$\begin{pmatrix} p_1^* \\ p_2^* \\ p_3^* \\ p_4^* \end{pmatrix} = \frac{\alpha}{4\,(4-7\gamma^2)} \begin{pmatrix} 2+6\phi-3\gamma-5\gamma^2+6\gamma^3-3\gamma\phi\left(1+5\gamma-4\gamma\phi-2\gamma^2+2\phi\right) \\ 2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi\,(1+\gamma+2\gamma\phi) \\ 2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi\,(1+\gamma+2\gamma\phi) \\ 2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi\,(1+\gamma+2\gamma\phi) \end{pmatrix}$$

and

$$\begin{pmatrix} X_1^* \\ X_2^* \\ X_3^* \\ X_4^* \end{pmatrix} = \frac{\alpha}{(4-19\gamma^2+21\gamma^4)} \begin{pmatrix} 2+6\phi-3\gamma-5\gamma^2+6\gamma^3-3\gamma\phi\left(1-4\gamma^2\phi-2\gamma^2+5\gamma+2\phi\right) \\ \left(1-2\gamma^2\right)\left[2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi\,(1+\gamma+2\gamma\phi)\right] \\ \left(1-2\gamma^2\right)\left[2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi\,(1+\gamma+2\gamma\phi)\right] \\ \left(1-2\gamma^2\right)\left[2+2\phi-\gamma+4\phi^2-3\gamma^2-3\gamma\phi\,(1+\gamma+2\gamma\phi)\right] \end{pmatrix}$$

It can easily be verified that, when γ is sufficiently small, the price and the aggregate demand of the star (central) variety is higher than that of the others.

4 Comparative statics

Let us now examine how the different parameters of the model affect the equilibrium prices. We will consider the effect of γ, the degree of product differentiation and ϕ, the spatial discount factor, on equilibrium prices.

4.1 A change in the degree of toughness of local competition γ

In general, how prices $\mathbf{p}^*$ and aggregate demand $\mathbf{X}^*$ vary with γ, which is the inverse measure of product differentiation, looks pretty complicated and is general ambiguous. However, it turns out to be possible to obtain a sharp result for the case when γ is close to zero.

4.1.1 Theoretical results when γ is close to zero

Plugging $\mathbf{p} = \mathbf{p}^*$ into the first-order conditions (25) and implicitly differentiating these conditions with respect to γ yields

$$2\frac{\partial b_{ii}}{\partial \gamma} p_i^* + 2b_{ii}\frac{\partial p_i^*}{\partial \gamma} + \sum_{j\neq i} \frac{\partial b_{ij}}{\partial \gamma} p_j^* + \sum_{j\neq i} b_{ij}\frac{\partial p_j^*}{\partial \gamma} = \sum_{j\in\mathcal{N}} \frac{\partial b_{ij}}{\partial \gamma}\overline{\alpha}_j. \tag{31}$$

When $\gamma = 0$, it follows from (25) and (11) that $p_i^* = \overline{\alpha}_i/2$, $b_{ii} = 1$ and $b_{ij} = 0$ for $i \neq j$. Furthermore, (11) implies that $\partial b_{ij}/\partial\gamma|_{\gamma=0} = -g_{ij}$. Thus, for $\gamma = 0$, (31) amounts to

$$\left.\frac{\partial p_i^*}{\partial \gamma}\right|_{\gamma=0} = -\frac{1}{4}\sum_{j\in\mathcal{N}} g_{ij}\overline{\alpha}_j = -\frac{1}{4}\sum_{j\in\mathcal{N}(i)} \overline{\alpha}_j < 0. \tag{32}$$

Assume now that varieties are highly differentiated, i.e. γ is positive but close to zero. Then, applying the continuity argument and using (32), we conclude that $\frac{\partial p_i^*}{\partial \gamma} < 0$. Thus, we have the following result.

Proposition 2. *For sufficiently low values of γ, we have $\partial\mathbf{p}^*/\partial\gamma < \mathbf{0}$, i.e. an increase in toughness of local competition γ leads to lower prices.*

The intuition behind Proposition 2 is as follows: when products are highly locally differentiated, a small reduction in the degree of differentiation makes competition tougher and reduces all prices. However, the magnitude of price reduction depends on both the network structure and the distance decay factor. To illustrate this, consider *relative* rather than absolute changes in prices in response to an increase in γ, we get

$$\left.\frac{\partial \ln p_i^*}{\partial \gamma}\right|_{\gamma=0} = -\frac{1}{2}\sum_{j\in\mathcal{N}(i)} \frac{\overline{\alpha}_j}{\overline{\alpha}_i}.$$

When consumers do not treat a specific variety as their "ideal" one (i.e. when $\phi = 1$), (32) boils down to

$$\left.\frac{\partial p_i^*}{\partial \gamma}\right|_{\gamma=0} = -\frac{\alpha}{4}N(i), \qquad \left.\frac{\partial \ln p_i^*}{\partial \gamma}\right|_{\gamma=0} = -\frac{1}{2}N(i). \tag{33}$$

where $N(i)$ is the degree of vertex i. Thus, in the special case of $\phi = 1$ both absolute and relative reductions in firm's price triggered by higher substitutability of varieties are *proportional* to the number of the firm's closest competitors.

Considering another extreme case, $\phi = 0$ (when each consumer focuses entirely on consuming one variety, like in the Hotelling model), we get

$$\left.\frac{\partial p_i^*}{\partial \gamma}\right|_{\gamma=0} = -\frac{\alpha}{4}\frac{N(i)}{N}, \qquad \left.\frac{\partial \ln p_i^*}{\partial \gamma}\right|_{\gamma=0} = -\frac{1}{2}N(i). \tag{34}$$

Comparing (33) with (34) yields an unexpected result: percentage change in price triggered by an increase in γ is the same when $\phi = 1$ as under $\phi = 0$. Since $\left.\frac{\partial \ln p_i^*}{\partial \gamma}\right|_{\gamma=0}$ in general depends on ϕ, it must be that the relationship between these two magnitudes is non-monotone. Thus, the interplay between different types of product differentiation is highly non-trivial.

To illustrate, assume the Chen-Riordan competition (Figure 1d), where the product-variety network is a star. Let $i = 1$ be the central node. Then, we have

$$\left.\frac{\partial \ln p_1^*}{\partial \gamma}\right|_{\gamma=0} = -\frac{N-1}{2}\frac{1+\phi+(N-2)\phi^2}{1+(N-1)\phi},$$

which is U-shaped in ϕ for any $N \geq 4$.

4.1.2 Numerical simulations

Let us now understand how γ affects equilibrium prices for larger values of γ. Because we cannot solve analytically this comparative statics analysis, we will resort to numerical simulations.

To illustrate how the equilibrium prices vary with γ, consider a *star-shaped network* (Chen-Riordan competition) with $N = 6$. Does the firm located in the star node always enjoy higher monopoly power than the other firms? The answer is no. The firm located at the star node has better access to the market than the periphery firms, but it also faces tougher competition (five direct competitors instead of one). Which of the two effects prevails depends on the value of the substitutability parameter γ. Figure 2 reports the prices of the star firm and a periphery firm, obtained by means of a simulation where $\phi = 0.6$, $\alpha = 1$, while γ varies between 0 and 0.2, the step of the grid being 0.01.

[*Insert Figure* 2 *here*]

As shown by Figure 2, the star firm charges a higher price than the periphery firms if and only if γ does not exceed a threshold value, which is approximately 0.15. When γ is above 0.15, the competition effect deprives the star firm of so much monopoly power that it starts charging a lower price compared to the periphery firms. We also performed simulations for *star-shaped* networks with a number of nodes different from 6. The results are qualitatively the same as in Figure 2.

We then perform the same exercise for the line or chain network (Hotelling competition as in Figure 1b) with $N = 5$. This network starts at node $i = 1$ and finishes at node $i = 5$, where node 3 is the midpoint, nodes 1 and 5 are the endpoints and nodes 2 and 4 are the between nodes. The results are displayed in Figure 3. We find results similar to that of Figure 2. Indeed, we see that there is a threshold value of γ (roughly $\gamma = 0.2$) for which prices of the endpoint nodes become higher than that of the midpoint nodes. This again highlights the trade off between better access to market and facing tougher competition mentioned above.

$[Insert\ Figure\ 3\ here]$

4.2 A change in spatial discount factor ϕ

We now turn to studying how the equilibrium $\mathbf{p}^*$ varies with ϕ. Just like in the preceding subsection, we are able to state clear-cut analytical results for low values of γ, while otherwise we proceed with simulations.[14]

4.2.1 Theoretical results when γ is close to zero

Differentiating (25) with respect to ϕ yields

$$2b_{ii}\frac{\partial p_i^*}{\partial \phi} + \sum_{j\neq i} b_{ij}\frac{\partial p_j^*}{\partial \phi} = \sum_{j\in\mathcal{N}} b_{ij}\frac{\partial \overline{\alpha}_j}{\partial \phi}, \tag{35}$$

where

$$\frac{\partial \overline{\alpha}_j}{\partial \phi} = \alpha \sum_{k=0}^{d} k\phi^{k-1}\frac{N_k(j)}{N}.$$

The following diagonal dominance condition holds for sufficiently low values of γ:

$$b_{ii}\frac{\partial \overline{\alpha}_j}{\partial \phi} > \sum_{j\neq i} |b_{ij}|\frac{\partial \overline{\alpha}_j}{\partial \phi}. \tag{36}$$

Observe that (36) has the same nature as (23), but uses $\partial\overline{\alpha}_j/\partial\phi$ instead of $\overline{\alpha}_j$ as weights.

