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ABSTRACT

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We develop a monopolistic competition model with two sectors and heterogeneous agents who self-select into entrepreneurship, depending on entrepreneurial ability. The effect of market size on the equilibrium share of entrepreneurs crucially hinges on properties of the lower-tier utility function for differentiated varieties – its elasticity of substitution and its Arrow-Pratt index of relative risk aversion. We show that the share of entrepreneurs, and the cutoff for self-selection into entrepreneurship, can increase or decrease with market size. The properties of the underlying ability distribution largely determine how income inequality changes with market size.

JEL Classification: D31, D43, L11 and L26
Keywords: entrepreneurship, heterogeneous agents, income inequality, market size and monopolistic competition

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1 Introduction

Market size matters. It matters for productivity, which increases by about 2–4% when the density of economic activity doubles (Rosenthal and Strange, 2004). These productivity gains can be ascribed to any combination of increasing returns to scale, indivisibilities, and a variety of agglomeration externalities – input-output linkages, labor market pooling, and knowledge spillovers – that operate more strongly in bigger and denser markets (Duranton and Puga, 2004). Size also matters for the location choices of heterogeneous individuals. The size elasticity of the share of college graduates in US metropolitan statistical areas is, for example, 6.8% in the 2000 Census data. More efficient firms and more talented workers sort into larger cities (Baldwin and Okubo, 2008; Combes, Duranton, and Gobillon, 2008; Behrens, Duranton, and Robert-Nicoud, 2014a), which partly explains those cities' productivity advantage.

The theoretical effects of market size on productivity and sorting are fairly well understood and documented by now. The same holds true for their empirical magnitudes. Little is still known, however, about the effects of size on firm selection, self-selection into entrepreneurship, and income inequality. Do larger and denser markets provide a tougher competitive environment where less efficient firms and entrepreneurs fail more often? Do these markets provide greater incentives to set up firms and engage in entrepreneurship? And how do competition and self-selection, driven by market size, translate into earnings inequality among heterogeneous individuals? These are the questions we theoretically address in this paper.

The existing empirical evidence linking market size to firm selection and entrepreneurship is inconclusive. Syverson (2004) documents that firm selection is tougher in larger markets in the US ready-mix concrete industry. Combes, Duranton, Gobillon, Puga, and Roux (2012), however, find that firm selection – measured as the left truncation of the productivity distribution – is tiny and explains little to nothing of the productivity advantage of large French cities. Their results show that a right shift in the productivity distribution, due to agglomeration effects that benefit all firms, is the key driver of higher productivity in larger cities. This right shift operates more strongly for more productive firms, thus dilating the productivity distribution. Earnings inequality should, therefore, rise with city size. That larger cities are indeed more unequal places has been documented by Glaeser, Tobio, and Resseger (2010), Baum-Snow and Pavan (2014), and Behrens and Robert-Nicoud (2014). The latter find that the size elasticity of the Gini coefficient is 1.7% for the 507 largest US metropolitan areas in 2007. The contribution of firm selection to inequality remains, however, unclear for now.

Turning to entrepreneurship, still less is known about the effects of market size and density on the ex ante decision to become an entrepreneur, and on the subsequent ex post survival probability. Behrens *et al.* (2014a) document that the share of self-employed – a proxy for entrepreneurship – is roughly constant across a sample of 276 US metropolitan statistical areas in the 2000 Census. Hence, there seems to be no link between market size and entrepreneurship.

Di Addario and Vuri (2010) find that the share of entrepreneurs grows when the population and employment density of Italian provinces increases. However, once individual characteristics and education are controlled for, the share of entrepreneurs decreases with market size. The probability of young Italian college graduates to be entrepreneurs three years after graduation decreases by 2–3 percentage points when the population density of a province doubles. About one third of this 'selection effect' seems to be explained by increased competition among entrepreneurs within industries. However, conditional on survival, successful entrepreneurs in dense provinces reap the benefits of agglomeration: their income elasticity with respect to city size is about 2–3%. Sato, Tabuchi, and Yamamoto (2010) find similar results for Japanese cities. Using survey data, they document that the ex ante share of individuals who desire to become entrepreneurs is higher in larger and denser cities: a 10% increase in density increases the share of prospective entrepreneurs by about 1%. It however reduces it ex post by more than that, so that the observed rate of entrepreneurship is lower in denser Japanese cities. To summarize, the empirical evidence suggests that larger markets have more prospective entrepreneurs (more entrants), but ex post only a smaller share of those entrants survive (tougher selection). Those who do survive in larger markets perform, however, significantly better.

The objective of our paper is to develop a monopolistic competition model of entrepreneurial choice and selection to investigate the effects of market size on entrepreneurship and inequality. To this end, we extend the model by Zhelobodko, Kokovin, Parenti, and Thisse (2012) to allow for heterogeneous agents à la Melitz (2003), who make an occupational choice à la Lucas (1978): they either become entrepreneurs and produce differentiated varieties under monopolistic competition, or they become workers that are hired by the entrepreneurs (as in Behrens *et al.*, 2014a). To derive general results, we make only few assumptions on agents' preferences and on the underlying distribution of types in the population. Within that framework, we show that the effects of market size on the share of entrepreneurs, the output of their firms, the prices they charge, and their incomes, crucially hinges on two properties of the subutility function for differentiated varieties – its elasticity of substitution and its Arrow-Pratt index of relative risk aversion. The ability cutoff for self-selection into entrepreneurship can either increase or decrease with market size, and so does the share of entrepreneurs and income inequality in the economy. We also find that the underlying distribution of types crucially determines whether or not measured inequality rises or falls with market size. When taken together, our results suggest that the effects of market size on entrepreneurship and inequality are theoretically ambiguous and crucially hinge on modeling choices. We thus cannot expect to get clear-cut results, which prompts us to be careful when interpreting the existing empirical evidence. Our findings also suggest that more empirical research on this topic is required. In the end, since theory can generate a large variety of responses of selection and inequality to changes in market size, the 'empirical razor' is needed to sort out the theoretical specifications consistent with the stylized facts.

Our work builds on – and extends – several recent theoretical contributions that aim to explain the links between market size, selection, and inequality. Behrens and Robert-Nicoud (2014) extend the model by Melitz and Ottaviano (2008) to an urban setting. They show that selection is tougher in larger markets, and that this tougher selection increases income inequality. In their model, inequality is driven by both those who fail – the bottom of the income distribution – and by those who succeed well – the top of the income distribution. Our model can generate similar patterns. However, whereas market size benefits disproportionately the most able agents in Behrens and Robert-Nicoud (2014), that effect can be reversed in our model, depending on the preference structure. Behrens *et al.* (2014a) build on the constant elasticity of substitution (henceforth, CES) model and investigate the effects of market size on agglomeration, sorting, and selection. One of their key findings is that selection is independent of market size, so that the productivity gains in larger cities are exclusively driven by agglomeration externalities and sorting along skills. Larger cities are also not more unequal in their framework. We show that the same results hold in our model for the CES case. Whereas little selection in response to differences in market size seems to align with empirical evidence, constant income inequality does not. Those two stylized facts cannot be easily reconciled within simple monopolistic competition models.

The remainder of this paper is organized as follows. Section 2 presents the model and derives some comparative static results for firm-level variables. Section 3 proves existence and uniqueness of equilibrium and discusses the intuition underlying the effects of market size on the self-selection cutoff for entrepreneurship. Section 4 contains our key results linking the share of entrepreneurs to market size. Section 5 then investigates and simulates the impacts of market size on inequality, both among entrepreneurs and for the economy in general. Finally, Section 6 concludes. All proofs and technical details – as well as extensions of the model and supplementary proofs – are relegated to an extensive set of appendices.

2 The Model

Consider a closed economy with a single production factor, labor. There are L agents, each endowed with one efficiency unit of labor. Hence, the maximum effective labor supply equals L. There are two sectors: a traditional sector and a modern one. Production in the traditional sector occurs under constant returns to scale and perfect competition. One unit of effective labor produces one unit of a homogeneous output. Firms in the modern sector produce a differentiated good under monopolistic competition and firm-level increasing returns to scale. Firms in that sector are run by entrepreneurs who differ by productivity.

Each agent is free to choose the sector he wants to work in, and labor is perfectly mobile across sectors. Although agents are homogeneous in their labor endowments, they differ in their innate entrepreneurial ability. The type of an agent is denoted by c, which we interpret as the marginal cost of a potential firm launched by an agent of this type. Hence, more able agents – with a higher $1/c$, which can be thought off as a measure of entrepreneurial ability – can organize more efficient firms that operate with lower marginal cost.[1] We assume that c is continuously distributed on $[\underline{c};\overline{c}]$ with an at least twice continuously differentiable cumulative distribution function $\Gamma(\cdot)$. We denote the associated density by $\gamma(\cdot)$.

In what follows, we choose the price p_A of the traditional good as the numeraire, and denote the wage in that sector by w_A. Perfect competition then implies that $w_A = p_A = 1$. Perfect mobility of workers across sectors further implies that wages are equalized: $w_A = w_M = w \equiv 1$, where w_M denotes the wage in the modern sector. Varieties in the modern sector can be costlessly differentiated so that, in equilibrium, each entrepreneur produces a distinct variety.[2] Each agent knows ex ante his entrepreneurial ability, c, and takes it into consideration when making his occupational choice: (i) to become an entrepreneur who operates a firm with marginal costs c; or (ii) to become a salaried worker supplying his unit of labor to the factor market to earn the market wage. Occupational choices are based on the highest income an agent can secure. Since a higher productivity $1/c$ maps into higher profit, the most productive agents – in terms of entrepreneurial ability $1/c$ – will operate as entrepreneurs, whereas the other agents will be workers. Formally, denote the optimal profit of an agent of type c by $\pi(c)$. Let $\widehat{c}$ denote the type of agent who is indifferent between being an entrepreneur or being a worker. For that type, $\pi(\widehat{c}) = w$, whereas $\pi(c) > w$ for all $\underline{c} \leq c < \widehat{c}$ and vice versa. The *self-selection cutoff for entrepreneurship*, $\widehat{c}$, is endogenous and depends on the characteristics of the economy – such as its size and its underlying ability distribution – and on consumers' preferences. Finally, the share of entrepreneurs in the economy is given by $\Gamma(\widehat{c})$, with an associated mass of firms $N \equiv L\int_{\underline{c}}^{\widehat{c}} \gamma(c)\mathrm{d}c$.

2.1 Preferences and Demand

We assume that all agents have identical quasi-linear preferences. We model the utility derived from the consumption of the differentiated good by a two-tier utility function, where the lower-tier is additively separable across varieties. This is a standard assumption in models of monopolistic competition (e.g., Dixit and Stiglitz, 1977; Matsuyama, 1995; Zhelobodko *et al.*, 2012). In our setting, there are $L\gamma(c)$ type-c entrepreneurs and, therefore, $L\gamma(c)$ type-c varieties. All type-c varietes are produced at the same marginal cost and enter utility symmetrically. They will thus be consumed in the same quantity and have the same price.

[1]The assumption that more able individuals become entrepreneurs is the same as in, e.g., Lucas (1978) or Behrens *et al.* (2014a). An alternative view is that entrepreneurs must be 'jacks-of-all-trades', i.e., have a balanced set of skills without necessarily being excellent anywhere (see Lazear, 2004, 2005). Poschke (2013) presents a model in which both high-ability and 'low-ability' agents become entrepreneurs. The same can occur in the model by Pokrovsky and Sharunova (2014), where agents are heterogeneous along two dimensions – their entrepreneurial ability and their productivity as a worker.

[2]Agents with the same productivity $1/c$ set up different firms that produce distinct varieties of the good.

Let $x_c \equiv x(c)$, $p_c \equiv p(x_c)$, and $\gamma_c \equiv \gamma(c)$ denote the quantity consumed and price charged for a variety of type c with density γ_c. The lower-tier utility from consuming all type-c varieties then equals $u(x_c)L\gamma_c$, and the expenditure for those varieties is $p_c x_c L\gamma_c$. Given our assumptions, utility is expressed as follows:

$$\mathcal{U} \equiv V\left(L\int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c\right) + A, \tag{1}$$

where A denotes the consumption of the traditional good; and where u is an increasing, concave, thrice continuously differentiable subutility function that satisfies $u'(x) < \infty$ for all $x > 0$. We also impose the natural condition that $u(0) = 0$ and assume that each agent has an endowment $\overline{A}$ of the traditional good. This endowment is assumed to be large enough for the consumption of the homogeneous good to be positive for all agents in equilibrium.

Turning to the upper-tier function, V, we assume that $V(\cdot) \equiv \ln(\cdot)$ in what follows (see, e.g., Oyama, Sato, Tabuchi, and Thisse, 2011; and Kukharskyy, 2012). In that case, the existence of an interior equilibrium, where production of the differentiated good takes place, is guaranteed. We explain this result in greater detail in Section 3 and show in Supplemental Appendix D.2 that our key results extend to an arbitrary function V, provided an interior equilibrium exists.

An agent with ability $\theta \in [\underline{c}; \overline{c}]$ has the following budget constraint:

$$L\int_{\underline{c}}^{\widehat{c}} p_c x_c \gamma_c \mathrm{d}c + A = I(\theta) + \overline{A}, \tag{2}$$

where his income $I(\theta)$ – which depends on his type θ – is given by

$$I(\theta) = \begin{cases} 1 & \text{if} \quad \theta \in (\widehat{c}; \overline{c}] \\ \pi(\theta) & \text{if} \quad \theta \in [\underline{c}; \widehat{c}] \,. \end{cases} \tag{3}$$

The inverse demand for a type-c variety is obtained from the consumer's first-order conditions. That inverse demand is proportional to the marginal utility of the variety, and inversely proportional to some market aggregate, μ, that acts as a demand shifter:

$$p(x_c) = \frac{u'(x_c)}{\mu}, \quad \text{for all} \quad c \in [\underline{c}; \widehat{c}], \quad \text{where} \quad \mu \equiv L\int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c. \tag{4}$$

Given the previous assumptions on u, it is clear that $p(x)$ decreases from some strictly positive value to zero with the consumption level x. Observe that the higher the value of μ, the lower the individual demand for each variety, and the more the consumers diversify their demand across varieties for any given income. This stronger demand for variety stimulates entrepreneurship by giving low-ability agents incentives to become entrepreneurs. Observe that μ is the same for all consumers. It is, therefore, a 'universal characteristic' – a sufficient statistic – of the market, which is taken into consideration by the producers when determining their optimal price for the varieties they produce. Since μ is a demand shifter, it is naturally linked to the degree of demand fragmentation, as in Zhelobodko *et al.* (2012). Yet, as shown later, a more fragmented demand does not necessarily imply that the market becomes 'more competitive' in the classical sense that the markups charged by firms are lower.

Some comments are in order. First, even though agents differ by incomes, their consumption of any type-c variety will be the same. This is due to our assumption of identical quasi-linear preferences, which rules out income effects and makes the model tractable. Second, a non-linear upper-tier utility function $V(\cdot)$ is required to guarantee that the substitution effect between the numeraire and the differentiated good is non-trivial. Observed that, in the general case, the demand shifter μ in equation (4) is given by $\mu \equiv 1/V'\left(L\int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c\right)$, which is independent of the cutoff $\widehat{c}$ and of market size L when V is linear. In that case, market size has no bearing on the toughness of competition and on inequality, two key questions we want to investigate in the present paper. Last, let us stress again that the cutoff type, $\widehat{c}$, is endogenously determined and corresponds to the type of agent who is indifferent between working as an employee and starting a business as an entrepreneur.

In what follows, the concavity properties of the lower-tier utility $u(\cdot)$ will prove important. We therefore introduce some notation and a few concepts. Define

$$r_f(x) \equiv -\frac{xf''(x)}{f'(x)}, \tag{5}$$

which is a measure of the concavity of f at x. When applied to the subutility function u, we obtain a measure of love-for-variety, given by the Arrow-Pratt index of relative risk aversion:

$$r_u(x) \equiv -\frac{xu''(x)}{u'(x)}. \tag{6}$$

Note that 'the more concave' is the lower-tier utility, the higher is the consumer's *relative love-for-variety* (henceforth, RLV). Let us further introduce the following notation for the elasticity of a function f with respect to its argument x: $\mathcal{E}_x[f(x)]$. Analoguously, we denote by $e_x[f(x,\cdot)]$ the *partial elasticity* of f with respect to x, holding all other variables constant. Using that notation, the elasticity of demand for any variety of the differentiated good – and, therefore, the markups that entrepreneurs can charge for that variety – are characterized by the Arrow-Pratt index r_u of the lower-tier utility function. Using equation (4), it can be expressed as the opposite of the elasticity of the marginal utility as follows:

$$r_u(x) = -\frac{u''(x)x}{u'(x)} = -\mathcal{E}_x[u'(x)] = \mathcal{E}_x[p(x)] = -\frac{p'(x)x}{p(x)}. \tag{7}$$

Expression (7) is the RLV calculated at the point of individual consumption, x. It measures the curvature of the function u at that point. Under standard assumptions on u, the value $r_u(x)$ is between 0 and 1. Its precise behavior – whether it is increasing or decreasing – depends on how 'different' u is from a power function. For the latter, it is well known that the RLV is constant. In the case of an increasing function r_u, we are in the presence of *increasing elasticity of demand* (henceforth, IED); whereas in the opposite case, we are in the presence of *decreasing elasticity of demand* (henceforth, DED). The case of a constant r_u for any value of x corresponds to the special case of CES preferences, which play a particular role as a borderline case.[3]

2.2 Production and Self-Selection into Entrepreneurship

Each entrepreneur takes his inverse demand function, $p(\cdot)$, the demand shifter μ, and the wage $w \equiv 1$ as given. An entrepreneur of type-c maximizes his profit $\pi(c) = [p(x_c) - c]Lx_c$ with respect to output, x_c.[4] The first-order condition $p'(x_c)x_c + p(x_c) = c$ yields the optimal price and output, the latter being proportional to the individual consumption of the variety. Rewriting the first-order condition in terms of the markup, $M(c) \equiv [p(x_c) - c]/p(x_c)$, and recalling the expression for the elasticity of inverse demand (7), we obtain a very simple condition for the profit-maximizing prices in terms of the RLV:

$$M(c) = r_u(x_c) \quad \Rightarrow \quad p(c) = \frac{c}{1 - r_u(x_c)}. \tag{8}$$

Using the first-order condition (4) of the consumer problem, and the first-order condition (8) of the producer problem, the individual consumption of a type-c variety is a solution to the following equation:

$$u'(x_c)\left[1 - r_u(x_c)\right] = \mu c \tag{9}$$

for any given value of μ. The latter is, of course, determined from the general equilibrium of the model. A sufficient second-order condition for profit maximization is given by

$$\frac{\partial^2 \pi(c)}{\partial x_c^2} = p''(x_c)x_c + 2p'(x_c) = \frac{u'''(x_c)x_c}{\mu} + 2\frac{u''(x_c)}{\mu} = \frac{u''(x_c)}{\mu}\left[2 + \frac{u'''(x_c)x_c}{u''(x_c)}\right] < 0,$$

which, recalling the definition (5) of r_f, can be rewritten as follows in terms of the RLV:

$$r_{u'}(x_c) < 2. \tag{10}$$

[3]Since IED and DED are 'local' concepts that depend on the consumption level x, a function may be IED over some range and DED over some other. We illustrate this point in the Supplemental Appendix D.3.

