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TOWARD A THEORY OF MONOPOLISTIC COMPETITION

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ABSTRACT

Toward a theory of monopolistic competition*

We propose a general model of monopolistic competition, which encompasses existing models while being flexible enough to take into account new demand and competition features. The basic tool we use to study the market outcome is the elasticity of substitution at a symmetric consumption pattern, which depends on both the per capita consumption and the total mass of varieties. We impose intuitive conditions on this function to guarantee the existence and uniqueness of a free-entry equilibrium. Comparative statics with respect to population size, GDP per capita and productivity shock are characterized through necessary and sufficient conditions. Finally, we show how our approach can be generalized to the case of a multisector economy and extended to cope with heterogeneous firms and consumers.

JEL Classification: D43, L11 and L13
Keywords: additive preferences, general equilibrium, homothetic preferences and monopolistic competition

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1 Introduction

The theory of general equilibrium with imperfectly competitive markets is still in infancy. In his survey of the various attempts made in the 1970s and 1980s to integrate oligopolistic competition within the general equilibrium framework, Hart (1985) has convincingly argued that these contributions have failed to produce a consistent and workable model. Unintentionally, the absence of a general equilibrium model with oligopolistic competition has paved the way to the success of the CES model of monopolistic competition developed by Dixit and Stiglitz (1977), which has been applied to an amazingly large number of economic problems. This has led many scholars to believe that the CES model was *the* model of monopolistic competition. For example, Head and Mayer (2014) observe that this model is "nearly ubiquitous" in the trade literature. However, owing to its extreme simplicity, the CES model of monopolistic competition dismisses several important effects that contradict basic findings in economic theory as well as empirical evidence. To mention a few, unlike what the CES predicts, markups and firm sizes are affected by entry, market size, consumer income, while markups vary with costs. In addition, tweaking the CES in the hope of obviating these difficulties, as done in many empirical papers, does not appear to be a satisfactory research strategy for at least two reasons. First, it does not permit a genuine comparison of results and, second, it hinders the development of new and more general models of monopolistic competition that could be brought to the data. In this paper, we concur with Mrázová and Neary (2013) that "assumptions about the structure of preferences and demand matter enormously for comparative statics in trade, industrial organization, and many other applied fields."

Our purpose is to build a general equilibrium model of monopolistic competition, which encompasses existing models and retains enough flexibility to take into account new demand and competition features. Since the 1970s, the literature in applied macroeconomics and international trade has imposed some strong structure on the demand side by using CES preferences. This research strategy has allowed one to introduce imperfect competition and increasing returns as well as heterogeneity across firms in the supply side. By using separable preferences, Zhelobodko et al. (2012) have obtained several comparative statics results under an arbitrary distribution of heterogeneous firm and variable marginal costs. However, this greater generality is obtained by imposing separability on the demand side, an assumption that has long been rejected by data in consumption analysis (Deaton and Muellbauer, 1999). Therefore, to bring back the demand side to the center stage, we have to pursue a different route. This is what we will undertake in this paper. Empirical evidence highlights the importance of market size, firms' productivity, income disparities, and trade liberalization in determining the market outcome as they all affect the elasticity of firms' demands. So far, however, those issues have been investigated in the case of very specific models. As a consequence, we do not know how robust the so-obtained results are. Our setting allows us to identify which findings hold true against alternative preference specifications, and those which depend on particular classes of preferences. In this paper, we characterize preferences through necessary and sufficient conditions for each comparative static effect to hold. This should be useful to the applied economists in discriminating between the different specifications used in their settings. The flip side of the coin is the need to reduce the complexity of the problem. This is why, in the baseline model, we will focus on competition among homogeneous firms. We see this as a necessary step toward the development and analysis of a fully general theory of monopolistic competition.

There are at least two reasons explaining why monopolistic competition is so popular in applied general equilibrium. First, the redistribution of firms' profits is at the root of the non-existence of an equilibrium in general equilibrium with oligopolistic competition. Since entry drives profits down to zero in monopolistic competition, we get rid of this feedback effect and end up with a consistent and analytically tractable model. Second, by modeling monopolistic competition as a noncooperative game with a continuum of players, we capture Chamberlin's central idea that the decision made by a firm has no impact on its competitors. The focus thus shifts from "strategic interactions" to "weak interactions," meaning that firms' behavior is influenced only by market aggregates which are themselves unaffected by the choices made by any single firm.[1]

Admittedly, the continuum assumption vastly simplifies the formal analysis. However, firms are still bound together through market aggregates that give rise to income and substitution effects in consumers' demand in response to firms' pricing behavior. This has a far-fetched implication: even though firms do not compete strategically, *our model is able to mimic oligopolistic markets* and to generate within a general equilibrium framework findings akin to those obtained in partial equilibrium settings. As will be shown, the CES is the only case in which all effects vanish.

To prove the existence and uniqueness of a free-entry equilibrium and to study its properties, we need to impose some restrictions on the demand side of our model. Rather than making new assumptions on preferences and demands, we tackle the problem from the viewpoint of the theory of product differentiation. To be precise, the key concept of our model is the elasticity of substitution among varieties. We then exploit the symmetry of preferences over a continuum of goods to show that, under the most general specification of preferences, at any symmetric outcome the elasticity of substitution between any two varieties is a function of two variables only: the per variety consumption and the total mass of firms. Combining this with the absence of the business-stealing effect of oligopoly theory reveals that, at the market equilibrium, *firms' markup is equal to the inverse of the equilibrium value of the elasticity of substitution.*

This result agrees with one of the main messages of industrial organization: the higher is the elasticity of substitution, the less differentiated are varieties, and thus the lower are firms' markup. It should then be clear that the properties of the symmetric free-entry equilibrium depends on how the elasticity of substitution function behaves when the per variety consumption and the mass of firms, which are both endogenous, vary with the parameters of the economy. The above

[1] The idea of using a continuum of firms was already discussed by Dixit and Stiglitz in their 1974 working paper, which has been published in Brakman and Heijdra (2004).

relationship, which links the supply and demand sides of the model in a very intuitive way, allows us to study the market outcome by means of simple analytical arguments. To be precise, by imposing plausible conditions to the elasticity of substitution function, we are able to disentangle the various determinants of firms' strategies. We will determine our preferred set of assumptions by building on what the theory of product differentiation tells us, as well as on empirical evidence.

Our main findings may be summarized as follows. First, using the concept of Frechet differentiability, we determine a general demand system, which includes a wide range of special cases such as the CES, quadratic, CARA, additive, indirectly additive, and homothetic preferences. At any symmetric market outcome, the individual demand for a variety depends only upon its consumption when preferences are additive. By contrast, when preferences are homothetic, the demand for a variety depends upon its relative consumption level and the mass of available varieties. Therefore, when preferences are neither additive nor homothetic, *the demand for a variety must depend on its consumption level and the total mass of available varieties.*

Second, to insulate the impact of various types of preferences on the market outcome, we focus on symmetric firms and, therefore, on symmetric free-entry equilibria. We provide necessary and sufficient conditions on the elasticity of substitution for the existence and uniqueness of a free-entry equilibrium. Our setting is especially well adapted to conduct detailed comparative static analyses in that we can determine the necessary and sufficient conditions for all the thought experiments undertaken. The typical experiment is to study the impact of market size. What market size signifies is not always clear because it compounds two variables, i.e. the number of consumers and their willingness-to-pay for the product under consideration. The impact of population size and income level on prices, output and the number of firms need not be the same because these two parameters affect firms' demand in different ways. An increase in population or in income raises demand, thereby fostering entry and lower prices. But an income hike also raises consumers' willingness-to-pay, which tends to push prices upward. The final impact is thus a priori ambiguous.

We show that a larger market results in a lower market price and bigger firms if and only if the elasticity of substitution responds more to a change in the mass of varieties than to a change in the per variety consumption. This is likely to be so in the likely case where the entry of new firms does not render varieties much more differentiated. Regarding the mass of varieties, it increases with the number of consumers if varieties do not become too similar when their number rises. Thus, like most oligopoly models, *monopolistic competition exhibits the standard pro-competitive effects associated with market size and entry.* However, anti-competitive effects cannot be ruled out a priori. Furthermore, an increase in individual income generates similar, but not identical, effects if and only if varieties become closer substitutes when their range widens. The CES is the only utility for which price and output are independent of both income and market size.