Using (35) and (36), it can be shown that the following result holds.

Proposition 3. *For sufficiently low values of γ, we have $\partial\mathbf{p}^*/\partial\phi > \mathbf{0}$, i.e. an increase in spatial discount factor ϕ leads to higher prices.*

The intuition behind this result is as follows: when ϕ increases, firm i will have less monopoly

[14] In Section 4.2.3 below, we reformulate our model in terms of transportation cost t instead of spatial discount factor ϕ and show that the comparative statics results with respect to equilibrium prices are the qualitatively the same.

power over consumer i, but better access to all the other consumers. The latter effect clearly dominates the former when γ is not too large. This result may seem puzzling, for a higher spatial discount factor may be viewed as a reduction in transportation costs. Ever since Hotelling (1929), models of spatial competition typically predict that this type of shocks leads to tougher competition, which entails a reduction in prices. This discrepancy of our results and the conventional wisdom is definitely worth discussing. However, we choose to postpone this discussion until Section 4.2.3, where we explicitly reformulate our model in terms of transportation costs, a reformulation that does not change our main results. We only observe here that Proposition 3 echoes the result obtained by Chen and Riordan (2008), who show that competition may be both price-decreasing and price-increasing.

4.2.2 Numerical simulations

Let us now understand how ϕ affects equilibrium prices for larger values of γ. Because we cannot solve analytically this comparative statics, we will resort to numerical simulations.

As above, we will consider a *star-shaped network* (Chen-Riordan competition) with $N = 6$. We look at how the equilibrium prices charged by firms located, respectively, at the "star" node and at the "periphery" nodes vary with the spatial discount factor ϕ. Figure 4 displays the results when $\gamma = 0.15$ (which means relatively *low substitutability* across neighboring varieties, i.e. softer local competition). Figure 5 gives the results when $\gamma = 0.3$ (which means relatively *high substitutability* across neighboring varieties, hence tough local competition). We can see that the results are qualitatively different between these two cases.

When $\gamma = 0.15$, we find that the "star" firm enjoys higher monopoly power (that is, charges higher price) than the "peripheral" firms if ϕ lies in an intermediate domain (from 0.1 to 0.625). Otherwise, peripheral firms price at a higher level. This may be explained by a considerable advantage the "star" firm gains from being a "star" in having better access to the markets. This advantage fades when ϕ is close to 0 (which means little access to markets other than the local market of a firm) or to 1 (all firms have almost complete access to all markets). Indeed, on the one hand, being a "star" does not yield better market access than the others have. On the other hand, the "star" competes directly with everyone (whereas a "peripheral" firm competes directly with the "star" only), thus bearing a burden of high competitive pressure. As a result, when ϕ is close to either 0 or 1, the price of the "star" is lower.

$[Insert\ Figure\ 4\ here]$

When $\gamma = 0.3$, being a "star" is even less of an advantage. Indeed, we see that the firm located at the center *always* charges a lower price than the "peripheral" firms. On top of it, we have here an example of non-monotone behavior of equilibrium prices with respect to ϕ, which confirms our results above, which hold when γ is "sufficiently small".

$[Insert\ Figure\ 5\ here]$

Finally, as above, we perform the same exercise for the line or chain network (Hotelling competition as in Figure 1b) with $N = 5$ and for $\gamma = 0.15$. This network starts at node $i = 1$ and finishes at node $i = 5$, where node 3 is the midpoint, nodes 1 and 5 are the endpoints and nodes 2 and 4 are the between nodes. The results are displayed in Figure 6. We find results similar to that of Figure 4, even though the picture is more complex. There is a non-monotonic relationship when ϕ increases and the mid-point firm tends to charge the highest prices when ϕ has intermediate values. When comparing the between nodes and the endpoint nodes, we see again that the former charge a higher prices only when ϕ takes intermediate values. This again highlights the trade off between better access to market and facing tougher competition mentioned above.

[*Insert Figure* 6 *here*]

4.2.3 Transportation cost versus spatial discount factor

An alternative formulation of the model suggests that consumers' preference for closer varieties is embodied not in the existence of spatial discount factors, but in the presence of positive transportation cost per unit of distance and per unit of consumption, which consumers bear to get access to each variety. Formally, instead of (1), consumer k's utility is now equal to

$$U(k, \mathbf{G}) = x_{0k} + \alpha \sum_{i \in \mathcal{N}} x_{ik} - \frac{1}{2} \left(\sum_{i \in \mathcal{N}} x_{ik}^2 + \gamma \sum_{i,j \in \mathcal{N}} g_{ij} x_{ik} x_{jk} \right), \tag{37}$$

while the budget constraint is given by

$$x_{0k} + \sum_{i \in \mathcal{N}} (p_i + t d_{ik}) x_{ik} = Y_k,$$

where t is transportation cost per unit of distance, Y_k is the income of individual k, and d_{ik} is the geodesic distance between nodes i and k. This formulation assumes that, whenever individual k consumes one unit of variety i, she has to pay a shopping (transportation) cost $t > 0$ per unit of distance (here distance is discrete, so each unit is the distance between two adjacent nodes).[15]

The inverse demand of consumer k for variety i is then given by:

$$p_i = \alpha - t d_{ik} - x_{ik} - \gamma \sum_{j \neq i} g_{ij} x_{jk}, \ i = 1, \ldots, N, \tag{38}$$

or, in vector-matrix form,

$$\mathbf{p} = \alpha \mathbf{1} - t \mathbf{d}_k - (\mathbf{I} + \gamma \boldsymbol{G}) \mathbf{x}_k,$$

[15] As in all the address models, t could also be interpreted as the cost of consuming a good different from the ideal variety.

where

$$\mathbf{1} \equiv \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}, \quad \mathbf{d}_k \equiv \begin{pmatrix} d_{1k} \\ \vdots \\ d_{Nk} \end{pmatrix}.$$

We can see that the inverse demand given by (38) is the same as the one given by (5) when $\alpha_{ik} = \alpha\phi^{d_{ik}} = \alpha - td_{ik}$.

In this new framework, consumer k's individual demands for all varieties are given by:

$$\mathbf{x}_k^* = (\mathbf{I} + \gamma \boldsymbol{G})^{-1} (\alpha \mathbf{1} - t\mathbf{d}_k - \mathbf{p}) = \mathbf{B}(-\gamma, \boldsymbol{G}) (\alpha \mathbf{1} - t\mathbf{d}_k - \mathbf{p}).$$

This leads to the following aggregate demands faced by firms:

$$\mathbf{X}^* = \mathbf{B}(-\gamma, \boldsymbol{G}) (N\alpha \mathbf{1} - t\mathbf{D}(\boldsymbol{G})\mathbf{1} - N\mathbf{p}), \tag{39}$$

where $\mathbf{D}(\mathbf{G}) \equiv (d_{ij})_{i,j \in \mathcal{N}}$ is the distance matrix of $(\mathcal{N}, \mathbf{G})$.

Let f_i stand for the *farness* of node i, $f_i \equiv \sum_{k \in \mathcal{N}} d_{ik}$. Since the *closeness centrality* C_i of node i is usually defined by $C_i \equiv 1/f_i$ (Jackson, 2008), the vector $\mathbf{f} \equiv \mathbf{D}(\boldsymbol{G})\mathbf{1}$ in (39) can be viewed as the vector of *inverse closeness centralities.* As a result, the aggregate demand faced by firm i can be written as:

$$X_i^* = N \left[\sum_{j \in \mathcal{N}} b_{ij} \left(\alpha - \frac{t}{N} f_j \right) - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j \right] \tag{40}$$

In order to differentiate the impact of willingness-to-pay α from that of transportation cost t on the demand faced by firm i, we rewrite (40) as follows:

$$X_i^* = N \left(\alpha b_i - \frac{t}{N} b_i^f - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j \right) \tag{41}$$

where $b_i \equiv \sum_{j \in \mathcal{N}} b_{ij}$ is the usual Bonacich centrality of node i ($\mathbf{u} = \mathbf{1}$) while $b_i^f \equiv \sum_{j \in \mathcal{N}} b_{ij} f_j$ is the weighted Bonacich centrality of node i, where the weights are defined by the farnesses, or, equivalently, the inverse closeness centralities , i.e. $\mathbf{u} = \mathbf{f}$. The interaction between Bonacich centrality and closeness centrality shapes the impact of transportation cost t on the aggregate demand faced by firm i. Observe that, in (41), the impact of the willingness-to-pay α on firm i's demand is fully captured by the Bonacich centrality $b_i(-\gamma, \mathbf{G}, \mathbf{1})$. We can determine the profit of firm i as follows:

$$\pi_i \equiv p_i X_i^* = N p_i \left[\alpha b_i - \frac{t}{N} b_i^f - b_{ii} p_i - \sum_{j \neq i} b_{ij} p_j \right] \tag{42}$$