[4]Choosing prices or outputs yields the same outcome under monopolistic competition with a continuum of firms (Vives, 2001).

Condition (10) states the classical condition that the demand function (4) must not be 'too convex' for optimal prices to exist. The maximized operational profit can be rewritten in terms of the RLV as follows:

$$\pi(c) = [p(x_c) - c]x_c L = \frac{r_u(x_c)}{1 - r_u(x_c)} cLx_c. \tag{11}$$

As in Lucas (1978), agents self-select into entrepreneurship when that activity is more profitable for them than being employees. A type-c agent compares his type with the cutoff type to make a decision regarding his occupational choice. Only agents whose marginal cost is lower than the cutoff, $c < \widehat{c}$, set up firms. The mass of entrepreneurs in the economy thus equals $N \equiv L\Gamma(\widehat{c})$. More formally, using (11), the indifference condition between entrepreneurship and salaried work can be expressed as:

$$\pi(\widehat{c}) = w \equiv 1 \quad \Leftrightarrow \quad \frac{r_u(x_{\widehat{c}})}{1 - r_u(x_{\widehat{c}})} \widehat{c} L x_{\widehat{c}} = 1. \tag{12}$$

Condition (12) pins down the equilibrium self-selection cutoff and, therefore, the share of entrepreneurs in the economy. The cutoff $\widehat{c}$ – which can be viewed as the ability threshold of active entrepreneurs – is another market aggregate that is taken as given by agents. The higher the value of $\widehat{c}$, the more entrepreneurs and thus product diversity is supported by the market in equilibrium. Yet, the higher $\widehat{c}$, the lower the average ability of entrepreneurs. The equilibrium trade-off between product diversity – a high value of $\widehat{c}$ – and consumption quantity – a low value of $\widehat{c}$ – will crucially determine how the share of entrepreneurs reacts to changes in market size. That trade-off fundamentally depends on consumers' preferences.

2.3 Entrepreneurial Talent and Firm-Level Outcomes

Before investigating in depth the effects of market size on the self-selection cutoff and on income inequality, we first look at firm-level equilibrium characteristics with respect to the ability of their entrepreneurs. Quite naturally, more able agents set up more efficient firms which produce larger output, charge lower prices, have larger market shares, and earn larger profits. Note, however, that it is not possible to derive clear results on markups: depending on the underlying preferences, more productive firms may charge either higher or lower markups.[5] Let $\pi_c \equiv \pi(c)$ for notational convenience. Formally, we can prove the following results:

Proposition 1 (Entrepreneurial Ability and Firm-Level Outcomes). Consider two entrepreneurs with abilities $c_1 < c_2$. The more productive entrepreneur, $1/c_1 > 1/c_2$, runs a firm that:

[5] In Melitz and Ottaviano (2008) and in Behrens, Mion, Murata, and Südekum (2014b), more productive firms charge higher markups that are decreasing in market size. As shown in the second paper, even if predictions on changes in markups are unambiguous for each market, it is difficult to make statements about how markups change in a multi-region trading world. For the case of 'increasing elasticity of substitution' preferences, where markups increase with market size, see Bertoletti, Fumagalli, and Poletti (2009) and Zhelobodko *et al.* (2012).

(i) produces more output, $x_{c_1} > x_{c_2}$; (ii) charges a lower price, $p(x_{c_1}) < p(x_{c_2})$; and (iii) earns higher profits, $\pi_{c_1} > \pi_{c_2}$, than the firm of the less productive entrepreneur. Markups are decreasing with firm-level productivity if r_u is decreasing, and increasing otherwise.

Proof. See Appendix B.1. □

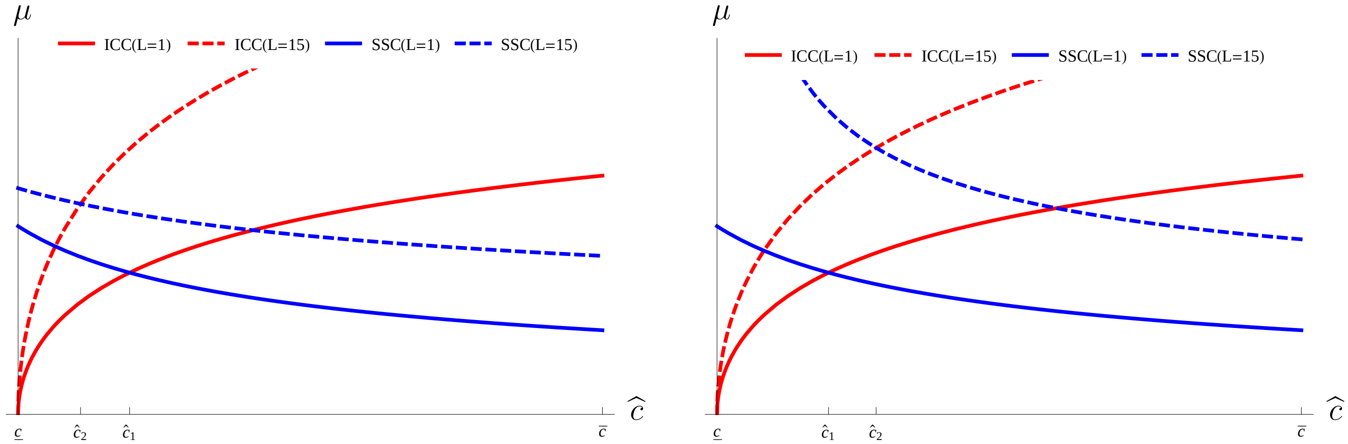
Agents with greater entrepreneurial ability organize firms with lower marginal cost, which allows them to charge lower prices. Since preferences are symmetric across varieties, demand is higher for cheaper goods, i.e., more efficient firms have a larger market share. The elasticity of demand for each variety being less than one, and given the properties of the subutility function u – concavity, zero utility at zero consumption – larger sales volumes translate into higher revenue: $\mathcal{E}_x\left[p(x)xL\right] = 1 - r_u(x) > 0$. Markups, however, can be either increasing or decreasing with the individual consumption of a variety produced by a particular firm, depending on the properties of r_u. Put differently, depending on preferences, it might turn out that more productive firms charge higher markups or the other way round.

Since markups and output sold may a priori move in opposite directions with productivity, the total effect of productivity on profits may be ambiguous. Nevertheless, we can show that even if r_u is a decreasing function, the effect of an increase in revenue exceeds the negative effect of the decrease in the markup. From (7) and from Corollary A.2 in Appendix A, it follows that $\mathcal{E}_x\left[\pi(x)\right] = \mathcal{E}_x\left[r_u(x)p(x)x\right] = 2 - r_{u'}(x) > 0$, where the last inequality holds because of the second-order conditions (10) for profit maximization. Hence, irrespective of whether more productive firms charge higher or lower markups, they earn higher profits.[6]

3 Equilibrium

An equilibrium is characterized by $\widehat{c}$, μ, $\{x_c\}_{c\in[\underline{c},\widehat{c}]}$, i.e., the two market statistics – the self-selection cutoff, $\widehat{c}$, and the demand shifter, μ – and outputs such that: (i) all consumers maximize utility; (ii) producers maximize profits conditional on their inverse demand $p(\cdot)$ and the two market statistics; (iii) agents optimally choose occupations, i.e., type-$\widehat{c}$ agents are indifferent between being workers or entrepreneurs, whereas all other agents pick the occupation that secures them the highest returns; and (iv) all markets clear. We can solve for equilibrium using the marginal utility of income (4), the first-order conditions (9) for consumers and producers, and the indifference condition (12). All the other variables – especially the prices and the consumption of the numeraire good – can then be retrieved from the first-order conditions of the consumer problem and from the budget constraint. We can prove the following result:

[6]Recent empirical evidence strongly suggests that more productive firms charge higher markups (see, e.g., De Loecker, Goldberg, Khandelwal, and Pavcnik, 2012, for the case of trade liberalization in India). This is also true in recent models of monopolistic competition with variable markups (Melitz and Ottaviano, 2008; Behrens *et al.*, 2014b), where the pass-through of productivity gains to consumers is less than 100%.



(a) Market size and tougher selection

(b) Market size and milder selection

Figure 1: Equilibrium Patterns and Changes in $\widehat{c}$ with Respect to L.

Proposition 2 (Existence and Uniqueness). Conditions (4), (9), and (12) determine an equilibrium in terms of individual consumptions, $\{x_c\}_{c\in[\underline{c};\widehat{c}]}$, the demand shifter, μ, and the self-selection cutoff, $\widehat{c}$. The equilibrium exists and is unique.

Proof. See Appendix B.2 □

Proposition 2 establishes that the equilibrium exists and is unique when the upper-tier utility function is logarithmic. Note that with an arbitrary upper-tier utility function, V, an interior equilibrium need not exist (see the Supplemental Appendix D.1). The reason is that, depending on preferences and on the underlying ability distribution, even the most productive entrepreneurs may not be able to make profits that exceed the wage they could secure as workers. In that case, no agent wants to become an entrepreneur and we have a corner equilibrium. We rule out this case as it is obviously not of great interest to our analysis.

Until now, we have only shown that the equilibrium exists and is unique. Clearly, in our monopolistic competition framework, its comparative static properties depend on preferences and on the distribution of entrepreneurial abilities among agents. To build intuition, we begin with a graphical analysis of the equilibrium. This analysis shows that the demand shifter μ, always increases as the market expands. The change in the self-selection cutoff $\widehat{c}$ – and thus in the share of entrepreneurs – can, however, go either way.[7] The two panels of Figure 1 depict the equilibrium in $(\widehat{c};\mu)$-space, at the intersection of the curves $\mu_1(\widehat{c} \mid L,\underline{c})$ – derived from (9), when combined with the market aggregate (4) – and $\mu_2(\widehat{c} \mid L)$ – derived from (9) when combined with equation (12). We refer to the former curve as the 'intensity of competition condition', ICC, and to the latter curve as the 'self-selection condition', SSC. Both depend on the

[7] Recall that the share of entrpreneurs equals $\Gamma(\widehat{c})$. When $\Gamma(\cdot)$ and $\underline{c}$ are given, changes in $\widehat{c}$ thus directly map into changes in the share of entrepreneurs. Things are more complicated when we consider changes in the distribution or in its support, two aspects that we will analyze in Section 5.

parameter L. As illustrated in both panels of Figure 1, an increase in market size L shifts both the ICC and the SSC curves upwards, thus increasing μ. We indeed show in the next section that an increase in market size, L, always increases μ since $\mathcal{E}_L[\mu] > 0$. As already stated, the change in the self-selection cutoff, $\widehat{c}$, in response to a change in L is more difficult to analyze. A detailed analysis of those changes is relegated to the next section, but Figure 1 already shows that the cutoff can either decrease (panel (a)) or increase (panel (b)).

To understand why we cannot generally expect unambiguous results, observe that the second equilibrium condition (9) depends on the marginal utility $u(\cdot)$ and the RLV. The third equilibrium condition (12) can be rewritten as $\mu = u(x_{\widehat{c}})\mathcal{E}_u[x_{\widehat{c}}]r_u(x_{\widehat{c}})L$, which depends on the scale elasticity and the RLV. The properties of the first condition (4) also depend on the properties of the subutility $u(\cdot)$. Thus, whether the share $\Gamma(\widehat{c})$ of entrepreneurs in the economy increases or decreases with market size L – which depends on the direction of the shift in the self-selection cutoff $\widehat{c}$ – crucially depends on preferences via r_u and $\mathcal{E}_u$. This is illustrated by panel (a) of Figure 1, which depicts the case where the share of entrepreneurs decreases with market size (because selection becomes 'tougher'), whereas panel (b) depicts the opposite case where the share of entrepreneurs increases (because selection becomes 'milder'). In the special case of CES preferences, both curves shift such that the cutoff remains unchanged.

The intuition underlying the two cases is as follows. Observe that an increase in population L corresponds to both a proportional increase in the number of producers of each type, as well as to a corresponding increase in the number of consumers. An increase in the number of available varieties leads to a decrease in their individual consumption and, therefore, forces the least efficient entrepreneurs out of the market. Yet, the increase in the number of consumers gives entrepreneurs with lower skills incentives to enter the market, because a larger market allows them to earn more than as a worker, even if they need to charge lower markups. We thus have two opposing forces due to market expansion, which influence the employment structure of the economy. An increase in market size shifts the ICC curve $\mu_1(\widehat{c} \mid L)$ along the SSC curve $\mu_2(\widehat{c} \mid L, \underline{c})$. Individual demands decrease with L as the demand shifter μ increases, and so does the selection cutoff $\widehat{c}$. At the same time, entry occurs 'from the bottom' with less efficient entrepreneurs. The intuition is that when consumers fragment their demand more strongly, there is 'a gap' for entrepreneurs who can thus enter the market (Nocke, 2006). Hence, $\widehat{c}$ increases because the SSC curve $\mu_2(\widehat{c} \mid L, \underline{c})$ now shifts along the ICC curve $\mu_1(\widehat{c} \mid L)$ so that their crossing point moves to the right. However, since the intensity of competition μ has changed, optimal prices have changed too. In particular, prices fall as more entrepreneurs enter the market, thus reducing demand further and increasing competition. Because of that change, survival becomes more difficult, and the strength of this effect determines how much the new self-selection cutoff will shift to the right. Note that the shift may be very small, so that the new equilibrium value $\widehat{c}$ can be smaller than the initial one (see pane (a)l of Figure 1). In other words, the share of entrepreneurs in the economy may fall as the market gets larger.

While the graphical analysis serves to build intuition, we show more formally in the next section under what conditions each case arises. In particular, we clarify how market size influences the share of entrepreneurs that operate in the economy, i.e., how it affects the self-selection of agents. We then show, in Section 5, how profits and incomes change. This allows us to trace out changes in income inequality in response to changes in the size of the economy. As usual, we can interpret an increase in market size as opening the economy to free trade, which makes our results on market size also useful for understanding the impact of international trade on inequality and selection under monopolistic competition with more general preference structures than those usually used in the literature.[8] Alternatively, our results are useful in a cross-sectional context, e.g., to understand why and how entrepreneurship and income inequality may vary across cities of different sizes.

4 Market Size and Entrepreneurship

We now analyze more formally how changes in market size – as measured by population L – affect the equilibrium of the model. We investigate, in particular, the influence of market size on changes in the self-selection cutoff, $\widehat{c}$. For a fixed distribution of types, changes in $\widehat{c}$ directly map into changes in the share of entrepreneurs and are, therefore, of particular interest to us. The demand shifter, μ, changes simultaneously with the self-selection cutoff in response to changes in market size. We show that μ always increases regardless of the preferences and of the distribution of entrepreneurial abilities. The change in the self-selection cutoff depends, however, on preferences, and can be either positive or negative. Put differently, larger markets can either exhibit a larger or a smaller share of entrepreneurs, and a higher or lower ability threshold for self-selection into entrepreneurship. We provide a full analytical characterization of the conditions under which each case occurs. As we will see, the case of CES preferences is bordeline in the sense that, given these preferences, the share of entrepreneurs is independent of market size (see, e.g., Behrens *et al.*, 2014a).

As we have illustrated graphically in Figure 1, changes in the two market statistics are crucial: the demand shifter, μ, and the self-selection cutoff, $\widehat{c}$. We thus present all our results on the effect of market size in terms of elasticities of $\widehat{c}$ and μ with respect to population L. The impact on all other endogeneous variables – prices, ouputs, profits – can then be retrieved from the first-order conditions, given μ and $\widehat{c}$, and we discuss this further in Section 5 below when focusing on the impacts of market size on income inequality. In Section 5, we also analyze in more detail how changes in the underlying ability distribution $\gamma(\cdot)$ affect the share of entrepreneurs and income inequality in the economy.

[8] See Behrens and Murata (2012) and Behrens *et al.* (2014b) for an analysis using CARA preferences, and Kichko, Kokovin, and Zhelobodko (2013) for a two-factor two-sector model using quasi-linear preferences.

4.1 CES Lower-Tier Utility Function

To make the analysis more transparent, we first consider the special case of CES preferences. These preferences turn out to be 'neutral', in the sense that there is a constant share of entrepreneurs in the economy. We do not provide formal proofs of that claim here because it is a particular case of Proposition 3 that we prove below when we characterize the possible changes in the cutoff $\widehat{c}$ depending on the properties of the lower-tier utility function.

Assume hence that the lower-tier utility function is given by $u(x) = x^\rho$, with $0 < \rho < 1$. As shown by Zhelobodko *et al.* (2012), given these preferences the selection cutoff in a Melitz-type model is constant. However, firms in the Melitz model are not linked to individuals and their decisions with respect to their occupational choice. This is done in, among others, Ohyama *et al.* (2011) and Kukharskyy (2012). In those two papers – which use CES as the lower-tier utility function, and the logarithm as the upper-tier utility function – market size does not influence the equilibrium share of entrepreneurs, the size of the firms, and prices. The same result holds true in Behrens *et al.* (2014a), who consider a model in which heterogeneous agents sort across cities and self-select into entrepreneurship based on a combination of their talent and a random productivity shock. In their model, preferences are CES but not quasi-linear, and there is no sunk cost of the Melitz-type. Instead, entrepreneurs self-select into the occupation that secures them the highest returns, as they do in our paper. More generally, selection is constant in one-sector CES models with income effects (see, e.g., Pokrovsky and Skolkova, 2013), and this is also true in our paper.[9]

It can readily be verified that when preferences are given by the following utility function $\mathcal{U} = \ln\left(\int_{\underline{c}}^{\widehat{c}} x_c^\rho L\gamma_c \mathrm{d}c\right) + A$, the cutoff $\widehat{c}$ is independent of market size. Neither the prices nor the output and – as a consequence – the share of entrepreneurs or their (relative) incomes vary with market size. This implies that there is also no effect of market size on income inequality, as we verify more formally in Section 5 below.

4.2 General Lower-Tier Utility Function

We now establish our key results concerning market size and entrepreneurship for the more general preference structure given by (1), with $V(\cdot) \equiv \ln(\cdot)$.[10] Hence, instead of CES prefer-

[9]Behrens *et al.* (2014a) show that with 'imperfect sorting' of types across cities, the self-selection cutoff varies even under CES preferences. This result is entirely due to the fact that cities attract more talented people, and it is the change in skill composition that can make selection tougher in bigger markets. However, there is no 'pure effect' of size on selection, and even if the selection cutoff does vary across cities, it does not vary substantially. We show in Section 5 that these results need to be qualified in non-CES models: differences in selection across markets may be sizable.