Our setting also allows us to study the impact of a cost change on markups. When all firms face the same productivity hike, we show that *the nature of preferences determines the extent of the pass-through.* Specifically, a decrease in marginal cost leads to a lower market price, but a higher markup, if and only if the elasticity of substitution decreases with the per capita consumption. In this event, there is incomplete pass-through. However, the pass-through rate need not be smaller than one.

Last, we discuss three major extensions of our baseline model. In the first one, we consider a multisector economy. The main additional difficulty stems from the fact that the sector-specific expenditures depend on the upper-tier utility. Under a fairly mild assumption on the marginal utility, we prove the existence of an equilibrium and show that many of our results hold true for the monopolistically competitive sector. This highlights the idea that our model can be used as a building block to embed monopolistic competition in full-fledged general equilibrium models coping various applications. The second extension focuses on Melitz-like heterogeneous firms. In this case, the market outcome is not symmetric anymore. In addition, when preferences are non-additive, the profit-maximizing price of a firm depends directly on the prices set by other types' firms. This requires the use of Tarski's fixed point theorem to prove the existence of an equilibrium. Our last extension addresses the almost untouched issue of consumer heterogeneity in love-for-variety models of monopolistic competition. Consumers may be heterogeneous because of taste or income differences. Here, we will restrict ourselves to the discussions of some special, but meaningful, cases.

Related literature. Different alternatives have been proposed to avoid the main pitfalls of the CES model. Behrens and Murata (2007) propose the CARA utility that captures both price and size effects, while Zhelobodko et al. (2012) use general additive preferences to work with a variable elasticity of substitution. Kuhn and Vives (1999), Vives (1999) and Ottaviano et al. (2002) show how the quadratic utility model obviates some of the difficulties associated with the CES model, while delivering a full analytical solution. More recently, Bertoletti and Etro (2013) consider an additive indirect utility function to study the impact of per capita income on the market outcome, but price and firm size are independent of population size in their setting. In sum, it seems fair to say that the state of the art looks like a scattered field of incomplete and insufficiently related contributions.

In the next section, we describe the demand and supply sides of our setting. The primitive of the model being the elasticity of substitution function, we discuss in Section 3 how this function varies with per variety consumption and the mass of varieties. In Section 4, we prove the existence and uniqueness of a free-entry equilibrium and characterize its various properties. The three extensions are discussed in Section 5, while Section 6 concludes.

2 The model and preliminary results

Consider an economy with a mass L of identical consumers, one sector and one production factor – labor, which is used as the numéraire. Each consumer is endowed with y efficiency units of labor, so that the per capita income y is given and the same across consumers. This will allow us to discriminate between the effects generated by the consumer income, y, and the number of consumers, L. Firms produce a horizontally differentiated good under increasing returns. Each firm supplies a single variety and each variety is supplied by a single firm.

2.1 Consumers

Let $\mathcal{N}$, an arbitrarily large number, be the mass of "potential" varieties, e.g. the varieties for which a patent exists. Very much like in the Arrow-Debreu model where all commodities need not be produced and consumed, all potential varieties are not necessarily made available to consumers. We denote by $N \leq \mathcal{N}$ the endogenous mass of available varieties.

A *potential consumption profile* $\mathbf{x} \geq 0$ is a Lebesgue-measurable mapping from $[0, \mathcal{N}]$ to $\mathbb{R}_+$. Since a market price profile $\mathbf{p} \geq \mathbf{0}$ must belong to the dual of the space of consumption profiles (Bewley, 1972), we assume that both $\mathbf{x}$ and $\mathbf{p}$ belong to $L_2([0, \mathcal{N}])$, which is its own dual. This implies that both $\mathbf{x}$ and $\mathbf{p}$ have a finite mean and variance. Furthermore, L_2 may be viewed as the most natural infinite-dimensional extension of $\mathbb{R}^n$. Indeed, as will be seen below, using L_2 allows us to write the consumer program in a simple way and to determine well-behaved demand functions by using the concept of Frechet-differentiability, which is especially tractable in L_2 (Dunford and Schwartz, 1988).

Each consumer is endowed with y efficiency units of labor whose price is normalized to 1. Individual preferences are described by a *utility functional* $\mathcal{U}(\mathbf{x})$ defined over $L_2([0, \mathcal{N}])$. In what follows, we make two assumptions about $\mathcal{U}$, which seem close to the "minimal" set of requirements for our model to be nonspecific while displaying the desirable features of existing models of monopolistic competition. First, for any N, the functional $\mathcal{U}$ is *symmetric* in the sense that any Lebesgue measure-preserving mapping from $[0, N]$ into itself does not change the value of $\mathcal{U}$. Intuitively, this means that renumbering varieties has no impact on the utility level.

Second, the utility function exhibits *love for variety* if, for any $N \leq \mathcal{N}$, a consumer strictly prefers to consume the whole range of varieties $[0, N]$ than any subinterval $[0, k]$ of $[0, N]$, that is,

$$\mathcal{U}\left(\frac{X}{k} I_{[0,k]}\right) < \mathcal{U}\left(\frac{X}{N} I_{[0,N]}\right) \tag{1}$$

where $X > 0$ is the consumer's total consumption of the differentiated good and I_A is the indicator of $A \sqsubseteq [0, N]$. Since (1) holds under any monotone transformation of $\mathcal{U}$, the nature of our definition of love for variety is ordinal. In particular, our definition does not appeal to any parametric measure such as the elasticity of substitution in CES-based models.

Proposition 1. *If* $\mathcal{U}(\mathbf{x})$ *is continuous and strictly quasi-concave, then consumers exhibit love for variety.*

The proof is given in Appendix 1. The convexity of preferences is often interpreted as a "taste for diversification" (Mas-Collel et al., 1995, p.44). Our definition of "love for variety" is weaker than that of convex preferences because the former, unlike the latter, involves symmetric consumption only. This explains why the reverse of Proposition 1 does not hold.

For any given N, the utility functional $\mathcal{U}$ is said to be *Frechet-differentiable* in $\mathbf{x} \in L_2([0, \mathcal{N}])$ when there exists a unique function $D(x_i, \mathbf{x})$ from $[0, N] \times L_2$ to $\mathbb{R}$ such that, for all $\mathbf{h} \in L_2$, the equality

$$\mathcal{U}(\mathbf{x} + \mathbf{h}) = \mathcal{U}(\mathbf{x}) + \int_0^N D(x_i, \mathbf{x})\, h_i \,\mathrm{d}i + \circ\left(||\mathbf{h}||_2\right) \tag{2}$$

holds, $||\cdot||_2$ being the L_2-norm. In what follows, we restrict ourselves to utility functionals that are Frechet-differentiable for all $\mathbf{x} \geq 0$ such that $D(x_i, \mathbf{x})$ is decreasing and differentiable with respect to the consumption x_i of variety i. The function $D(x_i, \mathbf{x})$ is the *marginal utility of variety* i when there is a continuum of goods. That $D(x_i, \mathbf{x})$ does not depend directly on $i \in [0, N]$ follows from the symmetry of preferences. Moreover, $D(x_i, \mathbf{x})$ strictly decreases with x_i if $\mathcal{U}$ is strictly concave.

The reason for restricting ourselves to decreasing Frechet-derivatives is that this property allows us to work with well-behaved demand functions. Indeed, maximizing the functional $\mathcal{U}(\mathbf{x})$ subject to (i) the budget constraint

$$\int_0^N p_i x_i \mathrm{d}i = y \tag{3}$$

and (ii) the availability constraint

$$x_i \geq 0 \text{ for all } i \in [0, N] \quad \text{and} \quad x_i = 0 \text{ for all } i \in]N, \mathcal{N}]$$

yields the following inverse demand function for variety i:

$$p_i = \frac{D(x_i, \mathbf{x})}{\lambda} \quad \text{for all } i \in [0, N] \tag{4}$$

where λ is the Lagrange multiplier of the consumer's optimization problem. Expressing λ as a function of y and $\mathbf{x}$, we obtain

$$\lambda(y, \mathbf{x}) = \frac{\int_0^N x_i D(x_i, \mathbf{x})\,\mathrm{d}i}{y} \tag{5}$$

which is the marginal utility of income at the consumption profile $\mathbf{x}$ under income y.[2]

The marginal utility function $D(x_i, \mathbf{x})$ also allows determining the Marshallian demand. Indeed, because the consumer's budget set is convex and weakly compact in $L_2([0, \mathcal{N}])$, while $\mathcal{U}$ is continuous and strictly quasi-concave, there exists a unique utility-maximizing consumption profile $\mathbf{x}^*(\mathbf{p}, y)$ (Dunford and Schwartz, 1988). Plugging $\mathbf{x}^*(\mathbf{p}, y)$ into (4) – (5) and solving (4) for x_i, we obtain the Marshallian demand for variety i:

[2] If we apply to $\mathcal{U}$ a monotonic transformation ψ, then $D(x_i, \mathbf{x})$ will change into $\psi'(\mathcal{U}(\mathbf{x}))\, D(x_i, \mathbf{x})$. However, (5) implies that λ is also multiplied by $\psi'(\mathcal{U}(\mathbf{x}))$. Thus, the inverse demand $D(x_i, \mathbf{x})/\lambda$ is invariant to a monotonic transformation of the utility functional.

$$x_i = \mathcal{D}(p_i, \mathbf{p}, y) \tag{6}$$

which is weakly decreasing in its own price.[3] In other words, when there is a continuum of varieties, *decreasing marginal utilities are a necessary and sufficient condition for the Law of demand to hold.*

Remark. Assume that preferences are asymmetric in that the utility functional $\mathcal{U}(\mathbf{x})$ is given by

$$\mathcal{U}(\mathbf{x}) = \tilde{\mathcal{U}}(\mathbf{a} \cdot \mathbf{x}) \tag{7}$$

where $\tilde{\mathcal{U}}$ is a symmetric Frechet-differentiable functional, $\mathbf{a} \in L_2([0, \mathcal{N}])$ a weight function, and $\mathbf{a} \cdot \mathbf{x}$ the function $(\mathbf{a} \cdot \mathbf{x})_i \equiv a_i x_i$ for all $i \in [0, \mathcal{N}]$. If $x_i = x_j$, $a_i > a_j$ means that all consumers prefer variety i to variety j, perhaps because the quality of i exceeds that of j.

The preferences (7) can be made symmetric by changing the units in which the quantities of varieties are measured. Indeed, for any $i, j \in [0, \mathcal{N}]$ the consumer is indifferent between consuming a_i/a_j units of variety i and one unit of variety j. Therefore, by using the change of variables $\tilde{x}_i \equiv a_i x_i$ and $\tilde{p}_i \equiv p_i/a_i$, we can reformulate the consumer's program as follows:

$$\max_{\tilde{\mathbf{x}}} \tilde{\mathcal{U}}(\tilde{\mathbf{x}}) \qquad \text{s.t.} \int_0^N \tilde{p}_i \tilde{x}_i \mathrm{d}i \leq y.$$

In this case, the equilibrium can be shown to be symmetric up to a rescaling of prices and quantities by the weights a_i.

To illustrate how preferences shape the demand system, consider the following examples of utility functionals satisfying the condition (2).

1. **Additive preferences**.[4] (i) Assume that preferences are additive over the set of available varieties (Spence, 1976; Dixit and Stiglitz, 1977):

$$\mathcal{U}(\mathbf{x}) \equiv \int_0^{\mathcal{N}} u(x_i) \mathrm{d}i \tag{8}$$

where u is differentiable, strictly increasing, strictly concave and such that $u(0) = 0$. The CES and the CARA utility (Bertoletti, 2006; Behrens and Murata, 2007) are special cases of (8).

It is straightforward to show that (8) satisfies (2). The marginal utility of variety i depends only upon its own consumption:

$$D(x_i, \mathbf{x}) = u'(x_i).$$

Thus, the inverse demand functions satisfy the property of independence of irrelevant alterna-

[3]Since D is continuously decreasing in x_i, there exists at most one solution of (4) with respect to x_i. If there is a finite choke price ($D(0, \mathbf{x}^*)/\lambda < \infty$), there may be no solution. To encompass this case, the Marshallian demand should be formally defined by $\mathcal{D}(p_i, \mathbf{p}, y) \equiv \inf\{x_i \geq 0 \mid D(x_i, \mathbf{x}^*)/\lambda(y, \mathbf{x}^*) \leq p_i\}$.

[4]The idea of additive utilities and additive indirect utilities goes back at least to Houthakker (1960).

tives, whereas the demand functions

$$x_i = (u')^{-1}(\lambda p_i) \tag{9}$$

do not because, as seen from (5), the multiplier λ captures information about the whole consumption profile.

(ii) Bertoletti and Etro (2013) have recently proposed a new approach to modeling monopolistic competition, in which preferences are expressed through the following indirect utility function:

$$\mathcal{V}(\mathbf{p}, y) \equiv \int_0^{\mathcal{N}} v(p_i/y)\mathrm{d}i \tag{10}$$

where v is differentiable, strictly decreasing and strictly convex. Using Roy's identity, the demand function for variety i is given by

$$x_i = \frac{v'(p_i/y)}{\int_0^N (p_k/y)v'(p_k/y)\mathrm{d}k} \tag{11}$$

where the denominator is an aggregate demand shifter that, by the envelope theorem, equals $-\lambda y$.

Clearly, the demand functions satisfy the property of independence of irrelevant alternatives, whereas the inverse demand functions

$$p_i = y(v')^{-1}(-\lambda y x_i)$$

do not. Thus, unlike the Marshallian demand (9) obtained under additive preferences, the Marshallian demand (11) now depends directly on y.

In brief, the link between a direct and an indirect additive utility goes through the demand functions under (10) and the inverse demand functions under (8), which both share the property of independence of irrelevant alternatives. Thus, we may already conclude that a direct and an indirect additive utility generate different market outcomes. This point is further developed in Sections 3 and 4.

2. Non-additive preferences. Consider first the quadratic utility proposed by Ottaviano et al. (2002):

$$\mathcal{U}(\mathbf{x}) \equiv \alpha \int_0^{\mathcal{N}} x_i \mathrm{d}i - \frac{\beta}{2}\int_0^{\mathcal{N}} x_i^2\, \mathrm{d}i - \frac{\gamma}{2}\int_0^{\mathcal{N}} \left(\int_0^{\mathcal{N}} x_i\, \mathrm{d}i\right) x_j \mathrm{d}j \tag{12}$$

where α, β,and γ are positive constants. In this case, the marginal utility of variety i is given by

$$D(x_i, \mathbf{x}) = \alpha - \beta\, x_i - \gamma \int_0^{\mathcal{N}} x_j \mathrm{d}j \tag{13}$$

which is linear decreasing in x_i. In addition, D also decreases with the aggregate consumption across varieties:

$$X \equiv \int_0^{\mathcal{N}} x_j \mathrm{d}j$$

which captures the idea that the marginal utility of every variety decreases with total consumption.

Another example of non-additive preferences, which also captures the idea of love for variety is given by the entropy-like utility proposed by Anderson et al. (1992):

$$\mathcal{U}(\mathbf{x}) \equiv U(X) + X \ln X - \int_0^{\mathcal{N}} x_i \ln x_i \mathrm{d}i$$

where U is increasing and strictly concave. The marginal utility of variety i is

$$D(x_i, \mathbf{x}) = U'(X) - \ln\left(\frac{x_i}{X}\right) \tag{14}$$

which decreases with x_i.

3. Homothetic preferences. A tractable example of non-CES homothetic preferences is the translog, as developed by Feenstra (2003). By appealing to the duality principle in consumption theory, these preferences are described by the following expenditure function:

$$\ln E(\mathbf{p}) = \ln \mathcal{U}_0 + \frac{1}{\mathcal{N}} \int_0^{\mathcal{N}} \ln p_i \mathrm{d}i - \frac{\beta}{2\mathcal{N}} \left[\int_0^{\mathcal{N}} (\ln p_i)^2 \mathrm{d}i - \frac{1}{\mathcal{N}} \left(\int_0^{\mathcal{N}} \ln p_i \mathrm{d}i \right)^2 \right].$$

A generalization of the translog is provided by the following expenditure function (Feenstra, 2014), which also portrays homothetic preferences:

$$E(\mathbf{p}) = \mathcal{U}_0 \cdot \left[\alpha \int_0^{\mathcal{N}} p_i^r \mathrm{d}i + \beta \left(\int_0^{\mathcal{N}} p_i^{r/2} \mathrm{d}i \right)^2 \right]^{1/r} \qquad r \neq 0.$$

A large share of the literature focusing on additive or homothetic preferences, we find it important to provide a full characterization of the corresponding demands (the proof is given in Appendix 2).