Firm i's first-order condition leads to:

$$2 b_{ii} p_i + \sum_{j \neq i} b_{ij} p_j = \alpha b_i - \frac{t}{N} b_i^f \tag{43}$$

Comparing (43) to (26), we see that the models (spatial discount factor and transportation cost) are very similar. Let us now investigate the comparative statics properties of prices with respect to transportation cost t and see if they are similar to that of prices and ϕ. By differentiating (43), we obtain:

$$2 b_{ii} \frac{\partial p_i^*}{\partial t} + \sum_{j \neq i} b_{ij} \frac{\partial p_j^*}{\partial t} = -\frac{b_i^f}{N}. \tag{44}$$

Hence, the comparative statics of the equilibrium prices $\mathbf{p}^*$ with respect to transportation cost t is fully captured by b_i^f. Along the lines of Section 4.2.1, we can show that, using (44), the following result holds:

Proposition 4. *For sufficiently low values of γ, we have $\partial \mathbf{p}^* / \partial t < \mathbf{0}$, i.e. an increase in transportation cost t leads to lower prices.*

In other words, with the transportation cost model, where $\alpha_{ik} = \alpha - t d_{ik}$, we obtain the same comparative-static result (Proposition 4) as in the spatial discount factor model (Proposition 3), where $\alpha_{ik} = \alpha \phi^{d_{ik}}$. Indeed, when ϕ increases or t decreases, each firm i exerts less monopoly power over consumer i but has better access to the market. When γ is small enough, the latter effect dominates the former and thus the impact of an increase of ϕ or a decrease of t on prices is positive.

Interestingly, these results are exactly the opposite to those obtained in the standard spatial competition models a la Hotelling where an increase in transportation cost t usually increases the local-monopoly power of firms, which, in turn, increase their prices. To understand why we have a different result, let us revisit the price game in the standard Hotelling's spatial duopoly model. For simplicity, assume that firms 1 and 2 are located at the endpoints of the product space $[0, 1]$ so that $x_1 = 0$ and $x_2 = 1$. The demand faced by, say, firm 1 is then given by

$$q_1 = \frac{p_2 - p_1}{2t}, \tag{45}$$

where, as above, $t > 0$ is the transportation cost per unit of distance. To understand the difference between our model and that of Hotelling, let us write the aggregate demand of firm 1 in our model when the network is composed of two firms (dyad). For that, let us rewrite equation (40) for $N = 2$ and for firm 1. We obtain:

$$X_1^* = 2\left[\alpha b_{11} + b_{22}\left(\alpha - \frac{t}{2}\right) - b_{11}p_1 - b_{12}p_2\right]. \tag{46}$$

We see that (45) and (46) are similar in many respects; in particular, they are both linear in prices. However, these two demand equations have one fundamental difference. On the one hand, by differentiating (45), we have:

$$-\frac{\partial q_1}{\partial p_1} = \frac{\partial q_1}{\partial p_2} = \frac{1}{2t},$$

i.e. the own price effect and the cross price effect balance each other. Put differently, in the standard Hotelling model, if both firms increase their prices by the same amount, the demand faced by firm 1 will not change. This indicates a strong substitution effect. On the other hand, if we differentiate (46), we obtain:

$$\frac{\partial X_1^*}{\partial p_1} = -2b_{11}, \qquad \frac{\partial X_1^*}{\partial p_2} = -2b_{12}. \tag{47}$$

The comparative statics result in the Hotelling model is not anymore true here. Furthermore, using (22), when the value of γ is close to zero, we have:

$$-\frac{\partial X_1^*}{\partial p_1} \gg \frac{\partial X_1^*}{\partial p_2}.$$

This is the main reason why, in Propositions 3 and 4, we obtain the opposite result compared to that of the Hotelling model. Indeed, the demand (45) may be viewed as a limiting case of the linear demand (46) with a *very high* substitution term, $\gamma \to 1$, while we prove our result for *sufficiently low* values of substitutability across varieties. This argument sheds light on the results of our numerical simulations shown in Figure 6. When γ is sufficiently high, with a product space (network) as in the Hotelling model, we obtain the opposing result of Proposition 3. In that case, we are back to the standard intuition of the spatial competition models a la Hotelling.

5 Symmetric equilibrium with regular networks

In the above section, we have studied how the market outcome varies with the degree of substitutability γ and the distance decay factor ϕ. Another question of interest is how the equilibrium responds to changes in the structure of the network. For example, what happens when competition gets less localized, i.e. when new links emerge without breaking the old ones? It is difficult to answer this question in the general case. To obtain clear-cut results, we now consider a special case when the equilibrium in the price-setting game is *symmetric*, i.e. when $p_i^* = p^*$ for all $i \in \mathcal{N}$. The necessary and sufficient condition for the equilibrium to be symmetric is given by:

$$\overline{\alpha}_i\left(\phi, \boldsymbol{G}\right) = \mu \sum_{j\in\mathcal{N}} g_{ij} b_{jj}, \tag{48}$$

where μ is a scalar independent of j.

The right-hand side of (48) is independent of ϕ, while the left-hand side is, by (19), a polynomial of ϕ. It can be shown that (48) holds for any $\phi \in (0,1)$ if and only if both $\overline{\alpha}_i\left(\phi, \boldsymbol{G}\right)$ and b_{ii} are the same for all $i \in \mathcal{N}$. Furthermore, $\overline{\alpha}_i\left(\phi, \boldsymbol{G}\right) = \overline{\alpha}_j\left(\phi, \boldsymbol{G}\right)$ is equivalent to the following condition:

$$\text{for any } k \leq d, \text{ and for any } i, j \in \mathcal{N}, \; N_k(i) = N_k(j). \tag{49}$$

Condition (49) tells us that, for a symmetric equilibrium to exist, the network $(\mathcal{N}, \mathbf{G})$ has to be "symmetric" in some sense. The weakest concept of symmetry for networks is *regularity.* Recall that a network is regular if all nodes have the same degree $r < N$, which is known as the *valency* of a regular graph. However, regularity of $(\mathcal{N}, \mathbf{G})$ is a necessary, but not sufficient condition for (49) to hold. A counterexample is given by the *Frucht graph* (Biggs, 1974, Chap. 15). The Frucht graph is a regular graph with $r = 3$, $N = 12$, a total of 18 edges, but it is *not symmetric.* Figure 7 displays the Frucht graph.

[*Insert Figure* 7 *here*]

If $(\mathcal{N}, \mathbf{G})$ is the Frucht graph, direct computation under $\gamma = 0.15$ and $\phi = 0.75$ yields

$$p = \begin{pmatrix} 0.230 & 0.232 & 0.225 & 0.239 & 0.240 & 0.235 & 0.230 & 0.231 & 0.245 & 0.234 & 0.235 & 0.235 \end{pmatrix}^{\mathrm{T}}$$

A sufficient condition for (49) is that the network is *vertex transitive.* Vertex transitivity means that, for any $i, \; j \in \mathcal{N}$, there exists an automorphism of $(\mathcal{N}, \mathbf{G})$ that maps i into j. Moreover, vertex transitivity implies that $b_{ii} = b_{jj}$ for all $i \in \mathcal{N}$ also holds. More intuitively, the network is *vertex transitive* if all nodes have the same centrality measure (this has to be true for any centrality measure).[16] Clearly, both the complete (Chamberlain-type) network and the circular (Salop-type) network are vertex transitive.

Observe that a network is regular if and only if $r^{-1}\mathbf{G}$ is a bistochastic matrix. Moreover, in this case $r = \lambda_{\max}(\mathbf{G})$. Thus, (8) boils down to $\gamma r < 1$. The unique interior *symmetric* equilibrium price p^* is then given by

$$p^* = \frac{\alpha}{N} \frac{1 + \phi r + \phi^2 N_2 + \ldots + \phi^d N_d}{1 + (1 + \gamma r) b}, \tag{50}$$

where $b \equiv b_{ii} = 1 + \gamma^2 r - \gamma^3 g_{ii}^{[3]} + \ldots$.