[10]In the Supplemental Appendix D.2, we extend our results to the case of an arbitrary upper-tier utility function. Hence, our results carry through to this more general case, provided that we have an interior equilibrium (see Appendix D.1 for a discussion). To simplify the exposition, we stick to log-preferences in the main text and relegate more general results to the appendix.

ences, we use an unspecific subutility $u(\cdot)$. As shown in Section 3, the shift in the ICC and in the SSC curves can lead to an increase or a decrease in the cutoff $\widehat{c}$. The shift in the curves depends on the characteristics of $u(\cdot)$, i.e., on the scale elasticity $\mathcal{E}_x[u]$ and on the change in the measure of concavity, $r_u(\cdot)$. Changes in the latter correspond either to: (i) decreasing elasticity of demand (DED; $r'_u < 0$); (ii) increasing elasticity of demand (IED; $r'_u > 0$); or constant elasticity of demand ($r'_u = 0$, which is the CES case we discussed before). It is important to emphasize from the start that, in general, all these properties are *local*, i.e., they depend on the equilibrium consumption level x. As shown in the Supplemental Appendix D.3, there exist well-behaved utility functions that are IED over some range, and DED over some other range. The function u can even have constant elasticity for a zero-measure set of points. However, constant elasticity everywhere is equivalent to $u(\cdot)$ being a power function. Also, the scale elasticity can increase over some range and decrease over some other range.

The impact of market size on the self-selection cutoff $\widehat{c}$, and thus on the share of entrepreneurs $\Gamma(\widehat{c})$, can be summarized as follows for any given equilibrium consumption x:

Proposition 3 (Market Size and the Share of Entrepreneurs). An increase in market size, L, changes the share of entrepreneurs in the following way:

(*pointwise at* x)	DED $r'_u < 0$	$r'_u = 0$	IED $r'_u > 0$
$\mathcal{E}'_x[u] > 0$	$\widehat{c}\,?$	$\widehat{c} = \text{const}$	$\widehat{c}\downarrow$
$\mathcal{E}'_x[u] = 0$	$\widehat{c} = \text{const}$	$\widehat{c} = \text{const}$	$\widehat{c} = \text{const}$
$\mathcal{E}'_x[u] < 0$	$\widehat{c}\uparrow$	$\widehat{c} = \text{const}$	$\widehat{c}\,?$

Proof. See Appendix B.4 □

To understand these results, observe that the consumers' willingness to pay for variety is characterized by the RLV (Zhelobodko *et al.*, 2012). The IED case corresponds to an increasing willingness of agents to pay for product diversity as their income and consumption level expand. As should be clear, an increasing willingness to pay for diversity provides strong market incentives for the provision of more variety, which ceteris paribus pushes towards a larger share of entrepreneurs in the economy. At the same time, a higher RLV raises markups and prices, which pushes down individual demands and, eventually, profits. In the latter case, it is tougher for the marginal entrepreneurs to survive. Given those two opposing channels, it is not surprising that the direction of change in the share of entrepreneurs depends precisely on the behavior of both the RLV and the scale elasticity of the utility function.

As mentioned before, under an increasing measure of concavity of the subutility function ($r'_u > 0$), a fall in individual consumption x due to an increase in market size L decreases ceteris paribus r_u and, therefore, prices. This requires that entrepreneurs must be able to sell a larger total quantity Lx to remain in the market. Since prices of more productive entrepreneurs fall relatively less than those of less productive entrepreneurs, the share of entrepreneurs decreases with L. The reason is that under IED relatively small increases in consumption map into relatively large changes in utility. In that case, agents have a strong incentive to increase their spendings on lower-priced varieties, which are precisely those provided by the more productive entrepreneurs. In a nutshell, this leaves a smaller market size for the less productive entrepreneurs, which then go out of business. In the opposite case, the outcome is symmetric. Under DED and a decreasing measure of concavity of the utility function, a fall in individual consumption x due to an increase in market size L increases r_u and, therefore, prices. This allows less productive entrepreneurs to profitably operate in the market so that the share of entrepreneurs increases.

A few additional comments are in order. First, as mentioned in the foregoing subsection, CES preferences are the border case between IED and DED. In that case, the occupational structure of the economy is independent of the size of the market for any equilibrium value of x. For other preferences, we may have 'locally' absence of an effect of size on selection, but this cannot hold generally. Second, there are two cases where an increase in market size has a priori an ambiguous impact on the share of entrepreneurs. The first one, corresponding to a decrease in the elasticity of the utility function – a satiation effect – and an increase in the elasticity of demand, seems the more plausible one. In that case, a larger L and an associated smaller x for each variety, increase the scale elasticity of utility, as well as decrease markups and prices. A higher scale elasticity pushed towards more demand for the more productive entrepreneurs, thus leaving less room for the less productive entrepreneurs. As a result, the self-selection cutoff decreases and the share of entrepreneurs falls.

4.3 Extensions

As shown in Appendix C.1 and Appendix C.2, the model can easily be extended to incorporate shocks to consumers' preferences and to entrepreneurs' productivity. Without going into details, we can note the following. First, common preference shocks that increase the value consumers attach to the differentiated good will, ceteris paribus, increase the share of entrepreneurs in the economy. At the same time, firms' sizes and entrepreneurs' profits also increase. As we will show in the next section, these changes translate into larger inequality between entrepreneurs, and between workers and entrepreneurs. Thus, economies where consumers attach more weight to the differentiated good produced by the entrepreneurs have a larger share of the latter and are also more unequal.[11]

Second, a common shock to the productivity of all entrepreneurs in equilibrium is formally equivalent to a change in the size of the market, L (see Appendix C.2 for the proof). Thus,

[11]This finding also suggests that the reorientation of consumer spending towards differentiated goods which are more intensive in high-skilled labor lead to increasing income inequality. The relationship between trade, demand shifts, and income inequality is an interesting avenue to explore in future research.

conditional on an initial equilibrium and a distribution of abilities, common changes to the ability of entrepreneurs and changes in market size have similar effects on the economy. A policy that would affect the productivity of all entrepreneurs in the same way could thus, e.g., be a perfect substitute for changes in market size due to migration of agents across countries or cities. It is important to point out that, since changes in market size have an ambiguous effect on the self-selection cutoff and the share of entrepreneurs, the same holds true for productivity shocks. Thus, a positive shock may very well lead to an increase or a decrease of the self-selection cutoff and the share of entrepreneurs, depending on the underlying preferences.

5 Market Size, Ability Distribution, and Income Inequality

We now investigate how changes in market size and in the ability distribution affect the distribution of income across heterogeneous agents. As shown in the previous section, changes in market size and the resulting changes in the market aggregates induce changes in the share of entrepreneurs. These changes in employment structure lead, in turn, to changes in the characteristics of active firms – their prices, outputs, and profits – which influence the distribution of income in the economy as a whole and among entrepreneurs in particular.

Following Behrens and Robert-Nicoud (2014), the effects of changes in market size on entrepreneurs' profits will be referred to as the *intensive margin*. Changes in the employment structure also affect the distribution of incomes via a composition effect – workers and entrepreneurs earn different incomes, and some entrepreneurs may switch to being workers or the other way round. We refer to this as the *extensive margin*.[12] We first investigate how entrepreneurs' profits change with market size. To this end, we look at how the whole profit distribution changes as the size of the market increases. The key finding is that larger markets can either increase or decrease the inequality in profits. A simple measure of inequality – the profit ratio at different percentiles of the ability distribution, which is aking to the 'interquartile range' – is analyzed later. Finally, we conclude by illustrating our model numerically for more complex measures of inequality like the Gini index, which subsume changes at both the intensive and the extensive margins.

5.1 Changes in Profits

Recall that the equilibrium profit of a type-c entrepreneur is equal to

$$\pi_c = (p_c - c)\, x_c L = \frac{r_u(x_c)}{1 - r_u(x_c)} c L x_c. \tag{13}$$

[12]See Behrens and Robert-Nicoud (2014) for the analysis of income inequality in an urban model based on Melitz and Ottaviano (2008). They decompose changes in income inequality into an extensive and an intensive margin, and show that both margins contribute to increase inequality in that model.

Hence, the elasticity of profits with respect to market size L is given by

$$\begin{aligned}\mathcal{E}_L[\pi_c] &= 1+\mathcal{E}_L[x_c]+\left(\mathcal{E}_x[r_u]-\mathcal{E}_x\left[1-r_u\right]\right)\mathcal{E}_L[x_c] \\ &= 1+\left(1+\mathcal{E}_x[r_u]+\frac{r_u}{1-r_u}\mathcal{E}_x[r_u]\right)\mathcal{E}_L[x_c], \end{aligned} \tag{14}$$

which depends, in particular, on the elasticity of individual consumption with respect to market size. Applying Corollary A.2 from Appendix A to (14), and using (B.8) from Appendix B.3, equation (14) can be rewritten as follows:

$$\mathcal{E}_L[\pi_c] = 1-\frac{1-\widehat{r}_u}{\widehat{r}_u}\frac{\widehat{r}_u}{r_u}\left(\frac{\widehat{r}_u}{1-\widehat{r}_u}-\mathcal{E}_L[\widehat{c}]\right) = \mathcal{E}_L\left[\pi_c \mid \mathrm{d}\widehat{c}/\mathrm{d}L=0\right]+\mathcal{S}_L[\pi_c]. \tag{15}$$

We refer to the first term of (15) as the *conditional effect* of market size on profits, for a given level of selection; whereas we refer to the second term as the *selection effect*. To understand the different effects of market size on profits, it is useful to consider the following thought experiment. Assume that market size expands whereas the self-selection cutoff $\widehat{c}$ is held constant. In that case, since $\mathcal{E}_L[\widehat{c}]=0$, we have

$$\mathcal{E}_L\left[\pi_c \mid \mathrm{d}\widehat{c}/\mathrm{d}L=0\right] = 1-\frac{r_u(x_{\widehat{c}})}{r_u(x_c)} \gtreqqless 0 \quad \Leftrightarrow \quad r_u' \gtreqqless 0, \tag{16}$$

where the last equivalence stems from the fact that $c\leq\widehat{c}$. Without selection effects, income inequality increases if and only if $r_u'\geq 0$. Indeed, when the cutoff is unchanged, then $\mathcal{E}_L[\pi \mid \mathrm{d}\widehat{c}=0]=1-\frac{\widehat{r}_u}{r_u}\gtreqqless 0$ is equivalent to $r_u'\gtreqqless 0$, since $\widehat{c}\geq c$. Hence, in the case of IED, profits increase with the conditional effect of market size, i.e., for any given values of $\widehat{c}$ and μ. As can be seen from (16), that effect is larger for more efficient firms (since x_c is decreasing in c), which already suggests that market size has an inequality-increasing effect when preferences are IED. In the case of DED, we naturally have the opposite result.

Until now, we have assumed that the cutoff does not change. Yet, this is generically only true in the CES case. In more general cases, the cutoff usually changes so that the selection effect and the conditional effect may have opposite signs. The total effect of market size on profits then depends on the relative strength of the two effects. The possible patterns of profit changes are illustrated in Figure 2. As can be seen, there exist cases where some firms increase their profits, but others lose profits, so that the impact of market size on inequality is ambiguous (see panels (a) and (d) of Figure 2 below). The magnifying or dampening impact of the selection effect, $\mathcal{S}_L[\pi_c]$, on profits depends on the direction of change in the share of entrepreneurs and in the RLV:

$$\mathcal{S}_L[\pi_c] = \frac{1-r_u(x_{\widehat{c}})}{r_u(x_{\widehat{c}})}\frac{r_u(x_{\widehat{c}})}{r_u(x_c)}\cdot\mathcal{E}_L[\widehat{c}].$$

If the share of entrepreneurs increases, the selection effect is positive and pushes towards more inequality. If r_u is increasing, we thus have quite naturally a positive effect of market size on inequality between workers and entrepreneurs (since all profits increase). As can be seen from panel (b) of Figure 2, inequality among entrepreneurs then is also likely to increase, though this claim needs to be qualified since it depends on the underlying ability distribution. It is true under a uniform distribution, but may fail to hold when there is a large density of types close to the cutoff. If the share of entrepreneurs decreases, the selection effect is negative and pushes towards more equality. If r_u is decreasing, we thus have a negative effect of market size on inequality between workers and entrepreneurs (since all profits fall). Panel (c) of Figure 2 illustrates this case. Again, it suggests that inequality among entrepreneurs should fall, but this claim must again be qualified since it depends on the underlying ability distribution. It is true under a uniform distribution of types, but may fail to hold when there is a small density of types close to the cutoff.

Note that even under a uniform distribution of abilities, the effect of market size on inequality cannot be clearly signed in general. In particular, when profits increase for some entrepreneurs, whereas they decrease for others, the total effect on inequality crucially depends on the underlying ability distribution (see panels (a) and (d) in Figure 2). We will thus use numerical methods later to investigate these effects in more detail.

Formally, our results concerning the shifts in the profit distribution and inequality among entrepreneurs can be summarized as follows:

Proposition 4 (Market Size and Entrepreneurs' Profits). The impact of changes in market size on entrepreneurs' profits is as follows:

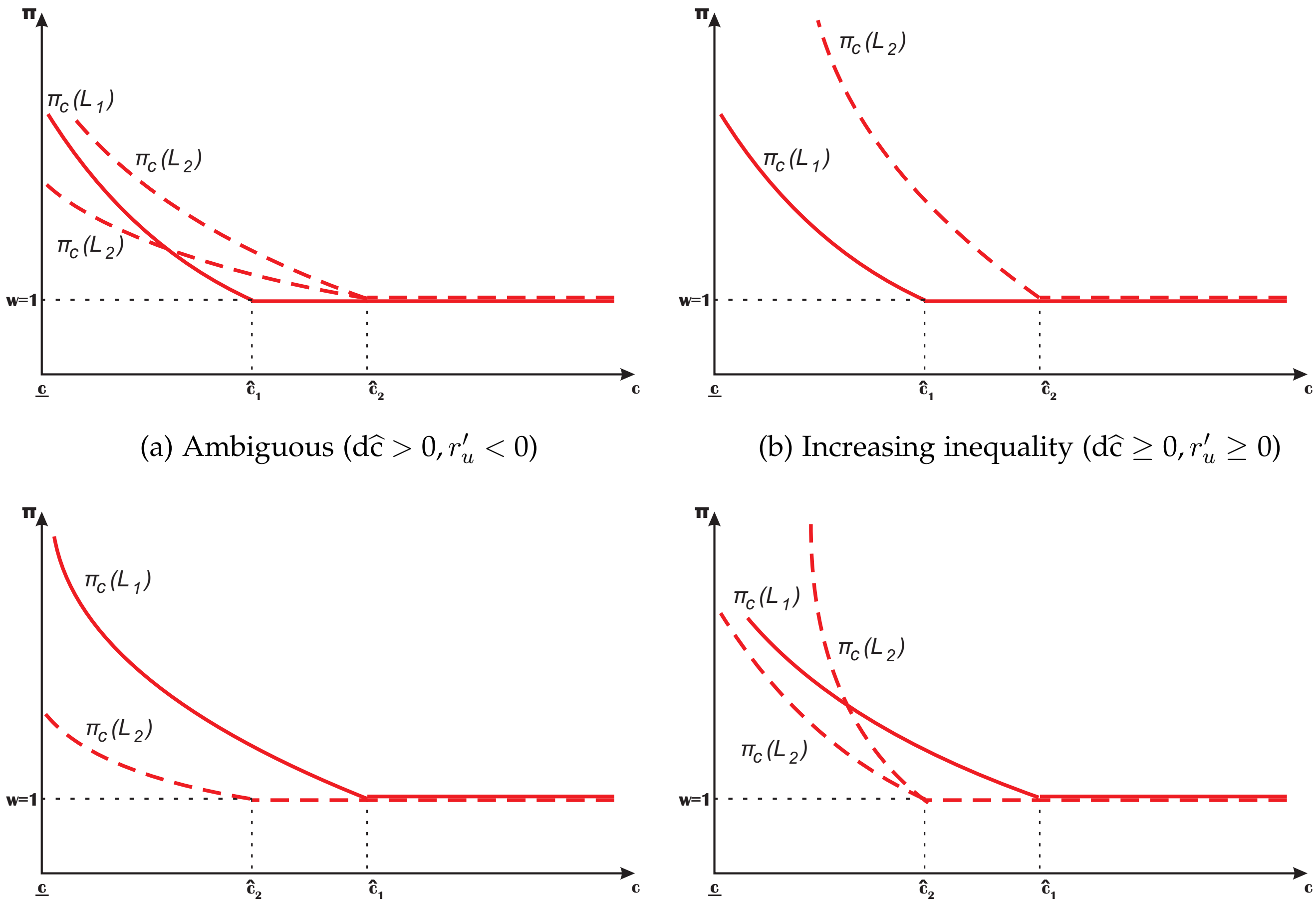
(*pointwise at* x)	DED $r_u' < 0$	$r_u' = 0$	IED $r_u' > 0$
$\widehat{c}\uparrow$	π_c ?	$\pi_c \uparrow$	$\pi_c \uparrow$
$\mathrm{d}\widehat{c} = 0$	$\pi_c \downarrow$	$\pi_c = \text{const}$	$\pi_c \uparrow$
$\widehat{c}\downarrow$	$\pi_c \downarrow$	$\pi_c \downarrow$	π_c ?

In the case of an increase in the self-selection cutoff, the influence of market size on profits is stronger for more efficient entrepreneurs under IED, and stronger for less efficient entrepreneurs under DED. When the self-selection cutoff decreases, the opposite results apply.

Proof. We provided all elements of the proof in the text before the proposition. □

The previous results can be understood as follows. Let us, for example, consider the DED case and assume that the share of entrepreneurs decreases. In that situation, prices and outputs go down (see expression (B.9) in Appendix B.3), so that the profit of each firm must fall. The two cases where the effect cannot be clearly signed are the same as for firms' outputs.[13] In

[13]Comparing (B.9) in Appendix B.3 and (14), we see that there is no difference between the effect of changes



(a) Ambiguous ($d\widehat{c} > 0, r'_u < 0$)

(b) Increasing inequality ($d\widehat{c} \geq 0, r'_u \geq 0$)

(c) Decreasing inequality ($d\widehat{c} \leq 0, r'_u \leq 0$)

(d) Ambiguous ($d\widehat{c} < 0, r'_u > 0$)

Figure 2: Different Patterns of Profit Changes.

those cases, one of the following two outcomes can be observed: (i) either the profits of all firms move in the same direction; or (ii) there is a threshold value $\widetilde{c}$ such that the influence of market size on profit is in one direction for all firms with $c < \widetilde{c}$, and in the opposite direction for all firms with $c > \widetilde{c}$. In particular, in the case where $r'_u < 0$ and where $\widehat{c}$ increases, the profits of all firms either increase, or the profits increase only for firms with $c > \widetilde{c}$ (see panel (a) of Figure 2). Conversely, when $r'_u > 0$ and $\widehat{c}$ decreases, the profits of all firms either decrease, or the profits decrease only for firms with $c > \widetilde{c}$ whereas it increases for the other firms (see panel (d) of Figure 2). In both cases, the ambiguity of results is due to the opposite signs of the conditional effect and the selection effect.