Proposition 2. *The marginal utility* $D(x_i, \mathbf{x})$ *of variety* i *depends only upon* (i) *the consumption* x_i *if and only if preferences are additive and* (ii) *the consumption ratio* $\mathbf{x}/x_i$ *if and only if preferences are homothetic.*

Proposition 2 can be illustrated by using the CES:

$$\mathcal{U}(\mathbf{x}) \equiv \left(\int_0^{\mathcal{N}} x_i^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\frac{\sigma}{\sigma-1}}$$

where $\sigma > 1$ is the elasticity of substitution across varieties. The marginal utility $D(x_i, \mathbf{x})$ is given by

$$D(x_i, \mathbf{x}) = A(\mathbf{x})\, x_i^{1/\sigma} = A(\mathbf{x}/x_i)$$

where $A(\mathbf{x})$ is the aggregate given by

$$A(\mathbf{x}) \equiv \left(\int_0^{\mathcal{N}} x_j^{\frac{\sigma-1}{\sigma}} \mathrm{d}j \right)^{-1/\sigma}.$$

The number of varieties as a consumption externality. In their 1974 working paper, Dixit and Stiglitz (1977) argued that the mass of varieties could enter the utility functional as a specific argument.[5] In this case, the number of available varieties has the nature of a consumption externality, the reason being that the value of N stems from the entry decisions made by firms.

An example is given by the augmented-CES, which is defined as follows:

$$\mathcal{U}(\mathbf{x}, N) \equiv N^{\nu} \left(\int_0^{\mathcal{N}} x_i^{\frac{\sigma-1}{\sigma}} \mathrm{d}i \right)^{\sigma/(\sigma-1)}. \tag{15}$$

In Benassy (1996), ν is a positive constant that captures the consumer benefit of a larger number of varieties. The idea is to separate the love-for-variety effect from the competition effect generated by the degree of product differentiation, which is inversely measured by σ. Blanchard and Giavazzi (2003) takes the opposite stance by assuming that $\nu = -1/\sigma(N)$ where $\sigma(N)$ increases with N. Under this specification, increasing the number of varieties does not raise consumer welfare but intensifies competition among firms.

Another example is the quadratic utility proposed by Shubik and Levitan (1971):

$$\mathcal{U}(\mathbf{x}, N) \equiv \alpha \int_0^{\mathcal{N}} x_i \mathrm{d}i - \frac{\beta}{2} \int_0^{\mathcal{N}} x_i^2 \, \mathrm{d}i - \frac{\gamma}{2N} \int_0^{\mathcal{N}} \left(\int_0^{\mathcal{N}} x_i \, \mathrm{d}i \right) x_j \mathrm{d}j. \tag{16}$$

The difference between (12) and (16) is that the former may be rewritten as follows:

$$\alpha X - \frac{\beta}{2} \int_0^{\mathcal{N}} x_i^2 \, \mathrm{d}i - \frac{\gamma}{2} X^2$$

which is independent of N, whereas the latter becomes

$$\alpha X - \frac{\beta}{2} \int_0^{\mathcal{N}} x_i^2 \, \mathrm{d}i - \frac{\gamma}{2N} X^2$$

which ceteris paribus strictly increases with N.

Introducing N as an explicit argument in the utility functional $\mathcal{U}(\mathbf{x}, N)$ may change the indifference surfaces. Nevertheless, the analysis developed below remains valid in such cases. Indeed,

[5] Note that N can be written as a function of the consumption functional $\mathbf{x}$ in the following way $N = \mu\{x_i > 0, \forall i \leq \mathbb{N}\}$. However, this raises new issues regarding the Frechet differentiability of the utility functional.

the marginal utility function D already includes N as an argument because the support of $\mathbf{x}$ varies with N.

2.2 Firms

There are increasing returns at the firm level, but no scope economies that would induce a firm to produce several varieties. Each firm supplies a single variety and each variety is produced by a single firm. Consequently, a variety may be identified by its producer $i \in [0, N]$. Firms are homogeneous: to produce q units of its variety, a firm needs $F + cq$ efficiency units of labor, which means that F is the fixed production cost and c the marginal production cost. Being negligible to the market, each firm chooses its output (or price) while accurately treating some market aggregates as given. However, for the market to be in equilibrium, firms must accurately guess what these market aggregates will be.

In monopolistic competition, unlike oligopolistic competition, Cournot and Bertrand competition yield the same market outcome (Vives, 1999). However, unless explicitly mentioned, we assume that firms are quantity-setters. Thus, firm $i \in [0, N]$ maximize its profits

$$\pi(q_i) = (p_i - c)q_i - F \tag{17}$$

with respect to its output q_i subject to the inverse market demand function $p_i = LD/\lambda$. Since consumers share the same preferences, the consumption of each variety is the same across consumers. Therefore, product market clearing implies $q_i = Lx_i$. Firm i accurately treats the market aggregates N and λ, which are endogenous, parametrically.

3 Market equilibrium

In this section, we characterize the market outcome when the number N of firms is exogenously given. This allows us to determine the equilibrium output, price and per variety consumption conditional upon N. In the next section, the zero-profit condition pins down the equilibrium number of firms.

When the number N of firms is given, a *market equilibrium* is given by the functions $\bar{\mathbf{q}}(N)$, $\bar{\mathbf{p}}(N)$ and $\bar{\mathbf{x}}(N)$ defined on $[0, N]$, which satisfy the following conditions: (i) no firm i can increase its profit by changing its output $\bar{q}_i(N)$, (ii) each consumer maximizes her utility subject to her budget constraint, (iii) the product market clearing condition

$$\bar{q}_i = L\bar{x}_i \qquad \text{for all } i \in [0, N]$$

and (iv) the labor market balance

$$c \int_0^N q_i \mathrm{d}i + NF = yL$$

hold.

3.1 Existence and uniqueness of a market equilibrium

Plugging D into (17), the program of firm i is given by

$$\max_{x_i} \pi(x_i, \mathbf{x}) \equiv \left[\frac{D\left(x_i, \mathbf{x}\right)}{\lambda} - c \right] L\, x_i - F. \tag{18}$$

Because firm i accurately treats λ as a parameter, the first-order condition for profit-maximization

$$x_i \frac{\partial D(x_i, \mathbf{x})}{\partial x_i} + D(x_i, \mathbf{x}) = \left[1 - \bar{\eta}(x_i, \mathbf{x})\right] D(x_i, \mathbf{x}) = \lambda c \tag{19}$$

where

$$\bar{\eta}(x_i, \mathbf{x}) \equiv -\frac{x_i}{D} \frac{\partial D}{\partial x_i}$$

is the elasticity of the inverse demand for variety i. For (19) to have at least one solution regardless of $c > 0$, it is sufficient to assume that, for any $\mathbf{x}$, the following Inada conditions hold:

$$\lim_{x_i \to 0} D = \infty \qquad \lim_{x_i \to \infty} D = 0. \tag{20}$$

Indeed, since $\bar{\eta}(0, \mathbf{x}) < 1$, (20) implies that $\lim_{x_i \to 0}(1 - \bar{\eta})D = \infty$. Similarly, since $0 < (1 - \bar{\eta})D < D$, it follows from (20) that $\lim_{x_i \to \infty}(1 - \bar{\eta})D = 0$. Because $(1 - \bar{\eta})D$ is continuous, it follows from the intermediate value theorem that (19) has at least one positive solution. Note that (20) is sufficient, but not necessary. If D displays a finite choke price exceeding the marginal cost, it is readily verified that (19) has at least one positive solution.

The first-order condition (19) is sufficient if the profit function π is strictly quasi-concave in x_i. To show under which condition this property holds, we use the concept of r-convexity: given a real number $r \neq 0$, a function $f(x)$ is *r-convex* if $[f(x)]^r$ is convex (Pearce et al., 1998).

For the profit function π to be strictly quasi-concave in x_i, the second derivative of π must be negative at any solution to the first-order condition:

$$x_i \frac{\partial^2 D}{\partial x_i^2} + 2 \frac{\partial D}{\partial x_i} < 0. \tag{21}$$

Solving (19) for x_i, plugging the result into (21) and multiplying both parts by $\partial D / \partial x_i$, we obtain:

$$2 \left(\frac{\partial D}{\partial x_i} \right)^2 - (D - \lambda c) \frac{\partial^2 D}{\partial x_i^2} > 0 \tag{22}$$

which necessarily holds if $D - \lambda c$ is a strictly (-1)-convex function for all $x_i < D^{-1}(\lambda c)$. Indeed, if

$$\frac{\partial^2}{\partial x_i^2}\left(\frac{1}{D-\lambda c}\right)=\frac{2\left(\frac{\partial D}{\partial x_i}\right)^2-(D-\lambda c)\frac{\partial^2 D}{\partial x_i^2}}{(D-\lambda c)^3}>0$$

then (22) also holds. Since

$$1+\frac{\lambda c}{D-\lambda c}=\frac{p_i}{p_i-c},$$

the strict convexity of $1/(D-\lambda c)$ is equivalent to strict convexity of $p_i/(p_i-c)$ in x_i. In other words, the profit function π is strictly quasi-concave in x_i if

(**A**) *the Lerner index* $(p_i-c)/p_i$ *is strictly* (-1)*-convex in* x_i.