Using equation (50), we can study how prices change when competition becomes *less localized*, which we model by adding new links to the network without removing the existing ones, and

[16] For example, the Frucht graph is regular but nodes do not have the same eigenvector centrality, which entails asymmetric distance distribution.

keeping the set of nodes unchanged. To be precise, assume that the network changes from $(\mathcal{N}, \mathbf{G})$ to $(\mathcal{N}, \mathbf{G}')$, where (i) both $\mathbf{G}$ and $\mathbf{G}'$ are regular, and (ii) $\mathbf{G}'$ is obtained from $\mathbf{G}$ by adding new links, without removing the old ones. In particular, this means that the valency r' of $\mathbf{G}'$ exceeds r, which is the valency of $\mathbf{G}$. We have the following result:

Lemma 2. *Assume that γ is sufficiently small while $\phi \in (0,1)$. Then, both the numerator and the denominator of (50) increase when $\mathbf{G}$ changes to $\mathbf{G}'$.*

The intuition behind this lemma is as follows. When competition gets less localized, two effects are at work. First, the numerator of (50) captures the *market access effect*, which results in skewing the *distribution of distances* (how many nodes are at distance 1 from i, at distance 2 from i, etc.) toward zero, thus bringing all consumers closer to each firm. Second, the numerator of (50) captures the *competition effect* since competition gets tougher as new links arise. The lemma basically states that the two effects work in the opposite directions: the market access effect always drives prices upwards, while competition effect leads to a reduction in prices. Which of the two effects dominates is a priori ambiguous. Indeed, the total change in prices is given by

$$\Delta p^* = \frac{\alpha}{N}\left[\frac{1+\phi r+\phi^2 N_2+\ldots+\phi^d N_d + MAE(\phi)}{1+(1+\gamma r)b+CE(\gamma)} - \frac{1+\phi r+\phi^2 N_2+\ldots+\phi^d N_d}{1+(1+\gamma r)b}\right],$$

where $MAE(\phi) \equiv \phi\Delta r + \phi^2\Delta N_2 + \ldots + \phi^d \Delta N_d$ is the *market access effect*, while $CE(\gamma) = (\gamma+\gamma)^2\Delta r + \mathcal{O}\left(\gamma^3\right)$ is the *competition effect.* Clearly, $\Delta p^* > 0$ if and only if $MAE(\phi) > CE(\gamma)Np^*/\alpha$. Whether this condition holds or not is a priori unclear. The following proposition gives an answer.

Proposition 5. *Assume that (i) the product variety network is regular, and (ii) γ is small enough. When competition becomes less localized (denser networks), there exist two threshold values, $\underline{\phi}, \overline{\phi} \in (0,1)$, of the distance decay factor ϕ, such that the competition effect dominates the market access effect if and only if $\underline{\phi} < \phi < \overline{\phi}$. Otherwise, the market access effect dominates the competition effect.*

This result concurs with that of Arie et al. (2015) who study the impact of multimarket contacts between firms on the toughness of competition. Although these authors work with a very different setting from ours, they find a non-monotonic relationship between firms' prices and markets shares. We differ from them by capturing a varying degree of overlap between markets served by different firms (closer firms compete more fiercely) while, in Arie et al. (2015), each market is either private or fully overlapping.

6 Further implications of our model

In this section, we further analyze some interesting properties of our model and put forward the importance of network structure on equilibrium outcomes.

6.1 Herfindahl index

Antitrust authorities often use the Herfindahl index to measure market concentration. The index is defined as $\sum_{i=1}^{n} s_i^2$ where s_i is firm i's market share. We follow Tirole (1988) and use firm's output to calculate market shares, so that the (equilibrium) market share of firm i in network $\mathbf{G}$ is given by:

$$s_i^*(\mathbf{G}) = \frac{q_i^*(\mathbf{G})}{Q^*(\mathbf{G})}$$

where $q_i^*(\mathbf{G})$ is the equilibrium quantity of variety i produced by firm i operating in network $\mathbf{G}$ and $Q^*(\mathbf{G})$ is the total equilibrium quantity of all varieties produced in network $\mathbf{G}$. Clearly, $\sum_{i=1}^{n} s_i^*(\mathbf{G}) = 1$. From equation (5), we can calculate the equilibrium quantity of variety i produced by firm i as follows:

$$q_i^*(\mathbf{G}) \equiv X_i^* = \sum_{k=1}^{n} x_{ik}^*$$

while $Q^*(\mathbf{G}) = \sum_{i=1}^{n} X_i^*$. We have seen that

$$X_i^* = N\left(\sum_{j\in\mathcal{N}} b_{ij}\overline{\alpha}_j - b_{ii}p_i^* - \sum_{j\neq i} b_{ij}p_j^*\right)$$

where the equilibrium prices p_i^* and p_j^* are determined in (28).

When a symmetric equilibrium exists (i.e. when a network is regular and vertex-transitive, see Section 5 above), all firms produce the same quantity. Hence, the Herfindahl index always attains its minimum value, which is $1/N$. For this reason, Chamberlinian product space (Fig. 1a) and Salop-type product space (Figure 1c) with the same number N of varieties/nodes always feature the same value $1/N$ of the Herfindahl index, which does not depend on toughness of local competition γ, nor on the spatial discount factor ϕ.

In order to better understand how industrial concentration varies with the network structure, we consider two simple irregular networks with $N = 4$ varieties and 4 consumers: the Hotelling network (Fig. 1b) and the Chen-Riordan network (Fig. 1d). Figure 8 shows how the Herfindahl index varies with γ in both these cases. We take $\gamma \in [0, 0.25]$, and $\phi = 0.5$.

[*Insert Figure* 8 *here*]

Interestingly, there is no clear prediction on which market is more concentrated. It depends on the parameters ϕ (spatial discount factor) and γ (which is the inverse measure of product differentiation) and on the network structure. As can be seen from Figure 8, if we compare Hotelling competition (chain network) with Chen-Riordan competition (star network) under $\phi = 0.5$, then, for γ between 0.044 and 0.192, the market is more concentrated in Hotelling competition than in the Chen-Riordan competition, while otherwise the opposite result holds true. This result is related to what we found in Section 4 where we show that, in a star-shaped network, the firm located in the star node does not always enjoy higher monopoly power than the peripheral firms.

6.2 Network versus non-network effects

What do we gain by having a network approach to modeling markets of differentiated goods? To shed light on this question, we illustrate here the difference between "network" and "non-network" effects. Assume $\phi = 1$. Then, in the *complete network* (Chamberlinian competition), we have pure *monopolistic competition* since

$$p^* = \frac{\alpha (1-\gamma)}{2(1-\gamma) + \gamma(n-1)} \tag{51}$$

This is exactly the result for the benchmark monopolistic competition model without network (see Combes. Mayer, Thisse, Chap. 3). If we differentiate (51), we obtain:

$$\frac{\partial p^*}{\partial \alpha} > 0 \ , \ \frac{\partial p^*}{\partial n} < 0 \text{ and } \frac{\partial p^*}{\partial \gamma} \gtreqqless 0 \Leftrightarrow \gamma n \lesseqqgtr 1.$$

In Section 4, we have seen that, with a more general approach in terms of networks, the comparative-statics results are quite different. For example, we have shown the importance of the spatial discount factor ϕ in evaluating the impact of γ on prices. Also, we have seen that these comparative-statics results crucially depend on the network structure and that the most "central" firms in a network do not always enjoy higher monopoly power than other firms and thus do not always charge higher prices. We have seen, for example, that for a *star-shaped network*, the firm located in the star node (the most "central" firm) does not always enjoy higher monopoly power than the other firms. This is because the firm located at the star node has better access to the market than the periphery firms, but it also faces tougher competition (all peripheral firms instead of one). Interestingly, Firgo et al. (2015) find empirically a similar result. Using data from the retail gasoline market of Vienna, Austria, they show that the relationship between centrality and pricing in a spatially differentiated market is not significant.[17] They explain their results by the existence of two countervailing effects: centrality implies a larger number of consumers (higher prices) but, at the same time, is associated with a larger number of direct competitors (lower prices). This is exactly what our model is also predicting but suggests a more precise test of this relationship. For example, they only use the *degree* centrality while, in our model any measure of centrality (see Jackson, 2008), such as betweenness or eigenvector centrality, could be used.

7 Conclusion

In this paper, we develop a new model of price competition, which combines features of both spatial and monopolistic competition, thus encompassing two very different aspects of product differentiation: love for variety and consumers' location-specific taste heterogeneity. As a consequence, our

[17]Their network is defined by geographical distances since gasoline stations are connected through a network of roads and intersections and can be characterized by different degrees of centrality (interconnectedness) within this network.

model allows for a rich set of regimes of imperfect competition. The salient feature of our setting is that we model the product space as a network, where link between two varieties exists if and only if they are direct substitutes, while consumer's willingness to pay decays with the geodesic distance of a specific variety from her ideal variety. Thus, consumers exhibit love for variety, as in monopolistic competition, but are willing to pay less for more distant varieties, like in the address approach. Chamberlin-type or Hotelling-type spatial structures are obtained as a limiting case when the substitutability network is a complete graph or a chain. We show that there exists a unique Nash equilibrium in prices where each equilibrium price is a function of the weighted Bonacich centrality of the firm. We also investigate how the degree of product differentiation and the spatial discount factor affect the equilibrium prices. We find that, when products are highly differentiated, a small reduction in the degree of differentiation makes competition tougher and reduces all prices. However, the magnitude of price reduction depends on both the network structure and the distance decay factor. If, for example, we consider a star-shaped network, we find that the firm located at the star node does not always enjoy higher monopoly power than the other firms. This is because the firm located at the star node has better access to the market than the periphery firms, but it also faces tougher competition (all peripheral firms instead of one). We also study symmetric equilibria and show how denser networks affect prices. Finally, we analyze some other implications of our model by calculating the Herfindahl index (which measures market competitiveness) and determining the role of networks in monopolistic competition.

We believe that this paper provides a methodological contribution by modeling firms' and consumers' heterogeneity by their position in the network of product varieties. We also believe that our model could be used to analyze issues related to economic geography, international trade, economic growth, etc. We finally hope that our contribution will spur further research in these directions.