Note, finally, that Figure 2 also sheds light on the distribution of profits *between* workers and entrepreneurs. Since wages of all workers are equal to one, the average profit $\overline{\pi}_c$ of entrepreneurs provides a measure of relative inequality between the two groups. In panels (b) and (c) of Figure 2, this 'between' measure of inequality increases – respectively decreases –

in market size on outputs and profits. The reason is that individual quantities fall with market size, so that the effect of market size on prices is unambiguous and depends only in the behavior of r_u.

irrespective of the distribution. Of course, this measure is based on the relative incomes of the two groups and it does not take into account the share of each group in the population. It is, therefore, not well suited to make statements about overall income inequality. We tackle that issue in more detail in Subsection 5.3 below.

5.2 Inequality Among Entrepreneurs

Our foregoing results establish how entrepreneurial profits shift in response to a change in market size. Yet, they do not tell us much on how income inequality changes, safe for two cases or when the underlying distribution of abilities is uniform. We now look in more detail at the 'intensive margin', i.e., income inequality among entrepreneurs, and derive additional results. In particular, we derive some inequality results that are independent of the underlying distribution since they compare only individuals at selected percentiles of the income distribution. Observe that looking at the ratio of profits for entrepreneurs of different abilities is formally equivalent to looking at the widely used interquartile range. Choosing, e.g., the entrepreneurs at the 25th and 75th percentile of the ability – and, therefore, income – distribution then provides a natural metric for gauging inequality among entrepreneurs.

Given our previous results, it should be clear that an increase or decrease in inequality among entrepreneurs with respect to changes in market size is related to the behavior of the RLV. Formally, we can prove the following result:

Proposition 5 (Market Size and Inequality Among Entrepreneurs). Consider two entrepreneurs with ability $1/c_1 > 1/c_2$. An increase in market size increases the relative profit of entrepreneur 1 in the IED case, whereas it decreases it in the DED case.

(*pointwise at* x)	DED $r'_u < 0$	$r'_u = 0$	IED $r'_u > 0$
$\pi(x_{c_1})/\pi(x_{c_2})$	$\downarrow$	const	$\uparrow$

Hence, income inequality among entrepreneurs – as measured by the profit ratio for selected percentiles – increases in the IED case, whereas it decreases it in the DED case.

Proof. See Appendix B.5. □

Several remarks are in order. First, Proposition 5 includes two cases. There is the case where the profits of the more productive entrepreneurs increase, whereas those of the less productive entrepreneurs decrease (see panel (d) of Figure 2). In other words, profits increase for all $c < \widetilde{c}$, whereas they decrease for all $c > \widetilde{c}$. In that case, inequality among entrepreneurs necessarily increases. Then, there are the cases where the profits of all entrepreneurs decrease (panels (c) and (d) of Figure 2), and the cases where the profits of all entrepreneurs increase (panels (a) and (b) of Figure 2). In these cases, inequality among entrepreneurs can either decrease (panel (c)) or increase (panel (d)), depending on whether the effect of a larger market is stronger for less or for more productive entrepreneurs. The direction of the change crucially hinges on the RLV, as shown by Proposition 5. In the case of DED, a larger market size benefits more the less productive entrepreneurs so that inequality among them decreases; whereas in the case of IED, a larger market size benefits more the more productive entrepreneurs so that inequality among them increases.[14] Second, as shown in Appendix C.1, a uniform shock to preferences that makes the differentiated good more desirable for consumers will increase the profits of all entrepreneurs and increase the self-selection cutoff. In other words, this case corresponds to the one depicted in panel (b) of Figure 2. Clearly, in that case, income inequality increases among entrepreneurs, and also between workers and entrepreneurs.

5.3 Income Inequality in the Economy

Until now, we have only looked at entrepreneurs and their profits. In so doing, we have disregarded the fact that the employment structure of the economy also changes with the size of the market. We thus now take into account the 'extensive margin' to gauge the impact of L on income inequality in the whole economy. Since all workers earn the same wage, there is no income inequality among them. However, as shown in Section 4, a change in market size generally has an effect on the self-selection cutoff. Thus, market size has both an effect on the distribution of income among entrepreneurs (see Section 5.2), as well as between entrepreneurs and workers. We now investigate how market size influences the overall distribution of income and measures of income inequality that account for both inequality *within* types (at least for entrepreneurs, since there is no inequality among workers) and *between* types. We first derive a number of results for unambiguous cases. As comprehensive analytical results are out of pencil-and-paper reach for the other cases, we resort to numerical illustrations to show how changes in market size and the ability support map into changes in inequality depending on the underlying distribution of types.

Using the results of Section 5.2 and Figure 2, there are two cases where the impact of market size on overall income inequality is clear. First, when the profits of all entrepreneurs increase – and especially so for the more productive ones – and the selection cutoff increases too, income inequality in the economy must increase (see panel (b) of Figure 2) irrespective of the underlying ability distribution. Second, when the profits of all entrepreneurs fall – and especially so

[14] Under CES preferences, market size has no influence on the cutoff $\widehat{c}$ and, thus, on relative profits. Hence, larger markets are not more unequal than smaller markets in that case. Yet, we know from the empirical evidence that, although selection cutoffs do not differ greatly across markets (Combes *et al.*, 2012), income inequality is increasing in city size. This suggests that other mechanisms – e.g., complementarities between talent and agglomeration economies, the so-called 'dilation' effect in Combes *et al.* (2012) – are drivers of a part of the observed inequality. Baum-Snow and Pavan (2014) come to the same conclusion. They show that "the rapid growth in within skill group inequality in larger cities has been by far the most important force driving" the rise in income inequality in US cities between 1979 and 2007. Hence, there is "an important role for agglomeration economies in generating changes in the wage structure" (Baum-Snow and Pavan, 2014, p.1).

for the more productive ones – and the selection cutoff decreases too, income inequality in the economy must fall (see panel (c) of Figure 2) irrespective of the underlying ability distribution. It is worth pointing out that in those two cases the between-group inequality of workers vs entrepreneurs – measured by the average profit of entrepreneurs relative to the workers' wage – unambiguously increases (respectively, decreases) for any underlying ability distribution. It is further worth emphasizing that, as shown in Appendix C.1, a uniform shock to preferences that makes the differentiated good more desirable for consumers will increase the profits of all entrepreneurs and increase the self-selection cutoff. In that case, inequality must also increase for the whole population, as we have just explained. Hence, economies where consumers value more strongly the differentiated good are, ceteris paribus, more unequal.

In all the remaining cases, it is a priori not possible to say something definitive about the change in inequality in the economy as a whole, or between the two groups of agents, even for simple distributions such as a uniform one. Some profits increase, whereas some profits decrease, and the employment structure changes at the same time. The overall effect on inequality is then ambiguous and depends on the properties of the RLV and on the underlying ability distribution. These are the cases illustrated by panels (a) and (d) of Figure 2.[15]

To better grasp how changes in market size and in the underlying ability distribution affect inequality, we hence resort to numerical methods to illustrate the model for different distributions. We are particularly interested in: (i) changes in the self-selection cutoff and in the share of entrepreneurs; (ii) changes in inequality in response to changes in market size; and (iii) changes in inequality in response to changes in the underlying ability support. Case (ii) is of interest since it will help us to better understand under what conditions larger markets – e.g., larger cities – are more unequal. Case (iii) may shed light on how certain policies that affect the ability support, e.g. migration restrictions or education policies, can affect the distribution of incomes. Both (ii) and (iii) are jointly especially relevant in a world where agents sort across markets of different sizes depending on their ability.

To cover a broad range of cases using a unified approach, we follow Zhelobodko *et al.* (2012) and make use of the following subutility function u:

$$u(x) = \frac{1}{\rho}\Big[(a+hx)^{\rho} - a^{\rho}\Big] + bx. \tag{17}$$

This function boils down to the CES when $a = b = 0$ and $h = 1$. It belongs to the class of IED functions when $a = h = 1$ and $b = 0$, whereas it belongs to the class of DED functions when $a = 0$ and $b = h = 1$. We can thus use (17) to cover all the cases that are of interest to us.[16] We

[15] Using quadratic-linear preferences, Behrens and Robert-Nicoud (2014) illustrate the case of panel (d) in Figure 2. In their model, the selection cutoff always falls, whereas profits of the more productive entrepreneurs always increase when compared to those of the less productive ones. Income inequality – as measured by the Gini coefficient – thus always increases in that case.

[16] The function (17) is IED or DED for all values of x for any given set of parameters. Hence, it cannot be used to

also choose three different ability distributions to illustrate the model. First, we use a uniform distribution on $[\underline{c}, \overline{c}]$. Second, we use a linear increasing distribution of c on that same support. Last, we consider the reverse case of a linear decreasing function of c. In the latter two cases, we thus have relatively more low (respectively, high) ability entrepreneurs in the economy.[17]

Table 1 summarizes our results for the three different versions of the subutility function, and for the three different distributions of types. The top part of the table contains the results for changes in the size L of the market (holding the ability distribution constant), whereas the bottom part of the table contains the results for multiplicative shifts in the ability distribution (holding market size L constant). We model the shift in the distribution as follows: the new density is given by $(1/k)\gamma(c/k)$, whereas the new support is given by $[k\underline{c}; k\overline{c}]$, with $k > 0$. When $k > 1$, we reduce the average ability of entrepreneurs in the economy, whereas when $k < 1$ we increase that average ability. In all cases, we report the value of the between-group inequality as measured by the average profit relative to the wage ($\overline{\pi}_c/w$); the share of entrepreneurs, $\Gamma(\widehat{c})$; and three inequality measures: the interquartile range IQR $= 100 \times [(\text{income}_{\text{p25}} - \text{income}_{\text{p75}})/\text{income}_{\text{p75}}]$, the coefficient of variation (CV) of income, and the Gini coefficient of income inequality.

As can be seen from the middle columns of Table 1, there is no change in the share of entrepreneurs and in income inequality in the case of CES preferences when either L or k change. We have shown this result for the case of L in the previous sections, and we prove it for the shift of the ability distribution in Supplemental Appendix D.4. Hence, selection and inequality cannot be meaningfully analyzed in the CES case since, conditional on the underlying ability distribution, neither size nor shifts matter (see also Behrens *et al.*, 2014a).[18] Table 1 reveals a number of interesting patterns. As can be seen from the top part (a) of the table, the share of entrepreneurs (and hence the self-selection cutoff) decreases with L in the IED case, whereas it increases in the DED case. In the CES case, the share is constant. Figure 3 depicts how the shares change with market size. These changes in shares map into increasing inequality – as measured by the interquartile range (IQR%) or the coefficient of variation (CV) – in the IED case, and into decreasing inequality in the DED case. Observe that the results for the IQR are those that we have formally established in Section 5.2.[19] Note also that the Gini coefficient decreases in *both* cases. The reason underlying that result is that inequality changes along two margins. In the IED case, selection is tougher and more agents become workers. Yet, workers all earn the same income, so that the extensive margin effect is inequality reducing. At the

illustrate the case where the function changes its behavior depending on x. See the Supplemental Appendix D.3 for an illustration of that case.

[17]We choose these distributions since they are easy to compute numerically. We could simulate the model with more complicated distributions such as the log-normal distribution at the cost of higher computational complexity.

[18]Of course, the results for selection and inequality in the CES case *do depend* on the underlying distribution $\gamma(\cdot)$. Yet, conditional on that distribution, they are invariant.

[19]It suffices to pick c_1 and c_2 associated with the 25th and the 75th percentiles of the income distribution, respectively, and to observe that the income associated with c_2 is constant if the agent is a worker.

Table 1: Numerical Simulations for Inequality Measures and Market Size (L) and Shift of the Ability Distribution (k).

	IED					CES					DED				
	$\overline{\pi}_c/w$	IQR%	CV	Gini	$\Gamma(\widehat{c})$	$\overline{\pi}_c/w$	IQR%	CV	Gini	$\Gamma(\widehat{c})$	$\overline{\pi}_c/w$	IQR%	CV	Gini	$\Gamma(\widehat{c})$
	(a) *Uniform distribution* ($k=1$)														
$L=10$	2.0421	83.6917	0.3319	0.0792	0.0890	1.3102	32.0465	0.1455	0.0491	0.1908	1.6965	50.7174	0.2019	0.0560	0.1331
$L=11$	2.0590	84.6366	0.3327	0.0782	0.0862	1.3102	32.0465	0.1455	0.0491	0.1908	1.6713	49.7332	0.1995	0.0560	0.1360
$L=12$	2.0743	85.4886	0.3332	0.0771	0.0837	1.3102	32.0465	0.1455	0.0491	0.1908	1.6495	48.8793	0.1973	0.0559	0.1384
$L=13$	2.0885	86.2628	0.3336	0.0762	0.0814	1.3102	32.0465	0.1455	0.0491	0.1908	1.6302	48.1302	0.1954	0.0558	0.1406
$L=14$	2.1014	86.9712	0.3339	0.0754	0.0793	1.3102	32.0465	0.1455	0.0491	0.1908	1.6131	47.4667	0.1936	0.0557	0.1425
	(b) *Uniform distribution* ($L=10$)														
$k=0.9$	2.0236	82.6336	0.3308	0.0803	0.0921	1.3102	32.0465	0.1455	0.0491	0.1908	1.7261	51.8670	0.2047	0.0560	0.1299
$k=1.0$	2.0421	83.6917	0.3319	0.0792	0.0890	1.3102	32.0465	0.1455	0.0491	0.1908	1.6965	50.7174	0.2019	0.0560	0.1331
$k=1.1$	2.0590	84.6366	0.3327	0.0781	0.0862	1.3102	32.0465	0.1455	0.0491	0.1908	1.6713	49.7332	0.1995	0.0559	0.1360
	(c) *Linear increasing distr.* ($L=10$)														
$k=0.9$	2.2305	97.5200	0.3861	0.0885	0.0854	1.3501	36.8629	0.1643	0.0540	0.1852	1.8624	61.0321	0.2372	0.0617	0.1219
$k=1.0$	2.2557	98.8331	0.3862	0.0884	0.0824	1.3501	36.8629	0.1643	0.0540	0.1852	1.8262	59.6064	0.2337	0.0618	0.1253
$k=1.1$	2.2785	100.006	0.3877	0.0873	0.0798	1.3501	36.8629	0.1643	0.0540	0.1852	1.7956	58.3865	0.2306	0.0617	0.1282
	(d) *Linear decreasing distr.* ($L=10$)														
$k=0.9$	1.6041	51.9727	0.2134	0.0559	0.1100	1.1983	19.9458	0.0952	0.0338	0.2086	1.4380	31.4223	0.1314	0.0393	0.1517
$k=1.0$	1.6132	52.6068	0.2136	0.0549	0.1063	1.1983	19.9458	0.0952	0.0338	0.2086	1.4212	30.7715	0.1298	0.0392	0.1548
$k=1.1$	1.6214	53.1719	0.2135	0.0540	0.1030	1.1983	19.9458	0.0952	0.0338	0.2086	1.4067	30.2141	0.1283	0.0392	0.1574

Notes: We use the function given by (17) and set $\underline{c}=1$, $\overline{c}=5$, and $\rho=1/2$. In the IED case, we set $a=h=1$ and $b=0$. In the DED case, we set $b=h=1$ and $a=0$. The CES case corresponds to $a=b=0$ and $h=1$. The linear distribution functions are parametrized by the intercept $\alpha_1=100$. The slope is set to $\alpha_2=20$ in the increasing case, and to $\alpha_2=-20$ in the decreasing case. We vary the population size L from 10 to 14 in the top part of the table. We then shift the ability distribution using k as a multiplicative factor in the bottom part of the table. We use a discrete approximation to compute the surface under the Lorenz curve (using 20 equally spaced points to generate the trapezoids) when computing the Gini coefficient.

same time, inequality among entrepreneurs increases, yet that intensive margin effect is not strong enough to offset the extensive margin effect. In the case of DED, inequality increases at the extensive margin – more agents become entrepreneurs – yet falls at the intensive margin. As the intensive margin dominates the extensive margin, the Gini coefficient falls again (in our example at least).

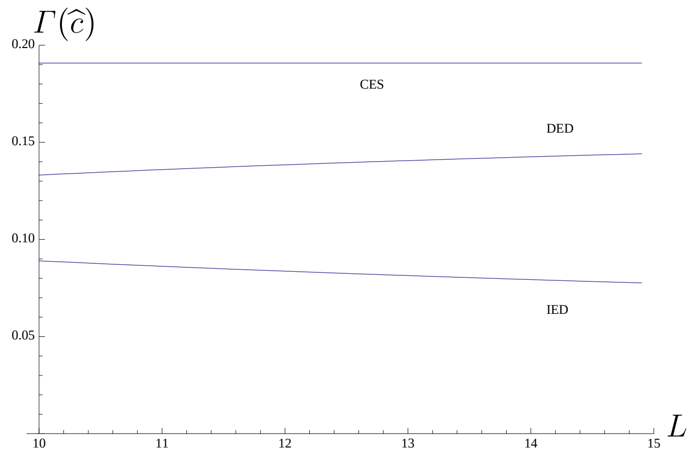


Figure 3: Share of Entrepreneurs as a Function of Market Size L.

It is further worth noting that when selection is not constant in the IED and in the DED cases, the effect of market size on selection can be sizable. An increase in market size by 40% increases the share of entrepreneurs by about 7% in the DED case, and decreases it by about 11% in the IED case.[20]

As can be seen from the bottom parts (b)–(d) of Table 1, changes in the underlying ability support can affect the share of entrepreneurs and inequality in a variety of ways. First, the CES case is again the borderline case where inequality and selection are not affected by the shift in the distribution. The reason is that, as in Behrens *et al.* (2014a), a multiplicative shift in the distribution leads to a proportional shift in the self-selection cutoff $\widehat{c}$, so that the share of entrepreneurs remains the same (see the Supplemental Appendix D.4). Second, as also shown in the Supplemental Appendix D.4, the impacts of a shift in the distribution are generally ambiguous when preferences are not CES, since it depends on how different the size elasticity of $\widehat{c}$ is from one. As shown by panels (c)–(d) of Table 1, an upwards shift decreases the share of entrepreneurs in the IED case but increases it in the DED case. This holds true in our examples regardless of whether there are relatively more high c types (panel (c)) or relatively more low c types (panel (d)). Yet, the impacts on inequality can go either way, depending on the underlying distribution and the measure that is actually used.

As should be clear from our results and discussion, it is virtually impossible to make precise predictions on how market size and the underlying productivity support affect the share of

[20]Behrens *et al.* (2014a) find that the selection cutoff does not vary greatly across cities of different sizes in the presence of sorting. They do, however, rely on CES preferences. As shown in this paper, the effect of size on selection can be much stronger with non-CES preferences, even without sorting along ability.