Note that **(A)** is a necessary and sufficient condition for the quasi-concavity of a firm's profit function. Yet, the most common assumption used in the literature is (see, e.g. Krugman, 1979):

(Abis) *the elasticity* $\bar{\eta}(x_i,x)$ *increases with* x_i or, equivalently, the superelasticity is positive.

It is readily verified that **(Abis)** is equivalent to

$$-x_i\frac{\partial^2 D/\partial x_i^2}{\partial D/\partial x_i}<1+\bar{\eta}. \tag{23}$$

Plugging $\lambda=D/p_i$ and (19) into (22), we may rewrite (**A**) as follows:

$$-x_i\frac{\partial^2 D/\partial x_i^2}{\partial D/\partial x_i}<2. \tag{24}$$

Since $\bar{\eta}<1$, **(Abis)** implies **(A)**. Note that (23) means that D cannot be "very" convex in x_i.

The condition (**A**) has the following important implication.

Claim. *Assume* (**A**). *For any given* $N\leq Ly/F$, *there exists a unique market equilibrium, which is symmetric.*

Each firm facing the same demand and being negligible, the function $\pi(x_i,\mathbf{x})$ is the same for all i. In addition, (**A**) implies that $\pi(x_i,\mathbf{x})$ has a unique maximizer for any $\mathbf{x}$. Therefore, the market equilibrium (if any) must be symmetric.

We now show that a market equilibrium exists when profits are uniformly distributed across consumers. The budget constraint is now given by

$$\int_0^N p_i x_i \mathrm{d}i=y+\frac{1}{L}\int_0^N \pi_i \mathrm{d}i.$$

Using $\pi_i\equiv(p_i-c)Lx_i-F$, this expression boils down to labor market balance:

$$cL\int_0^N x_i\mathrm{d}i+FN=yL \tag{25}$$

which captures the general equilibrium effects generated by the redistribution of profits through the budget constraint.

Since the equilibrium is symmetric, (25) yields the only candidate equilibrium for the per variety consumption:

$$\bar{x}(N) = \frac{y}{cN} - \frac{F}{cL} \tag{26}$$

which is unique and positive if and only if $N \leq Ly/F$. The product market clearing condition implies that the candidate equilibrium output is

$$\bar{q}(N) = \frac{yL}{cN} - \frac{F}{c}. \tag{27}$$

Plugging (27) into the profit maximization condition (31) shows that there is a unique candidate equilibrium price given by

$$\bar{p}(N) = c\frac{\sigma\left(\bar{x}(N), N\right)}{\sigma\left(\bar{x}(N), N\right) - 1}. \tag{28}$$

The condition (**A**) implies that $(\bar{q}(N), \bar{x}(N), \bar{p}(N))$ is the unique market equilibrium. Clearly, if $N > Ly/F$, there exists no equilibrium. This completes the proof of the claim.

3.2 The elasticity of substitution

In this section, we define the elasticity of substitution, which will be central in our equilibrium analysis. To this end, we extend the definition proposed by Nadiri (1982, p.442) to the case of a continuum of goods (see Appendix 3).

Consider any two varieties i and j such that $x_i = x_j = x$. We show in Appendix 3 that the elasticity of substitution between i and j, conditional on $\mathbf{x}$, is given by

$$\bar{\sigma}(x, \mathbf{x}) = -\frac{D(x, \mathbf{x})}{x}\frac{1}{\frac{\partial D(x,\mathbf{x})}{\partial x}} = \frac{1}{\bar{\eta}(x, \mathbf{x})}. \tag{29}$$

Because the market outcome is symmetric, we may restrict the analysis to symmetric consumption profiles:

$$\mathbf{x} = xI_{[0,N]}$$

and redefine $\bar{\eta}(x_i, \mathbf{x})$ and $\bar{\sigma}(x, \mathbf{x})$ as follows:

$$\eta(x, N) \equiv \bar{\eta}(x, xI_{[0,N]}) \qquad \sigma(x, N) \equiv \bar{\sigma}(x, xI_{[0,N]}).$$

Furthermore, (29) implies that

$$\sigma(x, N) = 1/\eta(x, N). \tag{30}$$

Hence, along the diagonal, our original functional analysis problem boils down into a two-dimensional one.

Rewriting the equilibrium conditions (19) along the diagonal yields

$$\bar{m}(N) \equiv \frac{\bar{p}(N) - c}{\bar{p}(N)} = \eta(\bar{x}(N), N) = \frac{1}{\sigma(\bar{x}(N), N)} \tag{31}$$

while

$$\bar{\pi}(N) \equiv (\bar{p}(N) - c)\bar{q}(N)$$

denotes the equilibrium operating profits made by a firm when there is a mass N of firms.

Importantly, (31) shows that, for any given N, *the equilibrium markup* $\bar{m}(N)$ *varies inversely with the elasticity of substitution*. The intuition is easy to grasp. It is well know from industrial organization that product differentiation relaxes competition. When the elasticity of substitution is lower (higher), varieties are worse (better) substitutes, thereby endowing firms with more (less) market power. Therefore, it is no surprise that firms have a higher (lower) markup when σ is lower (higher). It also follows from (31) that the way σ varies with x and N shapes the market outcome. In particular, this demonstrates that assuming a constant elasticity of substitution amounts to adding very strong restraints on the way the market works.

Combining (26) and (28), we find that the operating profits are given by

$$\bar{\pi}(N) = \frac{cL\bar{x}(N)}{\sigma\left(\bar{x}(N), N\right) - 1}. \tag{32}$$

It is legitimate to ask how $\bar{p}(N)$ and $\bar{\pi}(N)$ vary with the mass of firms? There is no simple answer to this question. Indeed, the expression (32) suffices to show that the way the market outcome reacts to the entry of new firms depends on how the elasticity of substitution varies with x and N. This confirms why static comparative statics under oligopoly yields ambiguous results.

So, to gain intuition about the behavior of σ, we give below the elasticity of substitution for the different types of preferences discussed in the previous section.

(i) When the utility is additive, we have:

$$\frac{1}{\sigma(x, N)} = r(x) \equiv -\frac{xu''(x)}{u'(x)} \tag{33}$$

which means that σ depends only upon the per variety consumption when preferences are additive. In this case, (31) yields

$$p = \frac{c}{1 - r(x)}.$$

(ii) When the indirect utility is additive, it is shown in Appendix 4 that σ depends only upon the total consumption $X = Nx$. Since the budget constraint implies $X = y/p$, (31) may be rewritten as follows:

$$\frac{p - c}{p} = \theta(X) \equiv -\frac{v'(1/X)}{v''(1/X)}X \tag{34}$$

so that the profit-maximizing price is given by

$$p = \frac{c}{1 - \theta(X)}. \tag{35}$$

(iii) When preferences are homothetic, it follows from Proposition 2 and (**??**) that

$$\frac{1}{\sigma(x, N)} = \varphi(N) \equiv \eta(1, N) \tag{36}$$

and thus the profit-maximizing price

$$p = \frac{c}{1 - \varphi(N)}$$

varies only with the mass of firms.

For example, under translog preferences, we have

$$\mathcal{D}(p_i, \mathbf{p}, \bar{y}(N)) = \frac{\bar{y}(N)}{p_i} \left(\frac{1}{N} + \frac{\beta}{N} \int_0^{\mathcal{N}} \ln p_j \mathrm{d}j - \beta \ln p_i \right)$$

where $\varphi(N) = 1/(1 + \beta N)$ where $\bar{y}(N) \equiv y + N\bar{\pi}(N)/L$.

(iv) In the CES case, the indirect utility is given by

$$\mathcal{V}(\mathbf{p}, \bar{y}(N)) = \int_0^N \left(\frac{p_i}{\bar{y}(N)} \right)^{-(\sigma - 1)} \mathrm{d}i.$$

Since both the direct and indirect CES utilities are additive, the elasticity of substitution is constant. Furthermore, since the CES is also homothetic, it must be that

$$r(x) = \theta(X) = \varphi(N) = \frac{1}{\sigma}.$$

It is, therefore, no surprise that σ is the only demand side parameter that drives the market outcome under CES preferences.