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APPENDIX

Proof of Lemma 1: Let us give a condition that ensures that the utility function (3) is strictly concave. This is equivalent to have that $\mathbf{I}+\gamma\mathbf{G}$ is a *positive definite matrix*. A necessary and sufficient condition for this to be true is that all eigenvalues of $\mathbf{I}+\gamma\mathbf{G}$ are strictly positive. Since $\mathbf{G}$ is symmetric, then all its eigenvalues are real and we can thus rank them. Since the eigenvalues of $\mathbf{I}+\gamma\mathbf{G}$ are equal to: $1+\gamma\lambda_1(\mathbf{G}),\ldots,1+\gamma\lambda_N(\mathbf{G})$, the utility function is strictly concave if and only if (4) holds.[18] ▮

Proof of Proposition 1. Restate (26) as follows:

$$\widetilde{p}_i^* = \frac{1}{2}\left(1+\sum_{j\in\mathcal{N},j\neq i}\frac{b_{ij}\overline{\alpha}_j}{b_{ii}\overline{\alpha}_i}(1-\tilde{p}_j)\right),\quad i\in\mathcal{N}, \tag{52}$$

where $\widetilde{p}_i \equiv p_i/\bar{\alpha}_i$. Denote by $\widetilde{\mathbf{p}}^*$ the linear mapping described by (52).

Condition (23) implies that $\widetilde{\mathbf{p}}^*$ is a contraction mapping. Indeed, by (23) we have

$$\sum_{j\in\mathcal{N},j\neq i}\left|\frac{b_{ij}\overline{\alpha}_j}{b_{ii}\overline{\alpha}_i}\right| < 1 \quad \text{for all} \quad i\in\mathcal{N},$$

which is the exact criterion for a linear mapping to be a contraction with respect to the L_1-norm in $\mathbb{R}^N$.

We now show that there exists a convex compact set $\Delta\subset\mathbb{R}_+^N$, such that (i) Δ is separated from the origin, and (ii) $\widetilde{\mathbf{p}}^*(\Delta)\subseteq\Delta$, i.e. $\widetilde{\mathbf{p}}^*$ maps Δ into itself.

Consider $\Delta\equiv\{\widetilde{\mathbf{p}}\in\mathbb{R}_+^N|\varepsilon\leq\widetilde{p}_i\leq 1-\varepsilon\}$, where $\varepsilon>0$ is small enough. What we mean by "small enough" is clarified below. As implied by (52), the necessary and sufficient condition for $\widetilde{\mathbf{p}}^*(\Delta)\subseteq\Delta$ is given by

$$-b_{ii}\overline{\alpha}_i \leq \frac{1}{1-2\varepsilon}\sum_{j\in\mathcal{N},j\neq i} b_{ij}\overline{\alpha}_j(1-\widetilde{p}_j)\leq b_{ii}\overline{\alpha}_i, \tag{53}$$

for all $\widetilde{p}_j\in[\varepsilon,1-\varepsilon]$. In other words, (53) says that $[-b_{ii}\overline{\alpha}_i,\, b_{ii}\overline{\alpha}_i]$ must contain the range in which $\frac{1}{1-2\varepsilon}\sum_{j\in\mathcal{N},j\neq i} b_{ij}\overline{\alpha}_j(1-\widetilde{p}_j)$ actually varies over Δ. Thus, (53) can be equivalently restated as follows:

$$-b_{ii}\overline{\alpha}_i \leq \frac{1}{1-2\varepsilon}\min_{\widetilde{\mathbf{p}}\in\Delta}\sum_{j\in\mathcal{N},j\neq i} b_{ij}\overline{\alpha}_j(1-\widetilde{p}_j) \quad \text{and} \quad \frac{1}{1-2\varepsilon}\max_{\widetilde{\mathbf{p}}\in\Delta}\sum_{j\in\mathcal{N},j\neq i} b_{ij}\overline{\alpha}_j(1-\widetilde{p}_j)\leq b_{ii}\overline{\alpha}_i. \tag{54}$$

[18]Observe that $\lambda_N(\mathbf{G})<0$ since the $tr(\mathbf{G})=\sum_{i=1}^n\lambda_i(\mathbf{G})=0$

Clearly, we have

$$\min_{\widetilde{\mathbf{p}}\in\Delta} \sum_{j\in\mathcal{N},j\neq i} b_{ij}\overline{\alpha}_j(1-\widetilde{p}_j) = \sum_{j\neq i:\, b_{ij}<0} b_{ij}\overline{\alpha}_j, \tag{55}$$

$$\max_{\widetilde{\mathbf{p}}\in\Delta} \sum_{j\in\mathcal{N},j\neq i} b_{ij}\overline{\alpha}_j(1-\widetilde{p}_j) = \sum_{j\neq i:\, b_{ij}<0} |b_{ij}|\overline{\alpha}_j. \tag{56}$$

Plugging (55)−(56) into (54) and slightly rearranging, we ultimately find that (53) holds if and only if

$$b_{ii}\,\overline{\alpha}_j \geq \frac{1}{1-2\varepsilon}\max\left\{\sum_{j\neq i:\, b_{ij}>0} b_{ij}\overline{\alpha}_j,\ \sum_{j\neq i:\, b_{ij}<0} |b_{ij}|\overline{\alpha}_j\right\}. \tag{57}$$

If we set

$$\varepsilon < \frac{1}{2} - \frac{1}{2}\frac{\max\left\{\sum_{j\neq i:\, b_{ij}>0} b_{ij}\overline{\alpha}_j,\ \sum_{j\neq i:\, b_{ij}<0} |b_{ij}|\overline{\alpha}_j\right\}}{\sum_{j\neq i}|b_{ij}|\overline{\alpha}_j},$$

then the diagonal dominance condition (23) implies (57), whence (53). Thus, $\widetilde{\mathbf{p}}^*(\Delta) \subseteq \Delta$.

To sum up, $\widetilde{\mathbf{p}}^*$ is a contraction mapping that maps Δ into itself, while Δ is separated from the origin. Hence, by the contraction mapping theorem, $\widetilde{\mathbf{p}}^*$ has a unique fixed point. In other words, there exists a unique interior equilibrium price vector $\mathbf{p}^*$, such that $p_i^* < \overline{\alpha}$. Since the best reply functions (26) are linear, a closed-form solution for the equilibrium prices $\mathbf{p}^*$ given by (28) is easily derived. This completes the proof. ▮

Proof of Lemma 2: We set $\Delta Ni \equiv N_i' - N_i$ for all $i \in \mathcal{N}$. The change of the denominator may be expressed as $(\gamma+\gamma)^2\,\Delta r + \text{calO}\left(\gamma^3\right)$, which is positive when γ is not too large, since the emergence of new links leads to $\Delta r > 0$. Thus, it remains to prove that the change in the numerator is also positive. The latter is given by

$$\phi\Delta r + \phi^2\Delta N_2 + \ldots + \phi^d\Delta N_d$$

where d is the *diameter* of the network $(\mathcal{N},\mathbf{G})$.[19] Observe that, since the set of nodes $\mathcal{N}$ remains unchanged, we have $r + N_2 + ... + N_d = r' + N_2' + ... + N_d' = N$. This can be restated in terms of changes as follows:

[19]Since $(\mathcal{N},\mathbf{G}')$ is obtained from $(\mathcal{N},\mathbf{G})$ by means of adding new links, we always have $d' \leq d$. When $d' < d$, we set by definition $N_k' \equiv 0$ for all $k > d'$.

$$\Delta r + \Delta N_2 + \ldots + \Delta N_d = 0. \tag{58}$$

Moreover, the following inequalities hold:

$$\Delta r > 0, \quad \Delta r + \Delta N_2 \geq 0, \quad ..., \quad \Delta r + \Delta N_2 + \ldots + \Delta N_{d-1} \geq 0. \tag{59}$$

Indeed, the magnitude $r + N_2 + \ldots + N_k$ shows the number of nodes which are not further than k from a fixed node. Because the distance between any two nodes either decreases or remains unchanged with the emergence of new links, it must be that (59) hold true.

Furthermore, since $\phi \in (0, 1)$, using (59) yields

$$\phi\Delta r + \phi^2\Delta N_2 > (\Delta r + \Delta N_2)\,\phi^2,$$

$$\phi\Delta r + \phi^2\Delta N_2 + \phi^3\Delta N_3 > (\Delta r + \Delta N_2)\,\phi^2 + \phi^3\Delta N_3 \geq (\Delta r + \Delta N_2 + \Delta N_3)\,\phi^3,$$

and so forth. Applying sequentially these inequalities, we end up with

$$\phi\Delta r + \phi^2\Delta N_2 + \ldots + \phi^d\Delta N_d > (\Delta r + \Delta N_2 + \ldots + \Delta N_d)\,\phi^d.$$

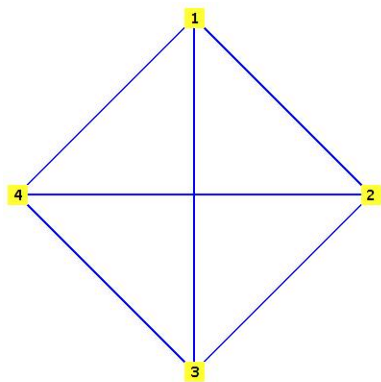
Combining this with (58), we have $\phi\Delta r + \phi^2\Delta N_2 + \ldots + \phi^d\Delta N_d > 0$, which completes the proof. ∎

Proof of Proposition 5: As stated by in Lemma 2, $MAE(\phi) > 0$ whenever $\phi \in (0, 1)$. What happens when either $\phi = 0$ or $\phi = 1$? Obviously, $MAE(0) = 0$. More interestingly, we also have $MAE(1) = 0$, which is implied by (58). This means that $MAE(\phi)$ is a non-monotone function of ϕ over $[0, 1]$. Hence, $\mathrm{d}(MAE)/\mathrm{d}\phi = 0$ has at least one internal solution. Denote by ϕ_0 and ϕ_1 the smallest and the largest of such solutions, respectively.[20] Since $MAE(\phi) \geq 0$, and $MAE(\phi) = 0$ if and only if $\phi \in \{0, 1\}$, it must be that $MAE(\phi)$ increases over $[0, \phi_0]$ and decreases over $[\phi_1, 1]$.