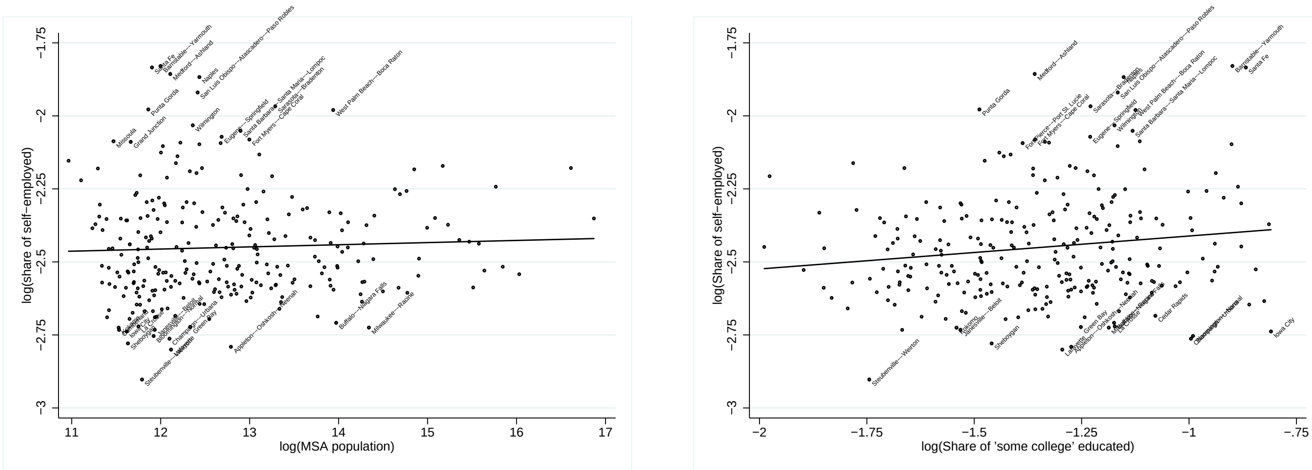


Figure 4: Size- and Education Elasticity of the Share of Self-Employed (US Census, 2000).

entrepreneurs, the self-selection cutoff, and income inequality. We may, therefore, ask what relationships we do see in the data? The left panel of Figure 4 shows that there is virtually no correlation between the population size of US MSAs in 2000 and the share of self-employed (a proxy for entrepreneurship). This result continues to hold true even when we control for a variety of socio-economic characteristics and include state fixed effects. Thus, there seems to be no strong relationship between 'entrepreneurship' and city size. As can be seen from the right panel of Figure 4, the share of self-employed in the US is, however, positively associated with a proxy for 'education', namely the share of workers having completed at least some college education.[21] The 'education elasticity' of the share of self-employed is 0.11, with p-value 0.017.[22] Thus, a 'better ability support' – proxied by a higher share of educated people – is positively associated with entrepreneurship. Given the natural assumption that the density of c is increasing, and given that bigger and more educated US cities have a higher Gini coefficient, the empirical evidence is in line with the IED case in panel (c) of Table 1.

6 Conclusions

We have developed a monopolistic competition model with two sectors and heterogeneous agents who self-select into entrepreneurship, depending on their entrepreneurial ability. To this end, we have extended the model by Zhelobodko *et al.* (2012) to allow for heterogeneous agents à la Melitz (2003), who make an occupational choice à la Lucas (1978). The effects of market size on the share of entrepreneurs and the distribution of incomes crucially hinges on two properties of the utility function – its elasticity of substitution and its Arrow-Pratt index of relative risk aversion. We have fully characterized the equilibrium and shown that the cutoff

[21]Using the share of people with a high-school degree or the share of college graduates yields similar results.

[22]Using as a proxy for education the share of highschool graduates we get an elasticity of 0.20, with p-value 0.002. Using the share of college graduates, we get an elasticity of 0.08, with p-value 0.023.

for self-selection into entrepreneurship, the share of entrepreneurs, and income inequality can increase or decrease with market size. Furthermore, the underlying ability distribution is crucial in tracing out changes in income inequality in response to changes in market size. In a nutshell, theory provides little clear guidance as to which effects we should expect to see in the data, and what their relative strength is likely to be. It should be clear that other mechanisms – like complementarities between skills and agglomeration economies (Combes *et al.*, 2012; Baum-Snow and Pavan, 2014) or sorting from 'both the top and from the bottom' of the ability distribution (Eeckhout, Pinheiro, and Schmidheiny, 2014) – are also likely to operate in the real world. Those mechanisms may affect the relationship between size and inequality that we see in the data. However, even without those mechanisms, we cannot make clear predictions without strong assumptions on the underlying preferences in the model.

Although some of our results are reminiscent of those contained in Zhelobodko *et al.* (2012) and in Behrens *et al.* (2014a), we have extended these models to allow for occupational choice and for more general preferences, respectively. We have also investigated in much more depth the impacts of market size on income inequality. We see this work as a step forward in the development of urban and trade models where either differences in city sizes or trade liberalization – which is often viewed in terms of market growth – play important roles. Understanding how sorting across cities or trade affect heterogeneous entrepreneurs in the presence of occupational self-selection with more general preference structures is certainly another interesting topic on our research agenda. This would allow us to tie our work more closely to the recent literature that investigates either the emergence of cities and the sorting of agents across cities, or to the literature on the differential effects of trade liberalization on heterogeneous agents. More work is called for here.

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Appendix

This extensive set of appendices is structured as follows. Appendix A contains various lemmas and their corollaries. Appendix B contains the proofs of the different propositions stated in the main body of the paper. Appendix C presents two extensions of the model that integrate preference shocks and productivity shocks, respectively. Finally, Appendix D contains supplemental material. In that appendix, we derive some more general results when the upper-tier utility function is not logarithmic, and we provide proofs for the impacts of a shift in the underlying ability distribution (in general, and for the CES case in particular).

Appendix A Lemmas

This appendix contains several lemmas that are required for proving the results in the paper.

Lemma A.1. This lemma establishes that $\mathcal{E}_x[\mathcal{E}_x[f]] = 1 + \mathcal{E}_x[f'] - \mathcal{E}_x[f]$.

Proof. The result can be established as follows:

$$\begin{aligned}
\mathcal{E}_x[\mathcal{E}_x[f]] &= \frac{\mathrm{d}}{\mathrm{d}x}[\mathcal{E}_x[f]] \cdot \frac{x}{\mathcal{E}_x[f]} = \left(\frac{f'x}{f}\right)' \frac{f}{\frac{f'x}{x}} \\
&= \left(\frac{f''x}{f} + \frac{f'}{f} - \frac{f'^2 x}{f^2}\right)\frac{f}{f'} = \frac{f''x}{f'} + 1 - \frac{f'x}{f} = 1 + \mathcal{E}_x[f'] - \mathcal{E}_x[f]. \qquad \text{(A.1)}
\end{aligned}$$

□

Corollary A.2. As a direct consequence of Lemma A.1, we can show that $\mathcal{E}_x[r_u] = 1 - r_{u'} + r_u$.

Proof. Since $r_u = -\mathcal{E}_x[u']$ and $r_{u'} = -\mathcal{E}_x[u'']$, we get the result by applying (A.1) to $f \equiv u'$. □

Corollary A.3. As a direct consequence of Corollary A.2 , we have the following equivalence:

$$\frac{2 - r_{u'}}{1 - r_u} \gtreqless 1 \quad \Leftrightarrow \quad r'_u \gtreqless 0. \qquad \text{(A.2)}$$

Proof. From Corollary A.2, we know that

$$\mathcal{E}_x[r_u] = 1 - r_{u'} + r_u = (2 - r_{u'}) - (1 - r_u) \gtreqless 0 \quad \Leftrightarrow \quad r'_u \gtreqless 0,$$

where the equivalence directly comes from the definition of the elasticity. Since $r_u \leq 1$ from the first-order conditions for profit maximization, it follows that

$$\frac{2 - r_{u'}}{1 - r_u} \gtreqless 1 \quad \Leftrightarrow \quad r'_u \gtreqless 0.$$

□

Lemma A.4. We have $\mathcal{E}_x[(1 - r_u)u'] = -\frac{2 - r_{u'}}{1 - r_u} r_u < 0$.

Proof. To obtain the result, take the derivative with respect to x to obtain:

$$\mathcal{E}_x[(1 - r_u)u'] = -\frac{r_u}{1 - r_u}\mathcal{E}_x[r_u] - r_u.$$

Using Corollary A.2, the result follows. Note that the elasticity is negative because $0 < r_u < 1$ and $r_{u'} < 2$ hold from the first- and second-order conditions for profit maximization. □

Lemma A.5. We have $r_{\ln u} = -\mathcal{E}_x[u'] + \mathcal{E}_x[u] = r_u + \mathcal{E}_x[u]$.

Proof. Applying the definition (5), straightforward computations show that

$$\begin{aligned} r_{\ln u} &= -\left(\frac{u'}{u}\right)' \frac{u}{u'}x = -\left(\frac{u''}{u} - \frac{u'^2 x}{u^2}\right)\frac{u}{u'}x \\ &= -\frac{u''}{u'}x - \frac{u'}{u}x = -\mathcal{E}_x\left[u'\right] + \mathcal{E}_x\left[u\right] = r_u + \mathcal{E}_x\left[u\right]. \end{aligned}$$

□

Corollary A.6. From Lemma A.5, we have $\mathcal{E}_x[\mathcal{E}_x[u]] = 1 - r_u - \mathcal{E}_x[u] = 1 - r_{\ln u}$.

Proof. Applying Lemma A.1 to the function u, taking into consideration that $r_u = -\mathcal{E}_x[u']$, and applying Lemma A.5, we have:

$$\mathcal{E}_x[\mathcal{E}_x[u]] = 1 + \mathcal{E}_x[u'] - \mathcal{E}_x[u] = 1 - r_u - \mathcal{E}_x[u] = 1 - r_{\ln u}.$$

□

Appendix B Proofs

This appendix contains the proofs of the different propositions stated in the main body of the paper.

Appendix B.1 Proof of Proposition 1

Proof. Consider two entrepreneurs with ability $1/c_1 > 1/c_2$. We will prove that entrepreneur 1: (i) produces a larger quantity ($x_{c_1} > x_{c_2}$); (ii) sets a lower price ($p(x_{c_1}) < p(x_{c_2})$); and (iii) earns higher profits ($\pi(x_{c_1}) > \pi(x_{c_2})$) than entrepreneur 2.

First, starting with (i), observe that the optimal output of a type-c firm, given by $y_c = x_c L$, is determined from the condition

$$u'(x_c)[1 - r_u(x_c)] = c\mu, \tag{B.1}$$

which combines the first-order conditions of consumers and producers. From (B.1), we obtain individual consumption of a type-c variety, x_c, as a function of c and μ. From Lemma A.4 in Appendix A, the left-hand side of (B.1) is a decreasing function of x. It then follows from (B.1) that it is also a decreasing function of c and of μ because

$$\mathcal{E}_c[x_c] = \mathcal{E}_\mu[x_c] = -\frac{1}{r_u}\frac{1 - r_u}{2 - r_{u'}} \leq 0. \tag{B.2}$$

Next, to obtain (ii), we use the optimal price for a type-c variety: $p_c = c/[1 - r_u(x_c)]$. Using the definition of the elasticity, applying the chain rule, making use of Lemma A.1 in Appendix A, and using (B.2), we have

$$\mathcal{E}_c[p_c] = e_c[p_c] + e_{x_c}[p_c] \cdot \mathcal{E}_c[x_c] = 1 + \mathcal{E}_{r_u}[p_c] \cdot \mathcal{E}_{x_c}[r_u] \cdot \mathcal{E}_c[x_c] = \frac{1 - r_u}{2 - r_{u'}} \geq 0.$$

Turning finally to (iii), the optimal profit of an entreprenuer, given by $\pi_c = (p_c - c)\, y_c$, is decreasing in c due to the envelope theorem since prices are set optimally (and since quantities depend on those optimal prices). □

Appendix B.2 Proof of Proposition 2

Before proving Proposition 2, we establish the following lemma.

Lemma B.7. The individual consumption of the cutoff variety, $x_{\widehat{c}}$, negatively depends on the cutoff $\widehat{c}$ and on market size L.

Proof. For simplicity, we adopt the following notation: $\widehat{x} \equiv x_{\widehat{c}}$, $\widehat{u} \equiv u(\widehat{x})$, $\widehat{u}' \equiv u'(\widehat{x})$, and $\widehat{r}_u \equiv r_u(\widehat{x})$. The self-selection condition (henceforth, ssc), given by

$$\frac{\widehat{r}_u}{1 - \widehat{r}_u} \widehat{x} L \widehat{c} = 1 \tag{B.3}$$

determines the individual consumption $\widehat{x}$ as a function of $\widehat{c}$ and L. Moving $L\widehat{c}$ to the right-hand side and taking the partial elasticity of both sides with respect to $\widehat{c}$, we get:

$$e_{\widehat{c}}\left[\frac{\widehat{r}_u \widehat{x}}{1 - \widehat{r}_u}\right] = \left\{\mathcal{E}_{\widehat{x}}[\widehat{r}_u] + \frac{\widehat{r}_u}{1 - \widehat{r}_u}\mathcal{E}_{\widehat{x}}[\widehat{r}_u] + 1\right\} e_{\widehat{c}}[\widehat{x}] = e_{\widehat{c}}\left[\frac{1}{L\widehat{c}}\right] = -1.$$

Using Corollary A.2 in Appendix A, we then get: $e_{\widehat{c}}[\widehat{x}] = -(1 - \widehat{r}_u)/(2 - \widehat{r}_{u'}) < 0$. By symmetry of (B.3) in the partial elasticity of $\widehat{x}$ with respect to L, we have exactly the same expression: $e_L[\widehat{x}] = -(1 - \widehat{r}_u)/(2 - \widehat{r}_{u'}) < 0$. □

We can now prove our main result, namely existence and uniqueness of equilibrium.

Proof. For simplicity, we will adopt the following notations:

$$\widehat{x} \equiv x_{\widehat{c}},\ \widehat{u} \equiv u(\widehat{x}),\ \widehat{u}' \equiv u'(\widehat{x}),\ \widehat{u}'' \equiv u''(\widehat{x}),\ \widehat{r}_u \equiv r_u(\widehat{x}),\ \widehat{r}_{u'} \equiv r_{u'}(\widehat{x}),\ \underline{u} = u(x_{\underline{c}}).$$

The equilibrium conditions are given by (4), (9), and (12):

$$u'(x_c)\left[1-r_u(x_c)\right] = \mu c, \quad \forall c \in [\underline{c};\widehat{c}] \tag{B.4}$$

$$\mu = L\int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c \tag{B.5}$$

$$\frac{\widehat{r}_u}{1-\widehat{r}_u}\widehat{c}L\widehat{x} = 1 \tag{B.6}$$

We start by showing that if an equilibrium exists, it is unique.

First, we know that individual consumption of a type-c variety, x_c, is a decreasing function of c and μ (see equation (B.2) in Appendix B.1). As also shown in Lemma A.4 of Appendix A, the left-hand side of the equilibrium condition (B.4) is a decreasing function of $\{x_c\}_{c\in[\underline{c};\widehat{c}]}$. Therefore, x_c is a decreasing function of c and of μ.

Second, the right-hand side of equation (B.5) positively depends on L, $\widehat{c}$ and x_c, but negatively depends on $\underline{c}$. By the properties of the demand functions x_c stated above, the right-hand side of (B.5) is a decreasing function of μ and $\underline{c}$, and an increasing function of $\widehat{c}$ and L. Applying the implicit function theorem to (B.5), we obtain an upward sloping curve $\mu(\widehat{c})$, parametrized by L and $\underline{c}$. We denote it by $\mu = \mu_1\left(\widehat{c} \mid L, \underline{c}\right)$. Observe that L shifts that curve up, while $\underline{c}$ shifts it down. This curve reflects the 'changing intensity of competition' for any given cutoff: the larger the demand shifter, μ, the smaller individual demand. We henceforth refer to μ_1 as the *intensity of competition condition* (ICC).

Third, we know from equation (B.6) that $\widehat{x}$ depends negatively on $\widehat{c}$ and L (see Lemma B.7 above). Combining (B.4) and (B.6), we obtain the following condition: $\mu = \widehat{r}_u\widehat{u}'L\widehat{x}$. Alternatively, using the definition of $\widehat{r}_u$, we obtain μ as a decreasing function of $\widehat{x}$: $\mu = -\widehat{x}^2\widehat{u}''L$. Indeed, the right-hand side of that condition is a decreasing function of L and an increasing function of $\widehat{x}$, because $\mathcal{E}_{\widehat{x}}[-\widehat{x}^2\widehat{u}''L] = 2-\widehat{r}_{u'} > 0$. Hence, due to the properties of $\widehat{x}$, we have a downward sloping curve $\mu(\widehat{c})$ that is parametrized by L. We denote it by $\mu = \mu_2(\widehat{c} \mid L)$. Observe that an increase in L shifts that curve up. The function μ_2 reflects the self-selection of agents into entrepreneurship under a given intensity of competition μ: the bigger the intensity of competition, the smaller the cutoff $\widehat{c}$ and the lower the share of entrepreneurs. We henceforth refer to μ_2 as the *self-selection condition* (SSC).

Because μ_1 is an increasing function of $\widehat{c}$, whereas μ_2 is a decreasing function of $\widehat{c}$, we have established that the equilibrium is unique whenever it exists.

We now show that the equilibrium exists and is unique. First, note that from the ICC curve (B.5) we clearly have $\lim_{\widehat{c}\to\underline{c}}\mu = 0$ since utility is finite-valued everywhere. Since $u(x_c)$ decreases in c, we can replace $u(x_c)$ with a smaller value $u(x_{\widehat{c}})$ everywhere to get

$$\mu(\widehat{c}) = L\int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma(c)\mathrm{d}c \geq Lu(x_{\widehat{c}})\Gamma(\widehat{c}).$$

Taking the limit $\widehat{c} \to \overline{c}$, we then have $\mu(\overline{c}) \geq Lu(x_{\overline{c}})$. Turning to the SSC curve, combining (B.4) and (B.6), it can be rewritten as $\mu(\widehat{c}) = \widehat{r}_u \widehat{x} \widehat{u}' L = \widehat{r}_u \mathcal{E}_{\widehat{x}}[\widehat{u}] \widehat{u} L$. Since $r_u \leq 1$, $\mathcal{E}_x[u] \leq 1$, we have $\mu \leq \widehat{u} L$. Put differently, the SSC curve starts from some positive value, is decreasing, and does not exceed $\lim_{\widehat{c} \to \overline{c}} \mu = Lu(x_{\overline{c}})$. Hence, $\mu(\overline{c}) \leq Lu(x_{\overline{c}})$.