(v) In the entropy utility case, it is readily verified that

$$\sigma(x, N) = U'(Nx) + \ln N \tag{37}$$

which decreases with x, whereas $\sigma(x, N)$ can be U-shaped in N according to the function form of U.

As illustrated in Figure 1, the CES is the *sole* function that belongs to the three classes of preferences. Furthermore, the expressions (33), (34) and (36) imply that the classes of additive, indirectly additive and homothetic preferences are disjoint, expect for the CES that belongs to the three of them.

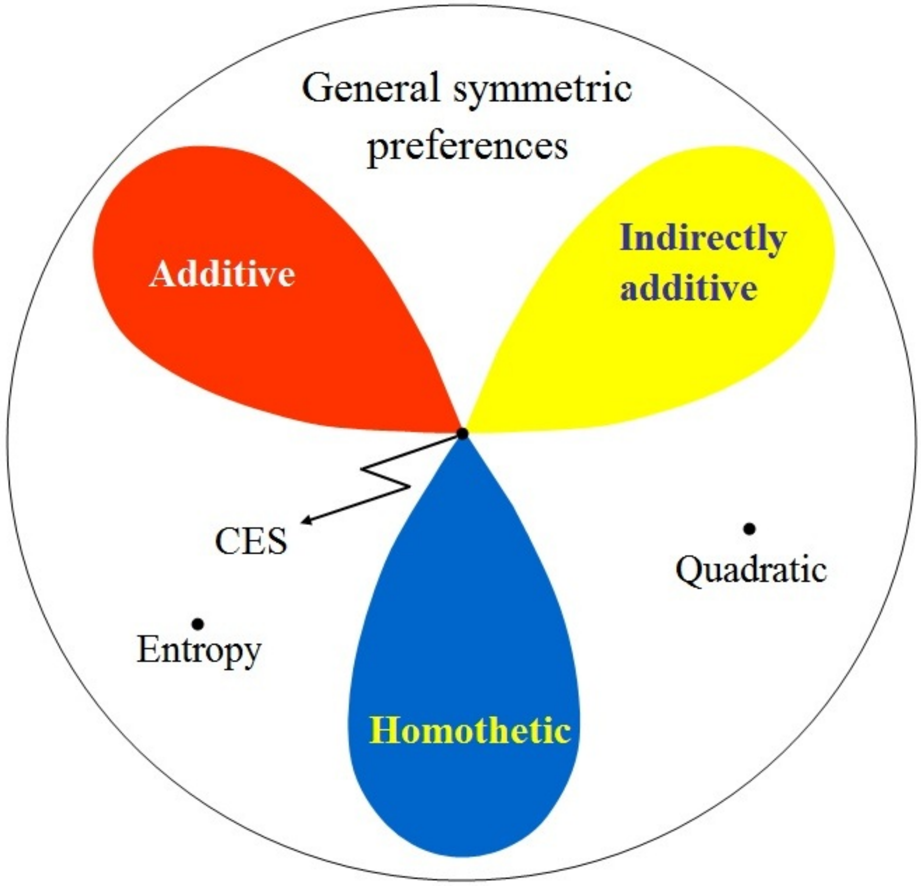

Fig. 1. The space of preferences

From now on, we consider the function $\sigma(x, N)$ as the *primitive* of the model. There are two reasons for making this choice. First, $\sigma(x, N)$ portrays what preferences are along the diagonal ($x_i = x > 0$ for all i). As a result, what matters for the equilibrium is how $\sigma(x, N)$ varies with x and N. Second, the properties of the market outcome can be characterized by necessary and sufficient conditions stated in terms of the elasticity of σ with respect to x and N, which are denoted $\mathcal{E}_x(\sigma)$ and $\mathcal{E}_N(\sigma)$. To be precise, the signs of these two expressions ($\mathcal{E}_x(\sigma) \gtrless 0$ and $\mathcal{E}_N(\sigma) \lesseqgtr 0$) and their relationship ($\mathcal{E}_x(\sigma) \gtrless \mathcal{E}_N(\sigma)$) will allow us to characterize *completely* the market equilibrium.

How σ varies with x is a priori not clear. Marshall (1920, Book 3, Chapter IV) has argued on intuitive grounds that the elasticity of the inverse demand $\bar{\eta}(x_i, \mathbf{x})$ increases in sales.[6] In our setting, (29) shows that this assumption amounts to $\partial\bar{\sigma}(x, \mathbf{x})/\partial x < 0$. However, this inequality does not tell us anything about the sign of $\partial\sigma(x, N)/\partial x$ because x refers here to the consumption of *all* varieties. When preferences are additive, as in Krugman (1979), Marshall's argument can be applied because the marginal utility of a variety depends only upon its own consumption. But this ceases to be true when preferences are non-additive. Nevertheless, as will be seen in Section 4.3, $\mathcal{E}_x(\sigma) < 0$ holds if and only if the pass-through is smaller than 100%. The literature on spatial pricing backs up this

[6] We thank Peter Neary for having pointed out this reference to us.

assumption, though it also recognizes the possibility of a pass-through exceeding 100% (Greenhut et al., 1987).

We now come to the relationship between σ and N. The literature in industrial organization suggests that varieties become closer substitutes when N increases, the reason being that adding new varieties crowds out the product space (Salop, 1979; Tirole, 1988). Therefore, assuming $\mathcal{E}_N(\sigma) > 0$ spontaneously comes to mind. As a consequence, the folk wisdom would be described by the following two conditions:

$$\mathcal{E}_x(\sigma) < 0 < \mathcal{E}_N(\sigma). \tag{38}$$

However, these inequalities turn out to be more restrictive that what they might seem at first glance. Indeed, they do not allow capturing some interesting market effects and to encompass some standard models of monopolistic competition. For example, when preferences are quadratic, Bertoletti and Epifani (2014) have pointed out that the elasticity of substitution decreases with N:

$$\sigma(x, N) = \frac{\alpha - \beta x}{\beta x} - \frac{\gamma}{\beta} N. \tag{39}$$

This should not come as a surprise. Indeed, although spatial models of product differentiation and models of monopolistic competition are not orthogonal to each other, they differ in several respects. In particular, when consumers are endowed with a love for variety, they are inclined to spread their consumption over a wider range of varieties at the expense of their consumption of each variety. By contrast, in spatial models every consumer has a unique ideal variety. Therefore, providing a reconciliation of the two settings is not an easy task (Anderson et al., 1992). In what follows, we propose to study the impact of N on σ under the assumption that a consumer's total consumption Nx is arbitrarily fixed, as in spatial models of product differentiation, while allowing the per variety consumption x to vary with N, as in love-for-variety models.

In this case, it is readily verified that the following two relationships must hold simultaneously:

$$\begin{aligned} \frac{\mathrm{d}x}{x} &= -\frac{\mathrm{d}N}{N} \\ \frac{\mathrm{d}\sigma}{\sigma} &= \frac{\partial\sigma}{\partial N}\frac{N}{\sigma}\frac{\mathrm{d}N}{N} + \frac{\partial\sigma}{\partial x}\frac{x}{\sigma}\frac{\mathrm{d}x}{x}. \end{aligned}$$

Plugging the first expression into the second, we obtain

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}N}\right|_{Nx=const} = \frac{\sigma}{N}\left(\mathcal{E}_N(\sigma) - \mathcal{E}_x(\sigma)\right).$$

In this event, the elasticity of substitution increases with N if and only if

$$\mathcal{E}_x(\sigma) < \mathcal{E}_N(\sigma) \tag{40}$$

holds. This condition is less stringent than $\partial\sigma/\partial N > 0$ because it allows the elasticity of substitution to decrease with N. In other words, entry may trigger more differentiation because the incumbents react by adding new attributes to their products. In addition, the evidence supporting the assumption $\mathcal{E}_x(\sigma) < 0$ being mixed, we find it relevant to investigate the implications of the opposite assumption $\mathcal{E}_x(\sigma) > 0$. Note that $\partial\sigma/\partial x = \partial\sigma/\partial N = 0$ in the CES case only.

4 Symmetric monopolistic competition

A *symmetric free-entry equilibrium* (SFE) is described by the vector (q^*, p^*, x^*, N^*), where N^* solves the zero-profit condition

$$\bar{\pi}(N) = F \tag{41}$$

while $q^* = \bar{q}(N^*)$, $p^* = \bar{p}(N^*)$ and $x^* = \bar{x}(N^*)$. The Walras Law implies that the budget constraint $N^*p^*x^* = y$ is satisfied. In what follows, we restrict ourselves to the domain of parameters for which $N^* < \mathcal{N}$.