Because the competition effect $CE(\gamma) = (\gamma + \gamma)^2\,\Delta r + \mathcal{O}\left(\gamma^3\right)$ is strictly positive, we have $MAE(\phi) < CE(\gamma)np^*/\alpha$ when ϕ is either close to zero or close to one. However, if γ is small enough for $CE(\gamma)np^*/\alpha < \min\limits_{\phi\in[\phi_0,\phi_1]} MAE(\phi)$ to hold, then the equation $CE(\gamma)np^*/\alpha = MAE(\phi)$ has a unique internal solution $\underline{\phi}$ over $[0, \phi_0]$, and a unique internal solution $\overline{\phi}$ over $[\phi_1, 1]$. Moreover, $CE(\gamma)p^* < MAE(\phi)$ for all $\phi \in \left(\underline{\phi}, \overline{\phi}\right)$, and $CE(\gamma)np^*/\alpha > MAE(\phi)$ otherwise. ∎

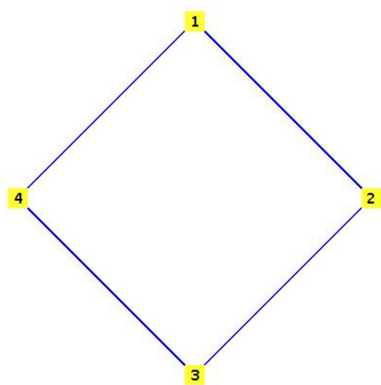
[20] Because $MAE(\phi)$ is a polynomial of ϕ, $\mathrm{d}MAE/\mathrm{d}\phi = 0$ has a finite number of internal solutions. Hence ϕ_0 and ϕ_1 are always well-defined.

Figure 1. Some commonly used types of competition

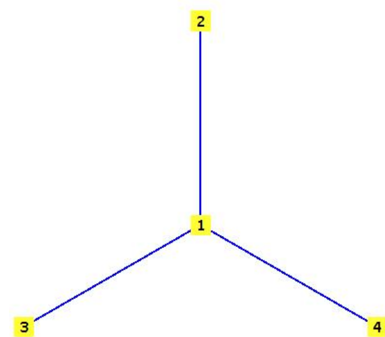


1.a. Chamberlinian type of competition:
G is a *complete graph*

1.b. Hotelling type of competition:
G is a *chain*



1.c. Salop type of competition:
G is a *cycle*



1.d. Chen-Riordan type of competition:
G is a *star*

Figure 2: Changes in equilibrium prices in the Chen-Riordan competition for N=6 and for ϕ=0.6

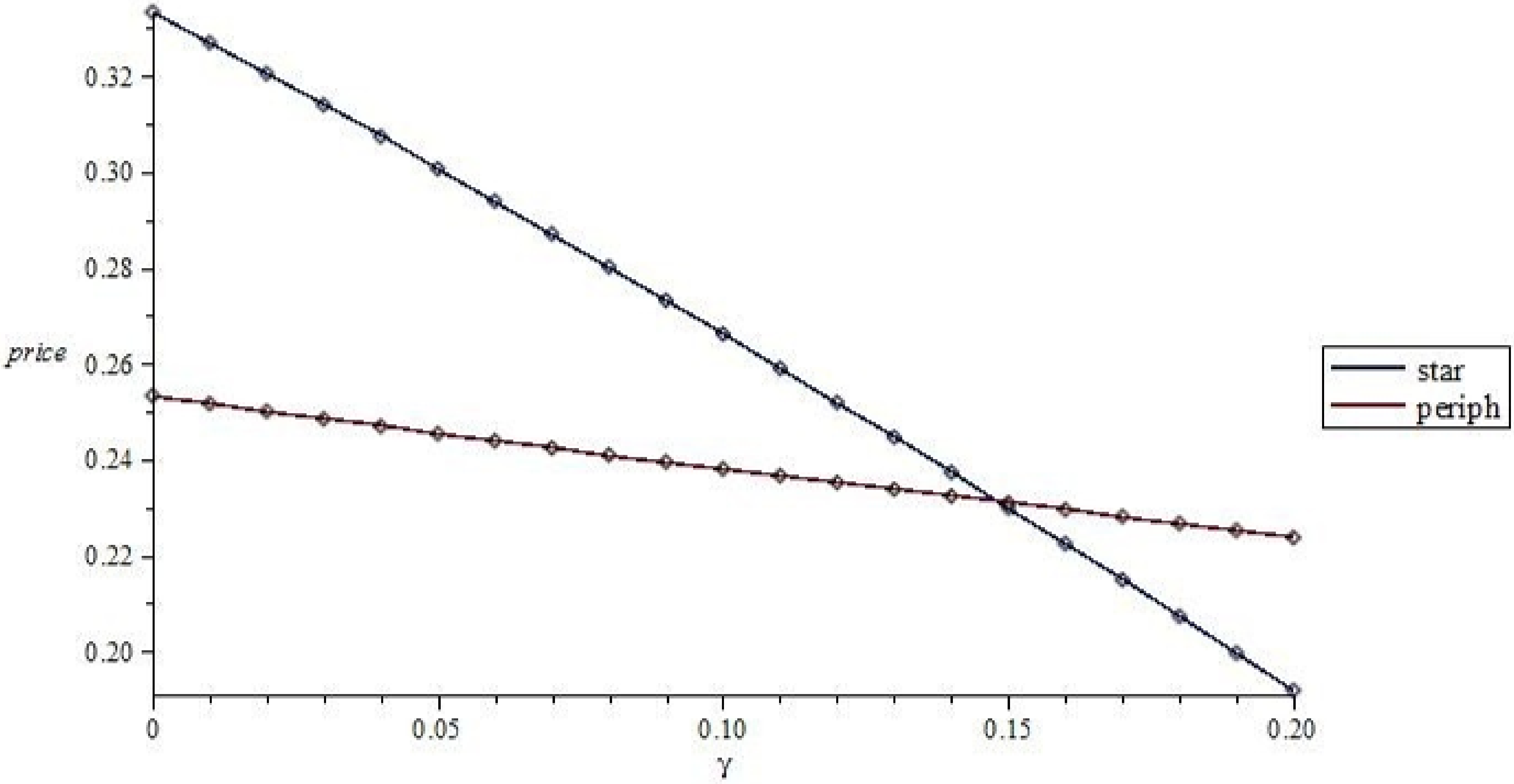


Figure 3: Changes in equilibrium prices in the Hotelling competition for N=5 and for ϕ=0.85