It follows from the foregoing that, by continuity, the ICC and SSC curves must intersect once (in the worst case, equilibrium is such that the cutoff $\widehat{c}$ coincides with the upper bound $\overline{c}$). $\square$

Appendix B.3 The Impact of Market Size on Firm Size

In this appendix, we characterize the elasticity of individual consumption, x_c, and of total output of a firm, $y_c = Lx_c$, with respect to market size L as a function of the elasticity of the cutoff $\widehat{c}$ with respect to market size.

Lemma B.8. The elasticity of the individual consumption $\widehat{x}$ of the cutoff variety with respect to market size is given by

$$\mathcal{E}_L[\widehat{x}] = -\frac{1 - \widehat{r}_u}{2 - r_{u'_c}} \cdot (1 + \mathcal{E}_L[\widehat{c}])\,. \tag{B.7}$$

The elasticity of individual consumption x of any variety with respect to L is given by

$$\mathcal{E}_L[x_c] = -\frac{1 - \widehat{r}_u}{\widehat{r}_u} \frac{\widehat{r}_u}{r_u(x_c)} \frac{1 - r_u(x_c)}{2 - r_{u'}(x_c)} \left(\frac{\widehat{r}_u}{1 - \widehat{r}_u} - \mathcal{E}_L[\widehat{c}] \right). \tag{B.8}$$

Proof. For simplicity, we use the following notation: $\widehat{x} \equiv x_{\widehat{c}}$, $\widehat{u} \equiv u\left(\widehat{x}\right)$, $\widehat{u}' \equiv u'\left(\widehat{x}\right)$, $\widehat{r}_u \equiv r_u\left(\widehat{x}\right)$. To prove (B.7), we can simply decompose the total elasticity $\mathcal{E}_L[\widehat{x}] = e_L[\widehat{x}] + e_{\widehat{c}}[\widehat{x}]\mathcal{E}_L[\widehat{c}]$ and use the expressions of the partial elasticities from Lemma B.7.

To prove (B.8), we make use of the condition $u'(x_c)[1 - r_u(x_c)] = c\mu$ applied to the individual consumption of the cutoff variety and the non-cutoff varieties to obtain the following relationship between them:

$$\frac{\left[1 - r_u\left(x_c\right)\right] u'\left(x_c\right)}{c} = \mu = \frac{\left(1 - \widehat{r}_u\right) \widehat{u}'}{\widehat{c}}$$

Taking the elasticity of the left-hand side with respect to L, using the result from Lemma A.4, and applying the chain rule, we have:

$$\mathcal{E}_L\left[\frac{\left(1 - r_u(x_c)\right) u'(x_c)}{c}\right] = \mathcal{E}_{x_c}\left[\left(1 - r_u(x_c)\right) u'(x_c)\right] \cdot \mathcal{E}_L[x_c] = -\frac{2 - r_{u'}(x_c)}{1 - r_u(x_c)} \cdot r_u(x_c) \cdot \mathcal{E}_L[x_c].$$

To obtain the elasticity of the right-hand side, we proceed in the same way but by taking into account the fact that the elasticity of $\widehat{c}$ with respect to L is non-zero. Using the expression for

$\mathcal{E}_L[\widehat{x}]$ from (B.7), we have:

$$\begin{aligned}\mathcal{E}_L\left[\frac{(1-\widehat{r}_u)\,\widehat{u}'}{\widehat{c}}\right] &= \mathcal{E}_{\widehat{c}}\left[(1-\widehat{r}_u)\,\widehat{u}'\right]\cdot\mathcal{E}_L[\widehat{x}]-\mathcal{E}_L[\widehat{c}] \\ &= \frac{2-\widehat{r}_{u'}}{1-\widehat{r}_u}\cdot\widehat{r}_u\cdot\frac{1-\widehat{r}_u}{2-\widehat{r}_{u'}}\cdot(1+\mathcal{E}_L[\widehat{c}])-\mathcal{E}_L[\widehat{c}]=\widehat{r}_u-(1-\widehat{r}_u)\cdot\mathcal{E}_L[\widehat{c}].\end{aligned}$$

Since the elasticities on both sides must be equal, we have:

$$-\frac{2-r_{u'}(x_c)}{1-r_u(x_c)}\cdot r_u(x_c)\cdot\mathcal{E}_L[x_c]=\widehat{r}_u-(1-\widehat{r}_u)\cdot\mathcal{E}_L[\widehat{c}].$$

Rearranging this expression, we get (B.8). □

Knowing the effect of market size on individual consumption, the total effect of market size on firm output $y_c = L_c x_c$ is simply given as follows:

$$\mathcal{E}_L[y_c]=1-\frac{1-\widehat{r}_u}{\widehat{r}_u}\frac{\widehat{r}_u}{r_u(x_c)}\frac{1-r_u(x_c)}{2-r_{u'}(x_c)}\left(\frac{\widehat{r}_u}{1-\widehat{r}_u}-\mathcal{E}_L[\widehat{c}]\right). \tag{B.9}$$

An increase in market size thus maps into the following changes in firms' output:

(*pointwise at* x)	DED $r'_u < 0$	$r'_u = 0$	IED $r'_u > 0$
$\widehat{c}\uparrow$	$y_c?$	$y_c\uparrow$	$y_c\uparrow$
$\mathrm{d}\widehat{c}=0$	$y_c\downarrow$	$y_c=\text{const}$	$y_c\uparrow$
$\widehat{c}\downarrow$	$y_c\downarrow$	$y_c\downarrow$	$y_c?$

If $\mathrm{d}r_{u'}(x)/\mathrm{d}x < 0$, a change in market size influences more strongly the output of the more efficient firms than that of the less efficient ones under IED. In the case of DED, the opposite result holds.

Appendix B.4 Proof of Proposition 3

In this appendix, we establish our main result which gives the elasticity of the self-selection cutoff, $\widehat{c}$, and the demand shifter, μ, with respect to market size L. Let us denote for simplicity:

$$\widehat{x}\equiv x_{\widehat{c}},\ \widehat{u}\equiv u(\widehat{x}),\ \widehat{u}'\equiv u'(\widehat{x}),\ \widehat{r}_u\equiv r_u(\widehat{x}),\ Y_{\widehat{c}}=L\int_{\underline{c}}^{\widehat{c}}u(x_c)\gamma_c\mathrm{d}c,$$

and

$$\widehat{\gamma}\equiv\gamma_{\widehat{c}},\ \widehat{\varGamma}\equiv\int_{\underline{c}}^{\widehat{c}}\gamma(c)\mathrm{d}c,\ U_{\widehat{c}}=\int_{\underline{c}}^{\widehat{c}}u(x_c)\gamma(c)\mathrm{d}c. \tag{B.10}$$

Using the condition $\mu\widehat{c} = (1-\widehat{r}_u)\widehat{u}'$ and $\mu = Y_{\widehat{c}}$, we get:

$$\mathcal{E}_L[\widehat{c}] = \mathcal{E}_L\left[(1-\widehat{r}_u)\widehat{u}'\right] - \mathcal{E}_L\left[Y_{\widehat{c}}\right], \tag{B.11}$$

so that

$$\mathcal{E}_L\left[\widehat{u}'(1-\widehat{r}_u)\right] = \mathcal{E}_{\widehat{x}}[\widehat{u}'(1-\widehat{r}_u)]\mathcal{E}_L[\widehat{x}].$$

Using Lemma A.4 from Appendix A and expression (B.7), we have

$$\mathcal{E}_L\left[\widehat{u}'\left(1-\widehat{r}_u\right)\right] = \widehat{r}_u\left(1+\mathcal{E}_L\left[\widehat{c}\right]\right) \tag{B.12}$$

Recalling that $\mu = Y_{\widehat{c}} = LU_{\widehat{c}}$, we have:

$$\mathcal{E}_L[\mu] = 1 + \mathcal{E}_L[U_{\widehat{c}}]. \tag{B.13}$$

We first decompose the full elasticity $\mathcal{E}_L[U_{\widehat{c}}]$ into two partial elasticities as follows:

$$\mathcal{E}_L[U_{\widehat{c}}] \quad = \quad e_L[U_{\widehat{c}}] + e_{\widehat{c}}[U_{\widehat{c}}]\cdot\mathcal{E}_L[\widehat{c}], \tag{B.14}$$

where

$$e_{\widehat{c}}[U_{\widehat{c}}] = \frac{\partial U_{\widehat{c}}}{\partial \widehat{c}}\cdot\frac{\widehat{c}}{U_{\widehat{c}}} = \frac{\widehat{u}\widehat{\gamma c}}{U_{\widehat{c}}} = \frac{\widehat{u}}{\widetilde{u}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}} \tag{B.15}$$

and where $\widetilde{u} \equiv \frac{U_{\widehat{c}}}{\widehat{\Gamma}}$ can be interpreted as the conditional expectation of utility. The first partial elasticity is given by

$$e_L[U_{\widehat{c}}] \quad = \quad \frac{\partial U_{\widehat{c}}}{\partial L}\cdot\frac{L}{U_{\widehat{c}}} = \frac{L}{U_{\widehat{c}}}\int_{\underline{c}}^{\widehat{c}} u'(x_c)\mathcal{E}_L[x_c]\frac{x_c}{L}\gamma_c \mathrm{d}c$$

Letting

$$J_{\widehat{c}} = \int_{\underline{c}}^{\widehat{c}} \frac{1}{r_u(x_c)} u(x_c)\mathcal{E}_{x_c}[u(x_c)]\cdot\frac{1-r_u\left(x_c\right)}{2-r_{u'}(x_c)} L\gamma_c \mathrm{d}c \tag{B.16}$$

and using (B.8),we have:

$$\begin{aligned}
e_L[U_{\widehat{c}}] \quad &= \quad \frac{1}{U_{\widehat{c}}}\int_{\underline{c}}^{\widehat{c}} u(x_c)\mathcal{E}_{x_c}[u(x_c)]\mathcal{E}_L[x_c]\gamma_c \mathrm{d}c \\
&= \quad \frac{1}{U_{\widehat{c}}}\int_{\underline{c}}^{\widehat{c}} u(x_c)\mathcal{E}_{x_c}[u(x_c)]\frac{1-\widehat{r}_u}{\widehat{r}_u}\frac{\widehat{r}_u}{r_u(x_c)}\frac{1-r_u(x_c)}{2-r_{u'}(x_c)}\left(\mathcal{E}_L[\widehat{c}] - \frac{\widehat{r}_u}{1-\widehat{r}_u}\right)\gamma_c \mathrm{d}c \\
&= \quad (1-\widehat{r}_u)\left(\mathcal{E}_L[\widehat{c}] - \frac{\widehat{r}_u}{1-\widehat{r}_u}\right)\frac{J_{\widehat{c}}}{Y_{\widehat{c}}}.
\end{aligned}$$

Using equations (B.16) and (B.17), we can express the equality (B.14) as follows:

$$\mathcal{E}_L[U_{\widehat{c}}] = \left[\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}} + (1 - \widehat{r}_u)\,\frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\right] \cdot \mathcal{E}_L\left[\widehat{c}\right] - \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\widehat{r}_u. \tag{B.17}$$

Using (B.13), we then have:

$$\mathcal{E}_L[\mu] = 1 + \left[\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}} + (1 - \widehat{r}_u)\,\frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\right] \cdot \mathcal{E}_L[\widehat{c}] - \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\widehat{r}_u. \tag{B.18}$$

Using (B.11), (B.12), and (B.18) we also have:

$$\mathcal{E}_L[\widehat{c}] = \widehat{r}_u + \widehat{r}_u \cdot \mathcal{E}_L[\widehat{c}] - 1 - \left[\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}} + (1 - \widehat{r}_u)\,\frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\right] \cdot \mathcal{E}_L[\widehat{c}] + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\widehat{r}_u.$$

Rearranging this expression, we get:

$$\mathcal{E}_L\left[\widehat{c}\right] = \frac{\widehat{r}_u - 1 + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\widehat{r}_u}{1 - \widehat{r}_u + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\left(1 - \widehat{r}_u\right) + \frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}} = \frac{\widehat{r}_u}{1 - \widehat{r}_u}\frac{1 - \frac{1}{\widehat{r}_u} + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}}{1 + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}} + \frac{1}{1 - \widehat{r}_u}\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}}. \tag{B.19}$$

As a result, we have:

$$\mathcal{E}_L\left[\widehat{c}\right] \gtreqless 0 \quad \Leftrightarrow \quad I_{\widehat{c}} \gtreqless 0, \quad \text{with} \quad I_{\widehat{c}} \equiv \left(\widehat{r}_u - 1\right) Y_{\widehat{c}} + \widehat{r}_u J_{\widehat{c}}.$$

Using expressions (B.10) and (B.16) for $Y_{\widehat{c}}$ and $J_{\widehat{c}}$, we then have

$$I_{\widehat{c}} = \int_{\underline{c}}^{\widehat{c}} \left[\left(\widehat{r}_u - 1\right) u(x_c) + u(x_c)\mathcal{E}_u[x_c]\frac{\widehat{r}_u}{r_u(x_c)}\frac{1 - r_u(x_c)}{2 - r_{u'}(x_c)}\right] \gamma_c \mathrm{d}c$$

It can be readily verified that

$$\underbrace{\frac{\mathcal{E}_x[u]}{1 - r_u}}_{A}\underbrace{\frac{1 - r_u}{2 - r_{u'}}\frac{1 - r_u}{1 - \widehat{r}_u}\frac{\widehat{r}_u}{r_u} \gtreqless 1}_{B} \quad \Rightarrow \quad I_{\widehat{c}} \gtreqless 0 \quad \Leftrightarrow \quad \mathcal{E}_L[\widehat{c}] \gtreqless 0.$$

Let us examine the term B first. From Corollary A.3 of Appendix A, we know that

$$\frac{1 - r_u}{2 - r_{u'}} \lesseqgtr 1 \quad \Leftrightarrow \quad r_u' \gtreqless 0,$$

and from Proposition 1, we know that $x_c > x_{\widehat{c}}$ for all $c < \widehat{c}$. Hence, $r'_u \gtreqless 0 \Leftrightarrow B \lesseqgtr 1$. Let us examine the term A next. From Corollary A.6 of Appendix A, we have:

$$(\mathcal{E}_x[u])' \gtreqless 0 \quad \Leftrightarrow \quad r_{\ln u} \lesseqgtr 1 \quad \Leftrightarrow \quad \frac{\mathcal{E}_x[u]}{1 - r_u} \lesseqgtr 1,$$

which implies that

$$(\mathcal{E}_x\left[u\right])' \gtreqless 0 \quad \Leftrightarrow \quad A \lesseqgtr 1.$$

Two possible cases can arise, depending on the signs of r'_u and $(\mathcal{E}_x\left[u\right])'$.

Case 1: $r'_u \leq 0$ and $(\mathcal{E}_x\left[u\right])' \leq 0$, in which case

$$A \geq 1, \; B \geq 1 \quad \Rightarrow \quad \mathcal{E}_L\left[\widehat{c}\right] \geq 0.$$

Case 2: $r'_u \geq 0$ and $(\mathcal{E}_x\left[u\right])' \geq 0$, in which case

$$A \leq 1, \; B \leq 1 \quad \Rightarrow \quad \mathcal{E}_L\left[\widehat{c}\right] \leq 0.$$

Note that only when $A = B = 1$, we have $\mathcal{E}_L[\widehat{c}] = 0$. This special case corresponds to u being a power function, i.e., the case of CES preferences. The ambiguous cases are when $A > 1$ and $B < 1$, which is equivalent to $(\mathcal{E}_x\left[u\right])' < 0$ and $r'_u > 0$; and when $A < 1$ and $B > 1$, which is equivalent to $(\mathcal{E}_x\left[u\right])' > 0$ and $r'_u < 0$. Recalling that $\mathcal{E}_L[\mu] = \mathcal{E}_L[Y_{\widehat{c}}]$, and combining (B.11), (B.12), and (B.19), we finally have:

$$\mathcal{E}_L[\mu] = \widehat{r}_u\left(1 + \mathcal{E}_L[\widehat{c}]\right) - \mathcal{E}_L[\widehat{c}] = \widehat{r}_u - \left(1 - \widehat{r}_u\right)\mathcal{E}_L[\widehat{c}] = \frac{1 + \frac{\widehat{r}_u}{1-\widehat{r}_u}\frac{\widehat{u}}{\widehat{\overline{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}}{1 + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}} + \frac{1}{1-\widehat{r}_u}\frac{\widehat{u}}{\widehat{\overline{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}} \geq 0. \tag{B.20}$$

Thus, the size elasticity of the demand shifter μ is bounded as follows:

$$\mathcal{E}_L[\mu] \in \begin{cases} \left[r_u(x_{\widehat{c}}); 1\right], & \text{if} \quad \mathcal{E}_L[\widehat{c}] \leq 0 \\ \left[0; r_u(x_{\widehat{c}})\right], & \text{if} \quad \mathcal{E}_L[\widehat{c}] \geq 0 \end{cases} \tag{B.21}$$

Appendix B.5 Proof of Proposition 5

Proof. We show that income inequality among entrepreneurs, measured by the 'interquartile range' between c_1 and c_2, increases if and only if $r'_u \geq 0$, whereas inequality does not change if and only if r_u is a constant, i.e., preferences are CES. To see this, consider the impact of market size on the ratio π_{c_1}/π_{c_2} of profits for two arbitrary types of entrepreneurs, where $1/c_1 > 1/c_2$

so that $x_{c_1} > x_{c_2}$. We then have

$$\mathcal{E}_L\left[\frac{\pi_{c_1}}{\pi_{c_2}}\right] = \mathcal{E}_L[\pi_{c_1}] - \mathcal{E}_L[\pi_{c_2}] = \left[\frac{1}{r_u\left(x_{c_2}\right)} - \frac{1}{r_u\left(x_{c_1}\right)}\right](1-\widehat{r}_u)\left(\frac{\widehat{r}_u}{1-\widehat{r}_u} - \mathcal{E}_L[\widehat{c}]\right). \tag{B.22}$$

Since $\mathcal{E}_L[\widehat{c}] \leq \frac{\widehat{r}_u}{1-\widehat{r}_u}$ (see equation (B.20) in Appendix B.4), and since $r_u \leq 1$, this expression is positive if and only if $r_u' \geq 0$. $\square$

Appendix C Shocks

In this appendix, we present two basic extensions of the model. In the first one, we consider a common shock to the weight consumers attach to the differentiated good. In the second one, we consider a common shock to the ability levels of entrepreneurs.

Appendix C.1 Preference Shocks

Assume that consumers attach a weight $\beta > 0$ to the consumption of the differentiated good, so that preferences are now given by:

$$\mathcal{U} \equiv \beta L \int_{\underline{c}}^{\widehat{c}} u\big(x(c)\big)\gamma_c \mathrm{d}c + A. \tag{C.1}$$

We first show that positive preference shocks – i.e., an increase in β – increase the share of entrepreneurs and the selection cutoff. We then show that they also increase individual consumption, firm size, firm profits, and income inequality in the economy.