Combining (31) and (41), we obtain a single equilibrium condition given by

$$\bar{m}(N) = \frac{NF}{Ly} \tag{42}$$

which means that, at the SFE, the equilibrium markup is equal to the share of the labor supply spent on overhead costs. When preferences are non-homothetic, (26) and (28) show that L/F and y enter the function $\bar{m}(N)$ as two distinct parameters. This implies that L/F and y have a different impact on the equilibrium markup, while a hike in L is equivalent to a drop in F.

4.1 Existence and uniqueness of a SFE

Differentiating (32) with respect to N, we obtain

$$\begin{aligned} \bar{\pi}'(N) &= \bar{x}'(N)\,\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{cLx}{\sigma\left(x, yL/(cLx+F)\right)-1}\right]\Bigg|_{x=\bar{x}(N)} \\ &= -\frac{y}{cN^2}\left(\sigma - 1 - x\frac{\partial\sigma}{\partial x} + \frac{cLx}{cLx+F}\frac{yL}{cLx+F}\frac{\partial\sigma}{\partial N}\right)\Bigg|_{x=\bar{x}(N)}. \end{aligned}$$

Using (26) and (41), the second term in the right-hand side of this expression is positive if and only if

$$\mathcal{E}_x(\sigma) < \frac{\sigma-1}{\sigma}(1 + \mathcal{E}_N(\sigma)). \tag{43}$$

Therefore, $\bar{\pi}'(N) < 0$ for all N if and only if (43) holds. This implies the following proposition.

Proposition 3. *Assume* (**A**). *There exists a unique free-entry equilibrium for all* $c > 0$ *if and only if* (43) *holds. Furthermore, this equilibrium is symmetric.*

Because the above proposition provides a necessary and sufficient condition for the existence of a SFE, we may safely conclude that the set of assumptions required to bring into play monopolistic competition must include (43). Therefore, throughout the remaining of the paper, we assume that (43) holds. This condition allows one to work with preferences that display a great of flexibility. Indeed, σ may decrease or increase with x and/or N. To be precise, varieties may become better or worse substitutes when the per variety consumption and/or the number of varieties rises, thus generating either price-decreasing or price-increasing competition. Evidently, (43) is satisfied when the folk wisdom conditions (38) hold.

Under additive preferences, (43) amounts to assuming that $\mathcal{E}_x(\sigma) < (\sigma - 1)/\sigma$, which means that σ cannot increase "too fast" with x. In this case, as shown by (42), there exists a unique SFE and the markup function $m(N)$ increases with N provided that the slope of m is smaller than F/Ly. In other words, *a market mimicking anti-competitive effects need not preclude the existence and uniqueness of a SFE* (Chen and Riordan, 2008; Zhelobodko et al., 2012). When preferences are homothetic, (43) holds if and only if $\mathcal{E}_N(\sigma)$ exceeds -1, which means that varieties cannot become too differentiated when their number increases, which seems reasonable.

We consider (43) *and* (40) *as our most preferred assumptions.* The former, which states that the impact of a change in the number of varieties on σ dominates the impact of a change in the per variety consumption, points to the importance of the variety range for consumers, while the latter is a necessary and sufficient for the existence and uniqueness of a SFE. Taken together, (43) and (40) define a range of possibilities which is broader than the one defined by (38). We will refrain from following an encyclopedic approach in which all cases are systematically explored. However, since (40) need not hold for a SFE to exist, we will also explore what the properties of the equilibrium become when this condition is not met. In so doing, we are able to highlight the role played by (40) for some particular results to hold.

4.2 Comparative statics

In this subsection, we study the impact of a higher gross domestic product on the SFE. A higher total income may stem from a larger population L, a higher per capita income y, or both. Next, we will discuss the impact of firm's productivity. To achieve our goal, it proves to be convenient to work with the markup as the endogenous variable. Setting $m \equiv FN/(Ly)$, we may rewrite the equilibrium condition (42) as a function of m:

$$m\sigma\left(\frac{F}{cL}\frac{1-m}{m}, \frac{Ly}{F}m\right) = F. \tag{44}$$

Note that (44) involves the four structural parameters of the economy: L, y, c and F. Furthermore, it is readily verified that the left-hand side of (44) increases with m if and only if (43) holds. Therefore, to study the impact of a specific parameter, we only have to find out how the corresponding curve is shifted.

Before proceeding, we want to stress that we provide below a complete description of the comparative static effects through a series of necessary and sufficient conditions. We acknowledge that some results are more plausible than others. However, the latter can be ruled out through empirical evidence only.

4.2.1 The impact of population size

Let us first consider the impact on the market price p^*. Differentiating (44) with respect to L, we find that the right-hand side of (44) is shifted upwards under an increase in L if and only if (40) holds. As a consequence, the equilibrium markup m^*, whence the equilibrium price p^*, decreases with L. This is in accordance with Handbury and Weinstein (2013) who observe that the price level for food products falls with city size. In this case, (44) implies that the equilibrium value of σ increases, which amounts to saying that varieties get less differentiated in a larger market, very much like in spatial models of product differentiation.

Second, the zero-profit condition implies that L always shifts p^* and q^* in opposite directions. Therefore, firm sizes are larger in bigger markets, as suggested by the empirical evidence provided by Manning (2010).

How does N^* change with L? Differentiating (32) with respect to L, we have

$$\left.\frac{\partial \bar{\pi}}{\partial L}\right|_{N=N^*} = \frac{cx}{\sigma\left(x,N\right)-1} + \frac{\partial \bar{x}(N)}{\partial L}\frac{\partial}{\partial x}\left.\left(\frac{cLx}{\sigma\left(x,N\right)-1}\right)\right|_{x=x^*,N=N^*}. \tag{45}$$

Substituting F for $\bar{\pi}(N^*)$ and simplifying, we obtain

$$\left.\frac{\partial \bar{\pi}}{\partial L}\right|_{N=N^*} = \left.\left[\frac{cx\sigma}{(\sigma-1)^3}\left(\sigma-1-\mathcal{E}_x(\sigma)\right)\right]\right|_{x=x^*,N=N^*}.$$

Since the first term in the right-hand side of this expression is positive, (45) is positive if and only if the following condition holds:

$$\mathcal{E}_x(\sigma) < \sigma - 1. \tag{46}$$

In this case, a population growth triggers the entry of new firms. Furthermore, restating (42) as $N/m(N) = Ly/F$, it is readily verified that the increase in $N^*$is less proportional than the population hike if and only if $m'(N) < 0$, which is equivalent to (40).

Observe that (43) implies (46) when preferences are (indirectly) additive, while (46) holds true under homothetic preferences because $\mathcal{E}_x(\sigma) = 0$.

It remains to determine how the per variety consumption level x^* varies with an increase in population L. Combining (28) with the budget constraint $x = y/(pN)$, we obtain

$$\frac{Nx\sigma(x,N)}{\sigma(x,N)-1} = \frac{y}{c}. \tag{47}$$

Note that L does not enter (47) as an independent parameter. Furthermore, it is straightforward to check that the left-hand side of (47) increases with x when (46) holds, and decreases otherwise. Combining this with the fact that (46) is also necessary and sufficient for an increase in L to trigger additional entry, the per variety consumption level x^* decreases with L if and only if the left-hand side of (47) increases with N, or, equivalently, if and only if

$$\mathcal{E}_N(\sigma) < \sigma - 1. \tag{48}$$

This condition holds if σ decreases with N or increases with N, but not "too fast," which means that varieties do not get too differentiated with the entry of new firms. Note also that (40) and (48) imply (46). Evidently, (48) holds for (i) additive preferences, for in this case $\mathcal{E}_N(\sigma) = 0$, while $\sigma > 1$; (ii) indirectly additive preferences, because, using (46) and $\sigma(x,N) = 1/\theta(xN)$, we obtain $1 + \mathcal{E}_N(\sigma) = 1 + \mathcal{E}_x(\sigma) < \sigma$; and (iii) any preferences such that σ weakly decreases with N.

The following proposition comprises a summary.

Proposition 4. *If $\mathcal{E}_x(\sigma)$ is smaller than $\mathcal{E}_N(\sigma)$, then a higher population size results in a lower markup and larger firms. Furthermore, if* (48) *holds, the mass of varieties increases less than proportionally with L, while the per variety consumption decreases with L.*

Note that the mass of varieties need not rise with the population size. Indeed, N^* falls when $\mathcal{E}_N(\sigma)$ exceeds $\sigma - 1$. In this case, increasing the number of firms makes varieties very close substitutes, which strongly intensifies competition among firms. Under such circumstances, the benefits associated with diversity are low, thus implying that consumers value more and more the volumes they consume. This in turn leads a fraction of existing firms to get out of business.