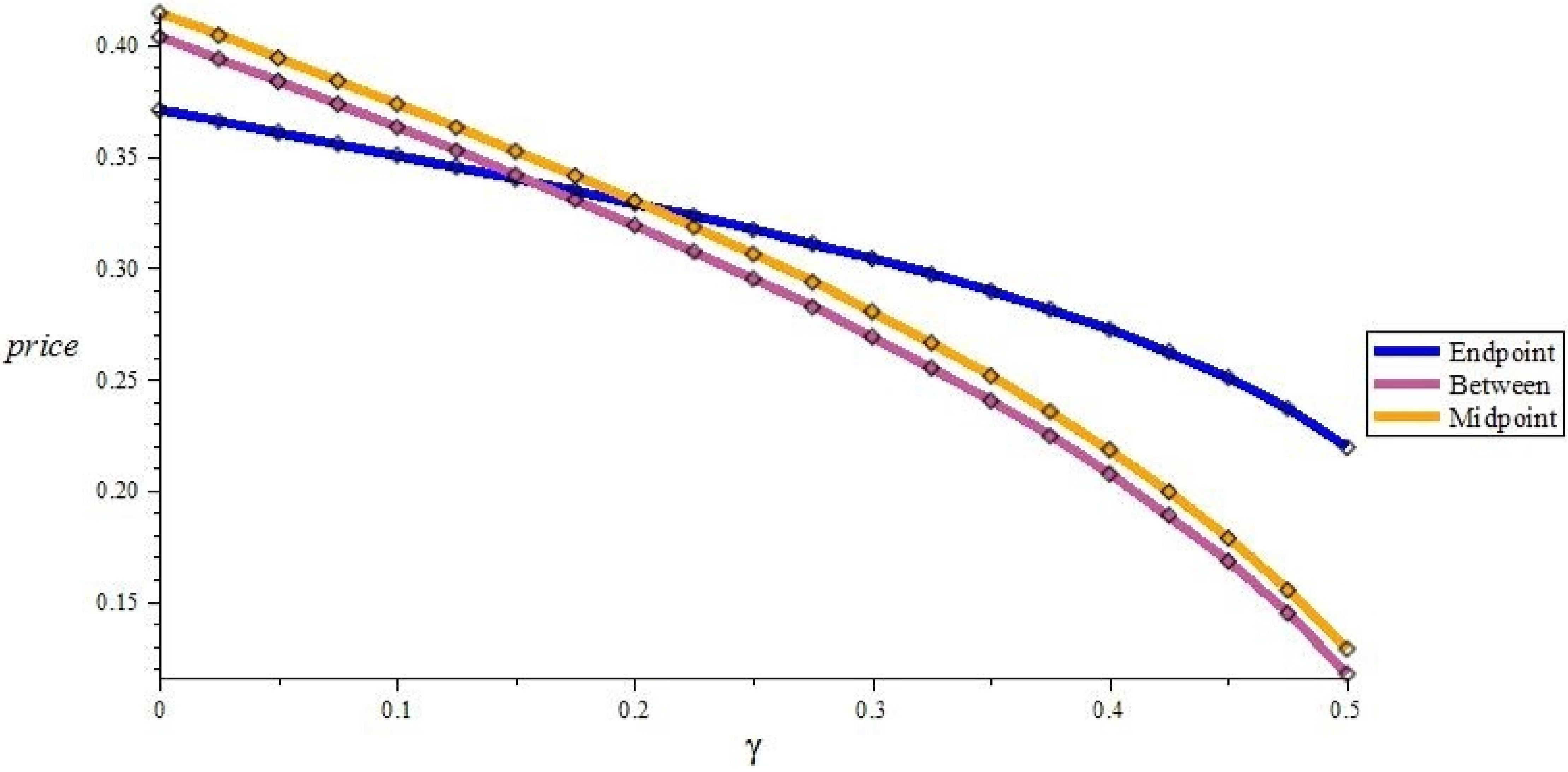


Figure 4: Changes in equilibrium prices in the Chen-Riordan competition for N=6 and for γ = 0.15

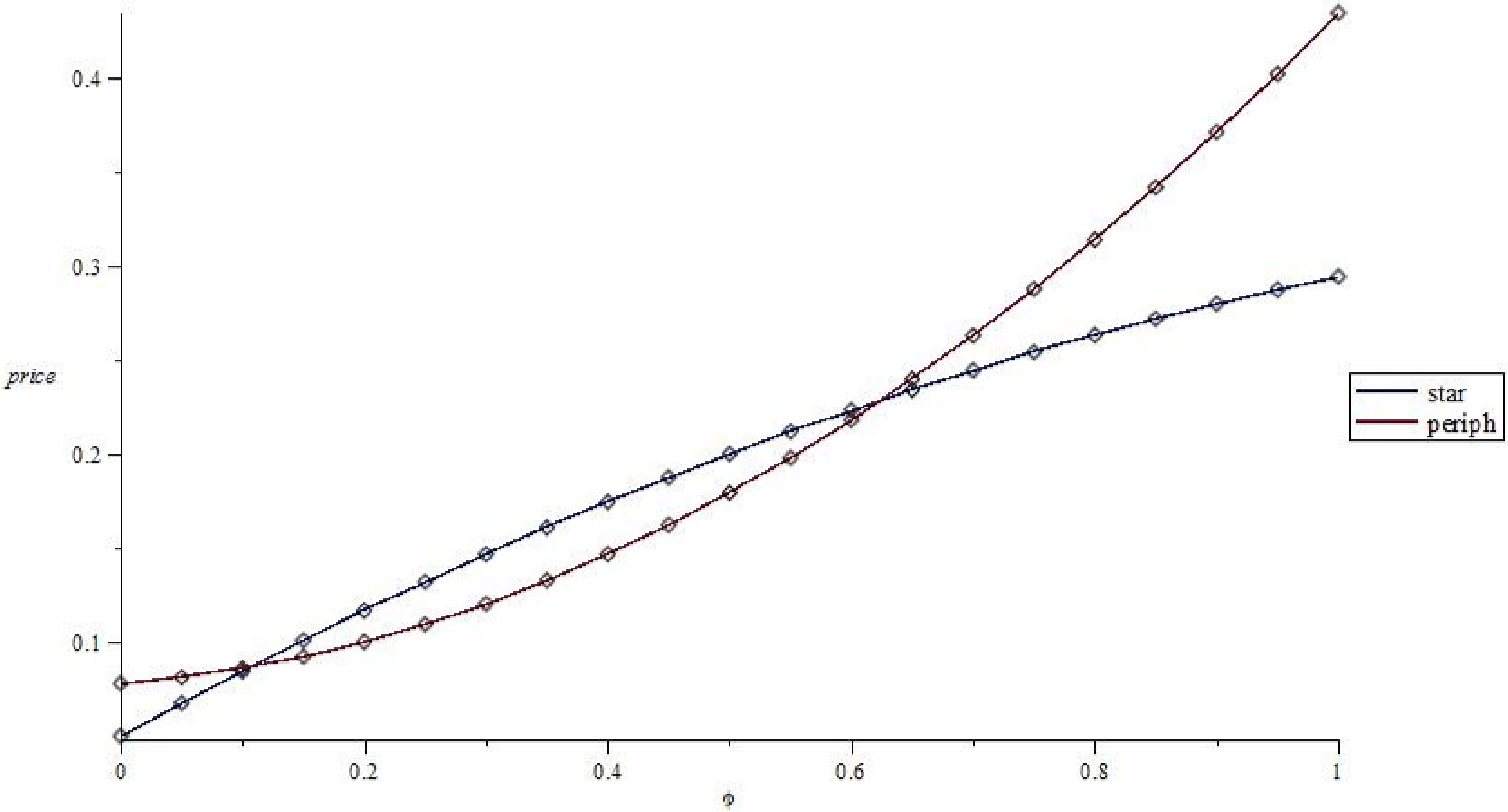


Figure 5: Changes in equilibrium prices in in the Chen-Riordan competition for N=6 and for $\gamma = 0.3$

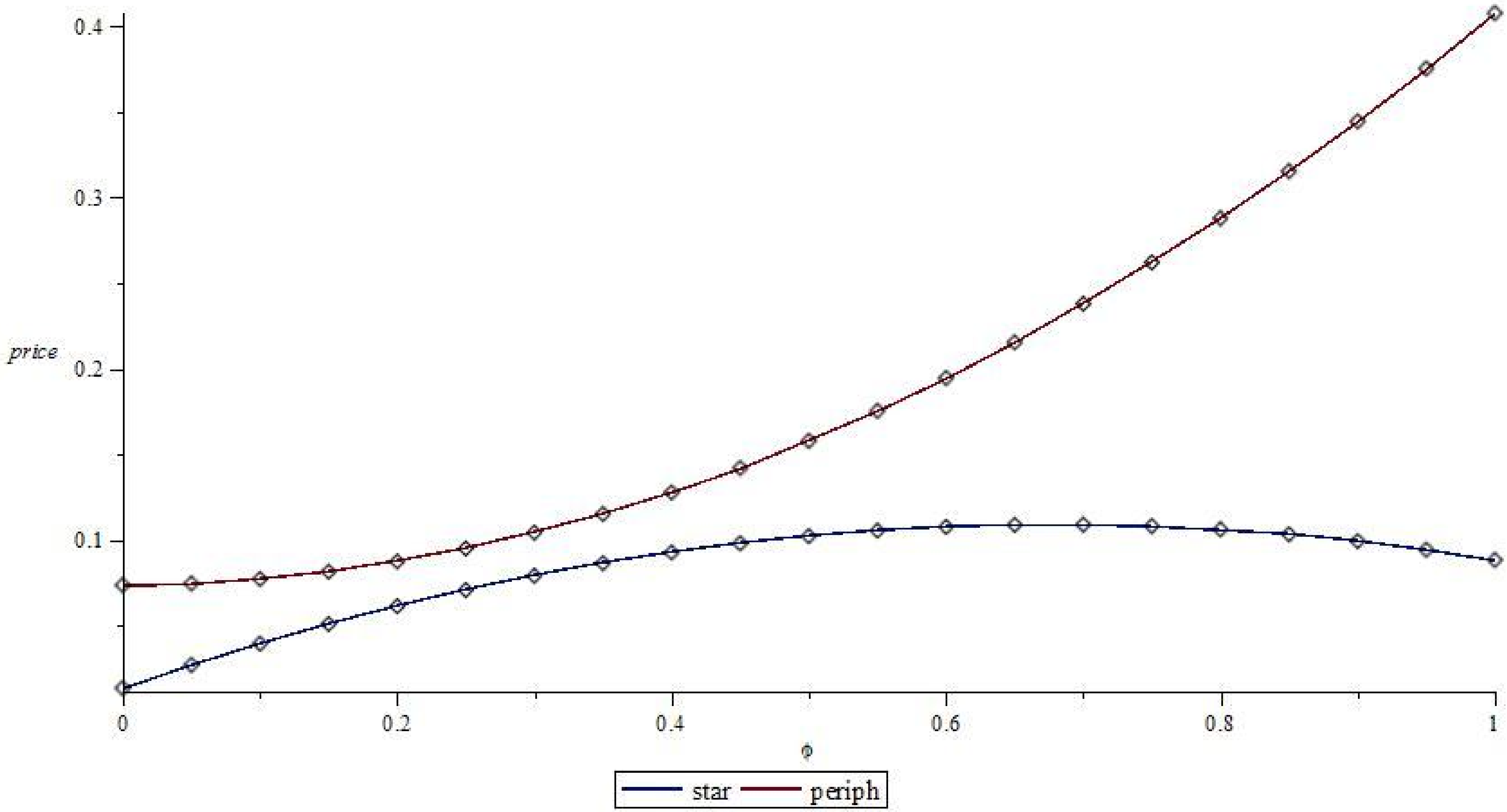


Figure 6: Changes in equilibrium prices in the Hotelling competition for N=5 and for $\gamma = 0.15$