Share of entrepreneurs. According to the self-selection condition $\frac{\widehat{r}_u}{1-\widehat{r}_u}\widehat{c}L\widehat{x} = 1$, the consumption $\widehat{x}$ of the cutoff variety is a decreasing function of $\widehat{c}$, parametrized by L: $\mathcal{E}_{\widehat{c}}[\widehat{x}] = -\frac{1-\widehat{r}_u}{2-\widehat{r}_{u'}} < 0$ (see Lemma B.7 in Appendix B.2). Hence, using the chain rule for elasticities, we have

$$\mathcal{E}_\beta[\widehat{x}] = -\frac{1-\widehat{r}_u}{2-\widehat{r}_{u'}}\mathcal{E}_\beta[\widehat{c}].$$

Straightforward calculations, using the results from Appendix B.3 and Appendix B.4, show that the condition $\frac{(1-r_u(x_c))u'(x_c)}{c} = \frac{(1-\widehat{r}_u)\widehat{u}'}{\widehat{c}}$ can be rewritten as follows in terms of elasticities:

$$\mathcal{E}_\beta\left[\frac{\left(1-r_u\left(x_c\right)\right)u'\left(x_c\right)}{c}\right] = -\frac{2-r_{u'}\left(x_c\right)}{1-r_u\left(x_c\right)}\cdot r_u\left(x_c\right)\cdot\mathcal{E}_\beta\left[x_c\right]$$

$$\mathcal{E}_\beta\left[\frac{\left(1-\widehat{r}_u\right)\widehat{u}'}{\widehat{c}}\right] = \mathcal{E}_{\widehat{c}}\left[\left(1-\widehat{r}_u\right)\widehat{u}'\right]\cdot\mathcal{E}_\beta[\widehat{x}] - \mathcal{E}_\beta[\widehat{c}] = \frac{2-\widehat{r}_{u'}}{1-\widehat{r}_u}\cdot\widehat{r}_u\cdot\frac{1-\widehat{r}_u}{2-\widehat{r}_{u'}}\mathcal{E}_\beta[\widehat{c}] - \mathcal{E}_\beta[\widehat{c}] = -(1-\widehat{r}_u)\cdot\mathcal{E}_\beta[\widehat{c}],$$

so that

$$\mathcal{E}_{\beta}\left[x_{c}\right]=\frac{1-r_{u}\left(x_{c}\right)}{2-r_{u'}\left(x_{c}\right)}\cdot\left(1-\widehat{r}_{u}\right)\cdot\mathcal{E}_{\beta}[\widehat{c}]. \tag{C.2}$$

We can then show that positive preference shocks increase the share of entrepreneurs in the economy. To see this, first use $\mu/\beta=(1-\widehat{r}_u)\widehat{u}'/\widehat{c}$ and $\mu=L\int_{\underline{c}}^{\widehat{c}}u(x_c)\gamma_c\mathrm{d}c$ to get

$$\mathcal{E}_{\beta}[\widehat{c}]=\mathcal{E}_{\beta}\left[(1-\widehat{r}_u)\widehat{u}'\right]-\mathcal{E}_{\beta}[\mu]+1. \tag{C.3}$$

Using the calculations and notations from Appendix B.4, we have:

$$\mathcal{E}_{\beta}\left[\widehat{c}\right]=\widehat{r}_u\mathcal{E}_{\beta}\left[\widehat{c}\right]-\mathcal{E}_{\beta}\left[U_{\widehat{c}}\right]+1, \tag{C.4}$$

where

$$\mathcal{E}_{\beta}[U_{\widehat{c}}]=e_{\beta}[U_{\widehat{c}}]+e_{\widehat{c}}[U_{\widehat{c}}]\cdot\mathcal{E}_{\beta}[\widehat{c}]=\int_{\underline{c}}^{\widehat{c}}u(x_c)\mathcal{E}_{x_c}[u(x_c)]\mathcal{E}_{\beta}[x_c]\frac{1}{\beta}\gamma_c\mathrm{d}c\cdot\frac{\beta}{U_{\widehat{c}}}+\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}\mathcal{E}_{\beta}[\widehat{c}].$$

Using (C.2), we have:

$$\begin{aligned}\mathcal{E}_{\beta}[U_{\widehat{c}}] &= \frac{1}{U_{\widehat{c}}}\int_{\underline{c}}^{\widehat{c}}u(x_c)\mathcal{E}_{x_c}[u(x_c)]\frac{1-r_u(x_c)}{2-r_{u'}(x_c)}\cdot(1-\widehat{r}_u)\cdot\mathcal{E}_{\beta}[\widehat{c}]\cdot\gamma_c\mathrm{d}c+\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}\mathcal{E}_{\beta}[\widehat{c}] \\ &= \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}(1-\widehat{r}_u)\cdot\mathcal{E}_{\beta}[\widehat{c}]+\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}\mathcal{E}_{\beta}[\widehat{c}]. \end{aligned} \tag{C.5}$$

Substituting (C.5) into (C.4), we then have:

$$\mathcal{E}_{\beta}[\widehat{c}]=\widehat{r}_u\mathcal{E}_{\beta}[\widehat{c}]-\frac{J_{\widehat{c}}}{Y_{\widehat{c}}}(1-\widehat{r}_u)\cdot\mathcal{E}_{\beta}[\widehat{c}]-\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}\mathcal{E}_{\beta}[\widehat{c}]+1,$$

which finally yields

$$\mathcal{E}_{\beta}[\widehat{c}]=\frac{1}{\left(1+\frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\right)(1-\widehat{r}_u)+\frac{\widehat{u}}{\widetilde{\widetilde{u}}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}}>0. \tag{C.6}$$

Firm size and consumption. A positive preference shock increases firm size and individual consumption. This can be directly seen from (C.2) and (C.6).

Firm profits and inequality. A positive preference shock increases firm profits. To see this, observe that the profit of a type-c entrepreneur at equilibrium is given by $\pi_c=(p_c-c)\,x_cL=$

$\frac{r_u(x_c)}{1-r_u(x_c)}cLx_c$. Hence, the elasticity of profit with respect to β is

$$\mathcal{E}_\beta[\pi] = \mathcal{E}_\beta[x] + \left(\mathcal{E}_x[r_u] - \mathcal{E}_x[1-r_u]\right)\mathcal{E}_\beta[x] = \left(1 + \mathcal{E}_x[r_u] + \frac{r_u}{1-r_u}\mathcal{E}_x[r_u]\right)\mathcal{E}_\beta[x] = \frac{1-r_u}{2-r_{u'}}\mathcal{E}_\beta[x]. \tag{C.7}$$

As shown before, a positive preference shock increases individual consumption (C.2) and increases the cutoff (C.6). Hence, the profit for any type of agent increases, and inequality will increase too.

Appendix C.2 Productivity Shocks

Assume that all entrepreneurs experience the same (ex post) shock a to their marginal cost, i.e., the productivity changes from $1/c$ to $1/ac$. We may view such a shock as a change in institutions or regulations that affect all active entrepreneurs in the market. We now show that such a shock is formally equivalent to an increase in market size L. To see this, observe that the optimal price for a type-c variety is given by: $p_c = \frac{ac}{1-r_u(x_c)}$. Hence, the optimal individual consumpion of a type-c variety is determined by $[1-r_u(x_c)]\, u'(x_c) = a\mu c$. The demand shifter μ is given as before by $\mu = L\int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c$. The new selection condition is now given by:

$$\frac{r_u\left(x_{\widehat{c}}\right) x_{\widehat{c}}}{1-r_u\left(x_{\widehat{c}}\right)} a\widehat{c}L = 1.$$

Thus, the system of equlibrium conditions with productivity shocks is of the following form:

$$\begin{cases} \dfrac{u'(x_c)\left(1-r_u(x_c)\right)}{c} = \dfrac{u'(x_{\widehat{c}})\left(1-r_u(x_{\widehat{c}})\right)}{\widehat{c}}, \quad \forall c \in [\underline{c}; \widehat{c}] \\ \dfrac{u'(x_{\widehat{c}})\left(1-r_u(x_{\widehat{c}})\right)}{\widehat{c}} = aL\displaystyle\int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c \\ \dfrac{r_u(x_{\widehat{c}})x_{\widehat{c}}}{1-r_u(x_{\widehat{c}})} = \dfrac{1}{a\widehat{c}L} \end{cases}$$

It is easy to see that the exogenous parameters a and L always jointly enter the equilibrium conditions, thus making them isomorphic. This proves our statement.

Appendix D Supplemental Material

In this appendix, we derive some additional results when the upper-tier utility function V is not logarithmic. We first explain why an interior equilibrium may not exist in that case. We then derive the effects of market size on the share of entrepreneurs assuming that an interior equilibrium exists. Finally, we establish some results concerning the impact of a shift in the underlying ability support on the share of entrepreneurs. We also prove that the share of entrepreneurs is invariant to such a shift in the distribution in the CES case, regardless of the underlying ability distribution.

Appendix D.1 Non-existence of an Interior Equilibrium

Let us adopt the following notation to ease the exposition:

$$\widehat{x} \equiv x_{\widehat{c}},\ \widehat{u} \equiv u(\widehat{x}),\ \widehat{u}' \equiv u'(\widehat{x}),\ \widehat{u}'' \equiv u''(\widehat{x}),\ \widehat{r}_u \equiv r_u(\widehat{x}),\ \widehat{r}_{u'} \equiv r_{u'}(\widehat{x}),\ \underline{u} = u(x_{\underline{c}}).$$

With an arbitrary upper-tier utility function, the system of equilibrium conditions is given as follows:

$$\begin{cases} u'(x_c)[1 - r_u(x_c)] = c\mu, & \forall c \in [\underline{c}; \widehat{c}] \\ \mu = 1/V'\left(L\int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c\right) & \\ \frac{\widehat{r}_u}{1-\widehat{r}_u}\widehat{x}L\widehat{c} = 1. & \end{cases} \tag{D.1}$$

Note that the only difference with the case where V is logarithmic is in the second equation. Since $V''(\cdot) < 0$, the term under $V(\cdot)$ on the right-hand side of the second equation positively depends on L, $\widehat{c}$ and x_c, but negatively on $\underline{c}$ just as in the case where $V \equiv \ln$. Hence, applying the implicit function theorem, we again have an upwarding curve of μ and $\widehat{c}$, parametrized by L and $\underline{c}$: $\mu = \mu_1\left(\widehat{c} \mid L; \underline{c}\right)$. From the other conditions, we derive the same curve $\mu_2 = \mu_2(\widehat{c} \mid L)$ as in the case with log preferences. As in the benchmark case with a logarithmic upper-tier utility, the functions μ_1 and μ_2 are increasing and decreasing in $\widehat{c}$, respectively, which ensures that the equilibrium is unique *if it exists*. The latter need, however, not be the case for a general upper-tier utility function $V(\cdot)$.

To understand why this is so, observe that if $V'(0) > 0$ then $\mu_1(\underline{c}) = 1/V'(0)$. The latter may be greater or equal to $\mu_2(\underline{c})$, which means that only a corner solution with $\widehat{c} = 0$ exists. In words, nobody wants to become an entrepreneur, because entrepreneurship is less profitable than salaried work irrespective of the types of the agents. Figure 5 illustrates a case where the μ_1 curve shifts up substantially so that the self-selection cutoff is very close to $\underline{c}$, i.e., the share of entrepreneurs is very small. It is easy to construct cases where the shift is even larger, in which case we hit a corner solution where no agent will produce the differentiated good since doing so is simply not profitable enough.

Appendix D.2 Market Size and the Share of Entrepreneurs

To simplify notation, we define the following expressions:

$$\widehat{x} \equiv x_{\widehat{c}},\ \widehat{u} \equiv u(\widehat{x}),\ \widehat{u}' \equiv u'(\widehat{x}),\ \widehat{r}_u \equiv r_u(\widehat{x}),\ \widehat{V'} \equiv V'(Y_{\widehat{c}}),\ \widehat{r}_V \equiv r_V(Y_{\widehat{c}})$$

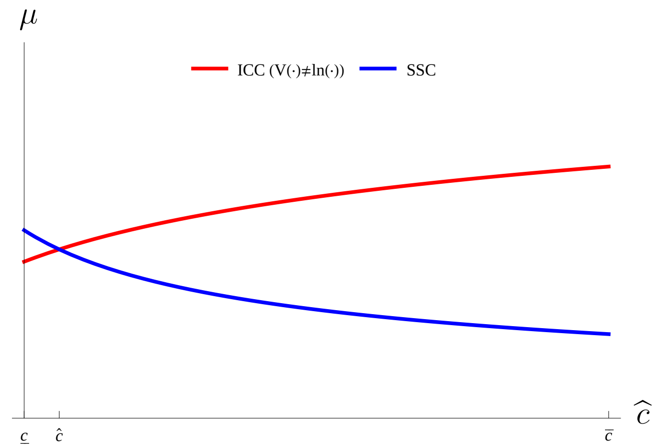

Figure 5: Possible Non-Existence of an Interior Equilibrium.

$$Y_{\widehat{c}} = L \int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma(c)\mathrm{d}c,\ \widehat{\gamma} \equiv \gamma_{\widehat{c}},\ \widehat{\Gamma} \equiv \int_{\underline{c}}^{\widehat{c}} \gamma(c)\mathrm{d}c,\ U_{\widehat{c}} = \int_{\underline{c}}^{\widehat{c}} u(x_c)\gamma(c)\mathrm{d}c. \tag{D.2}$$

We now show that our key results concerning market size and the share of entrepreneurs basically extend to an arbitrary upper-tier utility function V, provided an interior equilibrium exists. In that case, we can establish the following comparative static results of $\widehat{c}$ and μ with respect to market size L:

Proposition D.9 (Market Size and the Share of Entrepreneurs). The direction of change in the self-selection cutoff, $\widehat{c}$, depends on the properties of the upper- and the lower-tier utility functions, including the difference of their RLVs:

$$\mathcal{E}_L[\widehat{c}] \gtreqqless 0 \quad \Leftrightarrow \quad \frac{1}{r_u(x_{\widehat{c}})} - \frac{1}{r_V(Y_{\widehat{c}})} \lesseqqgtr \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}.$$

Both elasticities cannot change too fast since they are bounded as follows when $r_V(Y_{\widehat{c}}) \leq 1$:

$$\mathcal{E}_L[\widehat{c}] \in \left[-r_V(Y_{\widehat{c}}); \frac{r_u(x_{\widehat{c}})}{1 - r_u(x_{\widehat{c}})}\right] \tag{D.3}$$

$$\mathcal{E}_L[\mu] \in \begin{cases} \left[r_u(x_{\widehat{c}}); r_u(x_{\widehat{c}}) + (1 - r_u(x_{\widehat{c}}))\, r_V(x_{\widehat{c}})\right], & \mathcal{E}_L[\widehat{c}] \leq 0 \\ \left[0; r_u(x_{\widehat{c}})\right], & \mathcal{E}_L[\widehat{c}] \geq 0 \end{cases} \tag{D.4}$$

Proof. Using $\mu\widehat{c} = (1 - \widehat{r}_u)\,\widehat{u}'$ and $\mu = 1/\widehat{V'}$, we get:

$$\mathcal{E}_L[\widehat{c}] = \mathcal{E}_L\left[(1 - \widehat{r}_u)\,\widehat{u}'\right] + \mathcal{E}_L[\widehat{V'}]. \tag{D.5}$$

Decomposing the elasticity, we have $\mathcal{E}_L\left[\widehat{u}'(1 - \widehat{r}_u)\right] = \mathcal{E}_{\widehat{x}}\left[\widehat{u}'(1 - \widehat{r}_u)\right]\mathcal{E}_L[\widehat{x}]$. Using Lemma A.4

of Appendix A, as well as (B.7) of Appendix B.3, we have:

$$\mathcal{E}_L\left[\widehat{u}'(1-\widehat{r}_u)\right] = \widehat{r}_u\left(1+\mathcal{E}_L[\widehat{c}]\right). \tag{D.6}$$

Consider next the elasticity $\mathcal{E}_L[\widehat{V'}] = \mathcal{E}_{Y_{\widehat{c}}}[\widehat{V'}]\cdot\mathcal{E}_L[Y_{\widehat{c}}]$. Since $\mathcal{E}_{Y_{\widehat{c}}}[\widehat{V'}] = -\widehat{r}_V$ and $\mathcal{E}_L[Y_{\widehat{c}}] = 1+\mathcal{E}_L[U_{\widehat{c}}]$, we readily obtain

$$\mathcal{E}_{Y_{\widehat{c}}}[\widehat{V'}] = -\widehat{r}_V\cdot\left(1+\mathcal{E}_L[U_{\widehat{c}}]\right). \tag{D.7}$$

First, decompose the full elasticity $\mathcal{E}_L[U_{\widehat{c}}]$ into two partial elasticities as follows:

$$\mathcal{E}_L[U_{\widehat{c}}] = e_L[U_{\widehat{c}}] + e_{\widehat{c}}[U_{\widehat{c}}]\cdot\mathcal{E}_L[\widehat{c}], \tag{D.8}$$

where

$$e_{\widehat{c}}[U_{\widehat{c}}] = \frac{\partial U_{\widehat{c}}}{\partial\widehat{c}}\cdot\frac{\widehat{c}}{U_{\widehat{c}}} = \frac{\widehat{u\gamma c}}{U_{\widehat{c}}} = \frac{\widehat{u}}{\widetilde{u}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}}, \tag{D.9}$$

and where $\widetilde{u} = U_{\widehat{c}}/\widehat{\Gamma}$ is the conditional expectation of utility. We further have

$$e_L[U_{\widehat{c}}] \;=\; \frac{\partial U_{\widehat{c}}}{\partial L}\cdot\frac{L}{U_{\widehat{c}}} = \int_{\underline{c}}^{\widehat{c}} u'(x_c)\mathcal{E}_L[x_c]\frac{x_c}{L}\gamma_c\mathrm{d}c\cdot\frac{L}{U_{\widehat{c}}}.$$

Using (B.8) and (D.2), we have:

$$\begin{aligned}
e_L[U_{\widehat{c}}] &= \frac{1}{U_{\widehat{c}}}\int_{\underline{c}}^{\widehat{c}} u(x_c)\mathcal{E}_{x_c}[u(x_c)]\cdot\mathcal{E}_L[x_c]\cdot\gamma_c\mathrm{d}c \\
&= \frac{1}{U_{\widehat{c}}}\int_{\underline{c}}^{\widehat{c}} u(x_c)\mathcal{E}_{x_c}[u(x_c)]\cdot\frac{1-\widehat{r}_u}{\widehat{r}_u}\frac{\widehat{r}_u}{r_u(x_c)}\frac{1-r_u(x_c)}{2-r_{u'}(x_c)}\left(\mathcal{E}_L[\widehat{c}]-\frac{\widehat{r}_u}{1-\widehat{r}_u}\right)\cdot\gamma_c\mathrm{d}c \\
&= (1-\widehat{r}_u)\left(\mathcal{E}_L[\widehat{c}]-\frac{\widehat{r}_u}{1-\widehat{r}_u}\right)\frac{J_{\widehat{c}}}{Y_{\widehat{c}}},
\end{aligned}$$

where

$$J_{\widehat{c}} = \int_{\underline{c}}^{\widehat{c}}\frac{u_c(x_c)\mathcal{E}_u(x_c)}{r_u(x_c)}x_c\cdot\frac{1-r_u(x_c)}{2-r_{u'}(x_c)}L\gamma_c\mathrm{d}c. \tag{D.10}$$