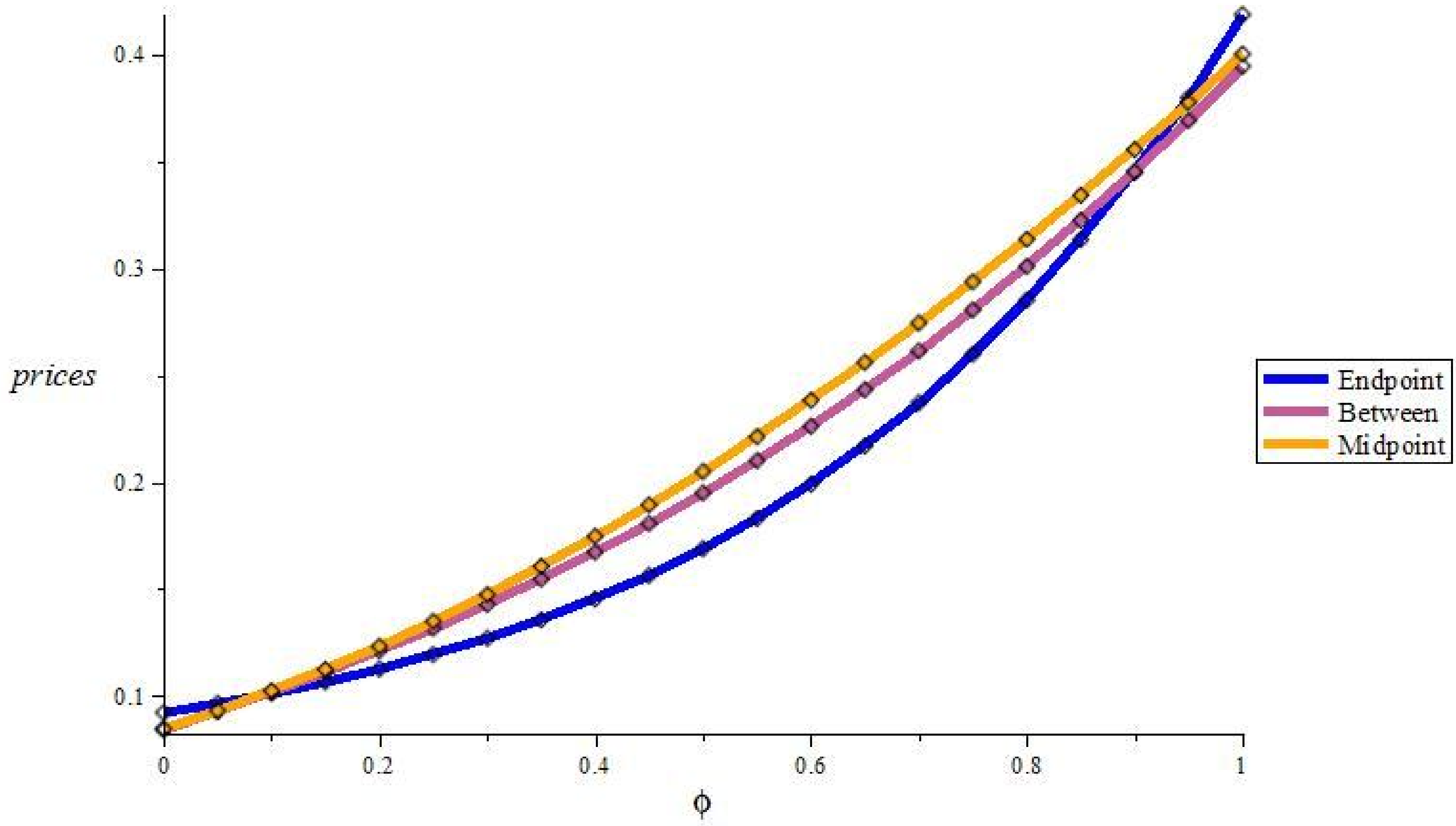


Figure 7: The Frucht graph

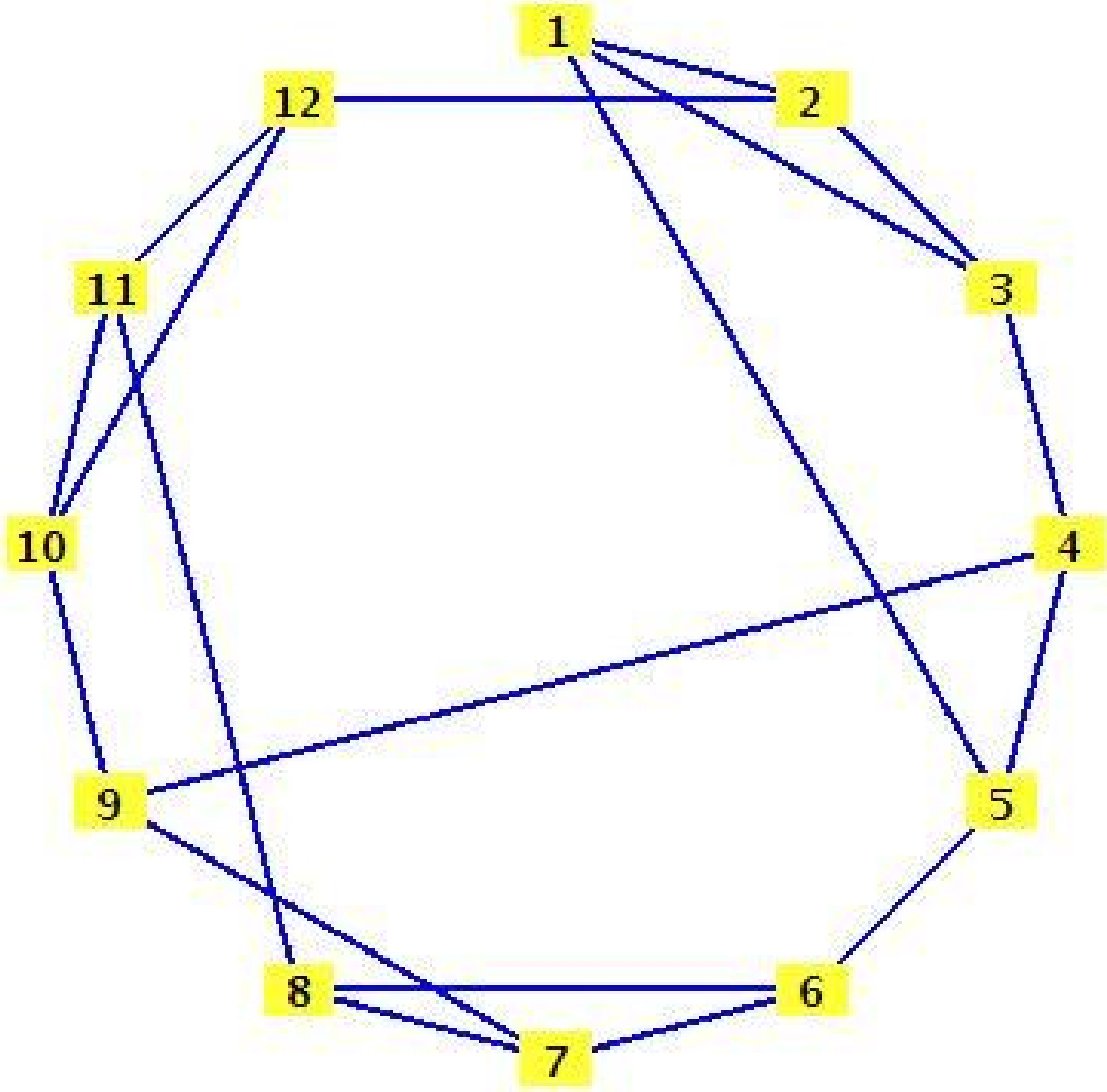


Figure 8: Herfindahl index in Hotelling and Chen-Riordan competition for ϕ=0.5 and $\gamma \in [0, 0.25]$