Using equations (D.9) and (D.10), we can express equation (D.8) in the following form:

$$\mathcal{E}_L[U_{\widehat{c}}] = \left[\frac{\widehat{u}}{\widetilde{u}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}} + (1-\widehat{r}_u)\frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\right]\cdot\mathcal{E}_L[\widehat{c}] - \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\widehat{r}_u. \tag{D.11}$$

Using (D.7), we then have:

$$\mathcal{E}_L[\widehat{V'}] = -\widehat{r}_V - \widehat{r}_V\cdot\left(\frac{\widehat{u}}{\widetilde{u}}\frac{\widehat{\gamma c}}{\widehat{\Gamma}} + (1-\widehat{r}_u)\frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\right)\cdot\mathcal{E}_L[\widehat{c}] + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\widehat{r}_u\widehat{r}_V. \tag{D.12}$$

Using (B.11), (D.6) and (D.12) we further have:

$$\mathcal{E}_L[\widehat{c}] = \widehat{r}_u + \widehat{r}_u \cdot \mathcal{E}_L[\widehat{c}] - \widehat{r}_V - \widehat{r}_V \cdot \left(\frac{\widehat{u}}{\widetilde{\widetilde{u}}} \frac{\widehat{\gamma c}}{\widehat{\Gamma}} + (1 - \widehat{r}_u) \frac{J_{\widehat{c}}}{Y_{\widehat{c}}} \right) \cdot \mathcal{E}_L[\widehat{c}] + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}} \widehat{r}_u \widehat{r}_V.$$

Rearranging this expression, we get:

$$\mathcal{E}_L[\widehat{c}] = \frac{\widehat{r}_u}{1 - \widehat{r}_u} \frac{\frac{1}{\widehat{r}_V} - \frac{1}{\widehat{r}_u} + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}}{\frac{1}{\widehat{r}_V} + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}} + \frac{1}{1-\widehat{r}_u} \frac{\widehat{u}}{\widetilde{\widetilde{u}}} \frac{\widehat{\gamma c}}{\widehat{\Gamma}}}. \tag{D.13}$$

In the case where $V(\cdot)$ is linear, i.e., where $r_V \equiv 0$, we have $\mathcal{E}_L[\widehat{c}] = \frac{\widehat{r}_u}{1-\widehat{r}_u}$. In general

$$\mathcal{E}_L[\widehat{c}] \gtreqqless 0 \quad \Leftrightarrow \quad \frac{1}{\widehat{r}_u} - \frac{1}{\widehat{r}_V} \lesseqqgtr \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}$$

and, therefore,

$$\mathcal{E}_L[\widehat{c}] - \frac{\widehat{r}_u}{1 - \widehat{r}_u} = -\frac{\widehat{r}_u}{1 - \widehat{r}_u} \frac{\frac{1}{\widehat{r}_u} + \frac{1}{1-\widehat{r}_u} \frac{\widehat{u}}{\widetilde{\widetilde{u}}} \frac{\widehat{\gamma c}}{\widehat{\Gamma}}}{\frac{1}{\widehat{r}_V} + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}} + \frac{1}{1-\widehat{r}_u} \frac{\widehat{u}}{\widetilde{\widetilde{u}}} \frac{\widehat{\gamma c}}{\widehat{\Gamma}}} \leq 0. \tag{D.14}$$

If $\widehat{r}_V \leq 1$ then:

$$\mathcal{E}_L[\widehat{c}] + \widehat{r}_V = \frac{\widehat{r}_u}{1 - \widehat{r}_u} \frac{\frac{1}{\widehat{r}_V} - 1 + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}} \left(1 + \left(\frac{1}{\widehat{r}_u} - 1\right) \frac{1}{\widehat{r}_V}\right) + \frac{1}{\widehat{r}_V} \frac{\widehat{u}}{\widetilde{\widetilde{u}}} \frac{\widehat{\gamma c}}{\widehat{\Gamma}}}{\frac{1}{\widehat{r}_V} + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}} + \frac{1}{1-\widehat{r}_u} \frac{\widehat{u}}{\widetilde{\widetilde{u}}} \frac{\widehat{\gamma c}}{\widehat{\Gamma}}} \geq 0. \tag{D.15}$$

□

Appendix D.3 Changes in the Curvature of $u(\cdot)$ as a Function of the Consumption Level x

As emphasized in the main text, our propositions are 'local' in the sense that they depend on the equilibrium consumption level. Hence, the equilibria of the model under the same preference structure may display qualitatively different behavior shoud $u(\cdot)$ not be IED or DED over the whole range of consumption levels.

As an example of a subutility function that switches regime for both the scale elasticity and the RLV, consider the following 'augmented CARA' function:[23] $u(x) = 1 - \mathrm{e}^{-ax} + bx$. It is readily verified that

$$r_u(x) = \frac{a^2 x \mathrm{e}^{-ax}}{b + a\mathrm{e}^{-ax}} \quad \text{and} \quad \mathcal{E}_u[x] = -\frac{(b + a\mathrm{e}^{-ax})x}{1 - \mathrm{e}^{-ax} + bx}. \tag{D.16}$$

[23] We thank Sergey Kokovin for finding this example.

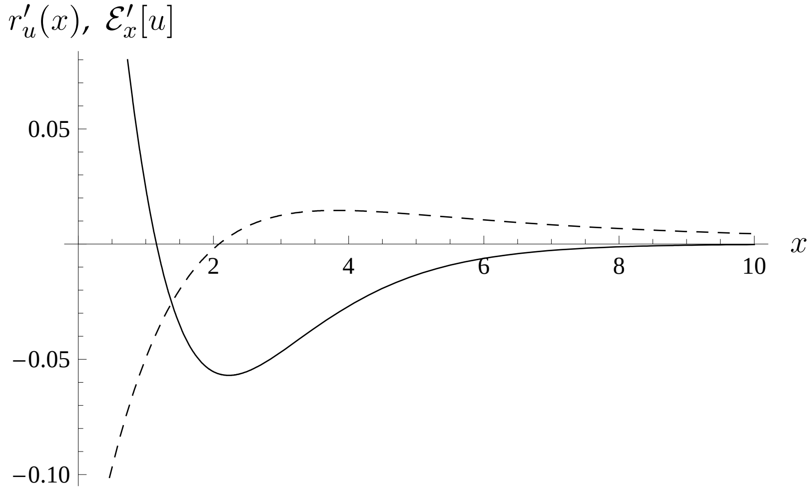

Figure 6: Changes in the Scale Elasticity and in the RLV Regimes in the ACARA Case.

As can be seen from Figure 6, which has been drawn using $a = 1$ and $b = 2$, this function exhibits the following regimes: (i) for $x \in [0, 1.15718]$, it has increasing $r_u(x)$ and decreasing $\mathcal{E}_u[x]$; (ii) for $x \in [1.15718, 2.08182]$ both $r_u(x)$ and $\mathcal{E}_u[x]$ decrease; and (iii) for $x > 2.08182$, $r_u(x)$ decreases whereas $\mathcal{E}_u[x]$ increases. Thus, should the equilibrium consumption fall into range (ii), we know that the self-selection cutoff $\widehat{c}$ is increasing with market size L; whereas we cannot clearly sign the effect of market size on the cutoff should the equilibrium consumption fall into the ranges (i) or (iii).

Appendix D.4 Shift of the Ability Distribution

In this appendix, we investigate how the self-selection cutoff $\widehat{c}$ changes with the underlying ability distribution. To derive clear results, we focus on a multiplicative transformation only (see, e.g., Behrens *et al.*, 2014a).[24] For simplicity, denote the initial distribution by g and the new distribution – obtained through a multiplicative shift – by γ_c. The new distribution of types is given by $\gamma_c \equiv g\left(\frac{c}{k}\right)\frac{1}{k}$, with support $[k\underline{c}; k\overline{c}]$. Note that

$$\mathcal{E}_k[\gamma_c] = \mathcal{E}_k\left[g\left(\frac{c}{k}\right)\frac{1}{k}\right] = -\frac{g'}{g}\frac{c}{k} - 1 = -\frac{\gamma'_c}{\gamma_c}\frac{c}{k} - 1,$$

an equation that will prove useful in what follows. Let

$$J_{\widehat{c}} = \int_{\underline{c}k}^{\widehat{c}} u(x_c)\frac{\mathcal{E}_x[u(x_c)]}{r_u(x_c)} \cdot \frac{1 - r_u(x_c)}{2 - r_{u'}(x_c)} L\gamma_c \mathrm{d}c, \quad \widetilde{u} = \frac{U_{\widehat{c}}}{\widehat{\Gamma}}, \quad \widehat{\Gamma} = \int_{\underline{c}k}^{\widehat{c}} \gamma_c \mathrm{d}c.$$

[24]We could also consider a simple shift in the bounds, but since the underlying distribution changes in that case the effects of changes in the ability support and changes in the distribution are conflated.

and

$$Z_{\widehat{c}} = \int_{\underline{c}k}^{\widehat{c}} u(x_c)\frac{\gamma_c'}{\gamma_c}\frac{c}{k}L\gamma_c \mathrm{d}c, \quad Y_{\widehat{c}} = \int_{\underline{c}k}^{\widehat{c}} u(x_c)L\gamma_c \mathrm{d}c, \quad U_{\widehat{c}} = \int_{\underline{c}k}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c.$$

Taking the elasticity of $U_{\widehat{c}}$, we have:

$$\begin{aligned}
\mathcal{E}_k[U_{\widehat{c}}] &= \int_{\underline{c}k}^{\widehat{c}} u'(x_c)\mathcal{E}_k[x_c]\frac{x_c}{k}\gamma_c \mathrm{d}c \cdot \frac{k}{U_{\widehat{c}}} + \int_{\underline{c}k}^{\widehat{c}} u(x_c)\mathcal{E}_k[\gamma_c]\frac{\gamma_c}{k}\mathrm{d}c \cdot \frac{k}{U_{\widehat{c}}} + \frac{\widehat{u}\widehat{\gamma}\widehat{c}}{U_{\widehat{c}}}\mathcal{E}_k[\widehat{c}] - \frac{\underline{u}\,\underline{\underline{\gamma}}\,\underline{c}k}{U_{\widehat{c}}} \\
&= \int_{\underline{c}k}^{\widehat{c}} u(x_c)\frac{\mathcal{E}_x[u(x_c)]}{r_u(x_c)} \cdot \frac{1 - r_u(x_c)}{2 - r_{u'}(x_c)}\gamma_c \mathrm{d}c \cdot \frac{(1 - \widehat{r}_u)\mathcal{E}_k[\widehat{c}]}{U_{\widehat{c}}} \\
&\quad - \int_{\underline{c}k}^{\widehat{c}} u(x_c)\frac{\gamma_c'}{\gamma_c}\frac{c}{k}\gamma_c \mathrm{d}c \cdot \frac{1}{U_{\widehat{c}}} - 1 + \frac{\widehat{u}\widehat{\gamma}\widehat{c}}{U_{\widehat{c}}}\mathcal{E}_k[\widehat{c}] - \frac{\underline{u}\,\underline{\underline{\gamma}}\,\underline{c}k}{U_{\widehat{c}}} \\
&= \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}(1 - \widehat{r}_u) \cdot \mathcal{E}_k[\widehat{c}] - \frac{Z_{\widehat{c}}}{Y_{\widehat{c}}} - 1 + \frac{\widehat{u}\widehat{\gamma}\widehat{c}}{\widetilde{u}\widehat{\Gamma}}\mathcal{E}_k[\widehat{c}] - \frac{\underline{u}\,\underline{\underline{\gamma}}\,\underline{c}}{\widetilde{u}\widehat{\Gamma}}k.
\end{aligned}$$

Hence, it follows that $\mathcal{E}_k[\widehat{c}] = \widehat{r}_u\mathcal{E}_k[\widehat{c}] - \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}(1 - \widehat{r}_u) \cdot \mathcal{E}_k[\widehat{c}] + \frac{Z_{\widehat{c}}}{Y_{\widehat{c}}} + 1 - \frac{\widehat{u}\widehat{\gamma}\widehat{c}}{\widetilde{u}\widehat{\Gamma}}\mathcal{E}_k[\widehat{c}] + \frac{\underline{u}\,\underline{\underline{\gamma}}\,\underline{c}}{\widetilde{u}\widehat{\Gamma}}k$, so that, after rearrangment, we finally have:

$$\mathcal{E}_k[\widehat{c}] = \frac{\frac{Z_{\widehat{c}}}{Y_{\widehat{c}}} + 1 + \frac{\underline{u}\,\underline{\underline{\gamma}}\,\underline{c}}{\widetilde{u}\widehat{\Gamma}}k}{(1 - \widehat{r}_u)\left(1 + \frac{J_{\widehat{c}}}{Y_{\widehat{c}}}\right) + \frac{\widehat{u}\widehat{\gamma}\widehat{c}}{\widetilde{u}\widehat{\Gamma}}}.$$

The share of entrepreneurs in the economy is given by $\Gamma(\widehat{c}) = \int_{\underline{c}k}^{\widehat{c}} \gamma_c \mathrm{d}c = \int_{\underline{c}}^{\frac{\widehat{c}}{k}} g(z)\mathrm{d}z$, and its derivative is equal to

$$\frac{\mathrm{d}\Gamma(\widehat{c})}{\mathrm{d}k} = g\left(\frac{\widehat{c}}{k}\right)\frac{1}{k}\left(\frac{\widehat{\mathrm{d}c}}{\mathrm{d}k} - \frac{\widehat{c}}{k}\right) = g\left(\frac{\widehat{c}}{k}\right)\frac{\widehat{c}}{k^2}\left(\mathcal{E}_k[\widehat{c}] - 1\right).$$

Clearly, the sign of this derivative – and hence the direction of change of the share of entrepreneurs – depends on the difference between the elasticity of $\widehat{c}$ from one. We can conjecture that under a quite 'large' transformation of the initial distribution, with a sufficiently large elasticity, the share of entrepreneurs will increase. Yet, theoretically both case are possible – we may have an increasing or a decreasing share of entrepreneurs, and the result crucially hinges on the underlying ability distribution.

Special case of CES preferences. We now show that the share of entrepreneurs is invariant to a multiplicative transformation in the case of CES preferences. Hence, CES preferences are again a borderline case of the model. Under CES preferences, we have $u(x) = x^\rho$, $u'(x) = \rho x^{\rho-1}$, $x_c^{\rho-1} = \mu c\rho^{-2}$, and $u(x_c) = (\mu c)^{-\frac{\rho}{1-\rho}}\rho^{\frac{2\rho}{1-\rho}}$. Plugging these expressions into the equilibrium conditions

$$\begin{cases} u'(x_c)\left[1-r_u(x_c)\right]=c\mu, \quad \forall c\in[k\underline{c};k\widehat{c}] \\ \mu = L\displaystyle\int_{\underline{c}k}^{\widehat{c}} u(x_c)\gamma_c \mathrm{d}c \\ \dfrac{r_u(x_{\widehat{c}})x_{\widehat{c}}}{1-r_u(x_{\widehat{c}})} = \dfrac{1}{L\widehat{c}}, \end{cases}$$

we obtain quite simple expressions for all the endogeneous variables of the model. From the second equation, we have the ICC curve

$$\mu = L\int_{\underline{c}k}^{\widehat{c}} (\mu c)^{-\frac{\rho}{1-\rho}}\,\rho^{\frac{2\rho}{1-\rho}}\gamma_c \mathrm{d}c \quad \Rightarrow \quad \mu^{\frac{1}{1-\rho}} = L\int_{\underline{c}k}^{\widehat{c}} c^{-\frac{\rho}{1-\rho}}\rho^{\frac{2\rho}{1-\rho}}\gamma_c \mathrm{d}c. \tag{D.17}$$

Inserting the expression for consumption of a $\widehat{c}$-type variety – given by $\widehat{x}^{\rho-1}=\mu\widehat{c}\rho^{-2}$ – into the third equation, we get the SSC curve:

$$\frac{1-\rho}{\rho}\widehat{x} = (L\widehat{c})^{-1} \quad \Rightarrow \quad \frac{1-\rho}{\rho}(\mu\widehat{c})^{-\frac{1}{1-\rho}}\rho^{\frac{2}{1-\rho}} = (L\widehat{c})^{-1} \quad \Rightarrow \quad \mu^{\frac{1}{1-\rho}} = L\rho^{\frac{2\rho}{1-\rho}}(1-\rho)\,\rho\widehat{c}^{-\frac{\rho}{1-\rho}}. \tag{D.18}$$

The equilibrium is at the intersection of the ICC and SSC curves, i.e., it satisfies

$$L\rho^{\frac{2\rho}{1-\rho}}(1-\rho)\,\rho\widehat{c}^{-\frac{\rho}{1-\rho}} = \int_{\underline{c}k}^{\widehat{c}} c^{-\frac{\rho}{1-\rho}}\rho^{\frac{2\rho}{1-\rho}}\gamma_c \mathrm{d}c \quad \Rightarrow \quad L(1-\rho)\,\rho = \widehat{c}^{\frac{\rho}{1-\rho}}\int_{\underline{c}k}^{\widehat{c}} c^{-\frac{\rho}{1-\rho}}\gamma_c \mathrm{d}c. \tag{D.19}$$

Define $\alpha \equiv \rho/(1-\rho)$ and $\delta \equiv (1-\rho)\,\rho L$. We can then express the equilibrium condition in the following form:

$$\delta = \widehat{c}^{\alpha}\int_{\underline{c}k}^{\widehat{c}} c^{-\alpha}\gamma_c \mathrm{d}c.$$

Recalling that $\gamma_c = g\left(\frac{c}{k}\right)\frac{1}{k}$ and using the change in variables $z=\frac{c}{k}$, the foregoing condition can be rewritten in the following form:

$$\delta = \left(\frac{\widehat{c}}{k}\right)^{\alpha}\int_{\underline{c}}^{\frac{\widehat{c}}{k}} c^{-\alpha}g(z)\mathrm{d}z. \tag{D.20}$$

Clearly, if $\widehat{c}$ is an equilibrium for $k=1$, then $k\widehat{c}$ is an equilibrium for $k\neq 1$, as can be seen from (D.20). It then follows from the definition of the shifted ability distribution γ_c that the share of entrepreneurs remains the same. It is worth noting that this result holds for any shift k and for any distribution of abilities in the CES case. Of course, depending on the underlying distribution, the shares of entrepreneurs will usually be different (see Table 1 in the paper).