When preferences are homothetic, σ depends upon N only. In this case, (47) boils down to

$$1 + \frac{N\varphi'(N)}{1-\varphi(N)} > 0.$$

When $\varphi'(N) < 0$, this inequality need not hold. However, in the case of the translog where $\varphi(N) = 1/(1+\beta N)$, (47) is satisfied, and thus x^* decreases with L.

What happens when $\mathcal{E}_x(\sigma) > \mathcal{E}_N(\sigma)$? In this event, (43) implies that (48) holds. Therefore, the above necessary and sufficient conditions imply the following result: If $\mathcal{E}_x(\sigma) < \mathcal{E}_N(\sigma)$, then a higher population size results in a higher markup, smaller firms, a more than proportional rise in the mass of varieties, and a lower per variety consumption. As a consequence, a larger market may generate anti-competitive effects that take the concrete form of a higher market price and less efficient firms producing at a higher average cost. Such results are at odds with the main body of industrial organization, which explains why (40) is one of our most preferred conditions.

4.2.2 The impact of individual income

We now come to the impact of the per capita income on the SFE. One expects a positive shock on y to trigger the entry of new firms because more labor is available for production. However, consumers have a higher willingness-to-pay for the incumbent varieties and can afford to buy each of them in a larger volume. Therefore, the impact of y on the SFE is not straightforward.

Differentiating (44) with respect to y, we see that the left-hand side of (44) is shifted downwards by an increase in y if and only if $\mathcal{E}_N(\sigma) > 0$. In this event, the equilibrium markup decreases with y.

To check the impact of y on N^*, we differentiate (32) with respect to y and get

$$\left.\frac{\partial \bar{\pi}(N)}{\partial y}\right|_{N=N^*} = \left.\left[\frac{\partial \bar{x}(N)}{\partial y}\frac{\partial}{\partial x}\left(\frac{cLx}{\sigma(x,N)-1}\right)\right]\right|_{x=x^*,N=N^*}.$$

After simplification, this yields

$$\left.\frac{\partial \bar{\pi}(N)}{\partial y}\right|_{N=N^*} = \left.\frac{L}{N}\frac{\sigma-1-\sigma\mathcal{E}_x(\sigma)}{(\sigma-1)^2}\right|_{x=x^*,N=N^*}.$$

Hence, $\partial\bar{\pi}(N^*)/\partial y > 0$ if and only if the following condition holds:

$$\mathcal{E}_x(\sigma) < \frac{\sigma-1}{\sigma}. \tag{49}$$

Note that this condition is more stringent than (46). Thus, if $\mathcal{E}_N(\sigma) > 0$, then (49) implies (43).Note that this condition is more stringent than (46). Thus, if $\mathcal{E}_N(\sigma) > 0$, then (49) implies (43).

As a consequence, we have:

Proposition 5. *If $\mathcal{E}_N(\sigma) > 0$, then a higher per capita income results in a lower markup and bigger firms. Furthermore, the mass of varieties increases with y if* (49) *holds and decreases with y otherwise.*

Thus, when entry renders varieties less differentiated, the mass of varieties need not rise with income. Indeed, the increase in per variety consumption may be too high for all the incumbents to stay in business. The reason for this is that the decline in prices is strong enough to lead to fewer firms operating at a much larger scale. As a consequence, a richer economy need not exhibit a wider array of varieties.

Evidently, if $\mathcal{E}_N(\sigma) < 0$, the markup is higher and firms are smaller when the income y rises. Furthermore, (43) implies (49) so that N^* increases with y. Indeed, since varieties get more differentiated when entry arises, firms exploit consumers' higher willingness-to-pay to sell less at a higher price, which goes together with a larger mass of varieties.

Propositions 4 and 5 show that an increase in L is not a substitute for an increase in y and vice versa, except, as shown below, in the case of homothetic preferences. This should not come as a surprise because an increase in income affects the shape of individual demands when preferences are non-homothetic, whereas an increase in L shifts upward the market demand without changing its shape.

Finally, observe that using (indirectly) additive utilities allows capturing the effects generated by shocks on population size (income), but disregard the impact of the other magnitude. Propositions 4 and 5 thus extend results obtained by Zhelobodko et al. (2012) and Bertoletti and Etro (2013). If preferences are homothetic, it is well known that the effects of L and y on the market variables p^*, q^* and N^* are exactly the same. Indeed, m does not involve y as a parameter because σ depends solely on N. Therefore, it follows from (42) that the equilibrium price, firm size, and number of firms depend only upon the total income yL.

4.2.3 The impact of firm productivity

Firms' productivity is typically measured by their marginal costs. To uncover the impact on the market outcome of a productivity shock common to all firms, we conduct a comparative static analysis of the SFE with respect to c and show that *the nature of preferences determines the extent of the pass-through*. In particular, we establish that the pass-through is lower (higher) than 100% if and only if σ decreases (increases) with x, i.e.

$$\mathcal{E}_x(\sigma) < 0 \qquad (0 < \mathcal{E}_x(\sigma)) \tag{50}$$

holds.

Figure 2 depicts (42). It is then straightforward to check that, when σ decreases (increases) with x, a drop in c moves the vertical line rightward (leftward) while the p^*-locus is shifted downward. As a consequence, the market price p^* decreases with c. But by how much does p^* decrease relative to c?

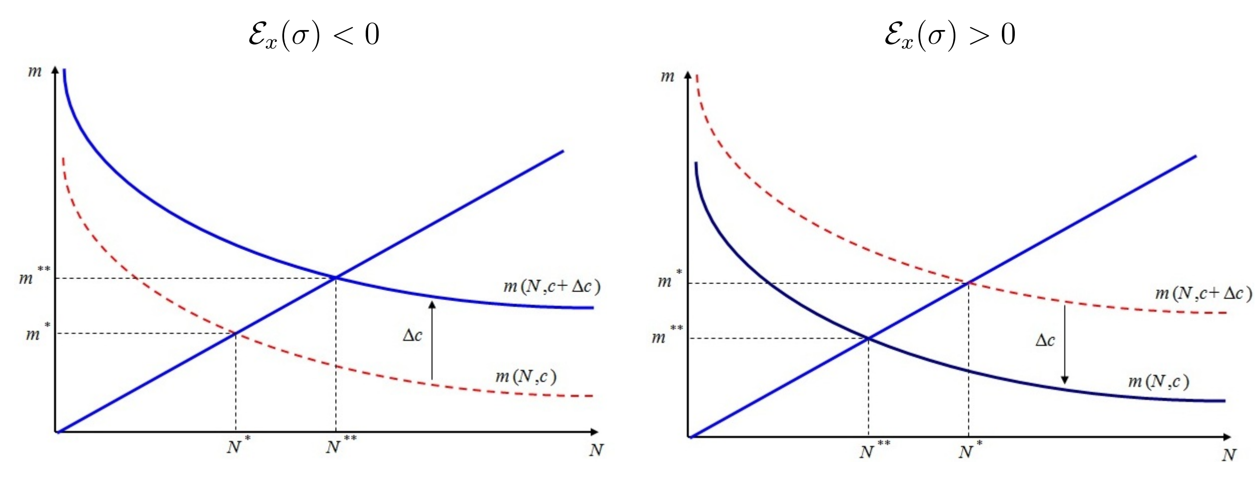

Fig. 2. Productivity and entry.

The left-hand side of (44) is shifted downwards (upwards) under a decrease in c if $\mathcal{E}_x(\sigma) < 0$ ($\mathcal{E}_x(\sigma) > 0$). In this case, both the equilibrium markup m^* and the equilibrium mass of firms

$N^* = (yL/F) \cdot m^*$ increases (decreases) with c. In other words, when $\mathcal{E}_x(\sigma) < 0$, *the pass-through is rate smaller than* 1 because varieties becomes more differentiated, which relaxes competition. On the contrary, when $\mathcal{E}_x(\sigma) > 0$, the markup and the mass of firms decrease because varieties get less differentiated. In other words, competition becomes so tough that p^* decreases more than proportionally with c. In this event, *the pass-through rate exceeds* 1.

Under homothetic preferences, ($\mathcal{E}_x(\sigma) = 0$), $\bar{p}(N)$ is given by

$$\bar{p}(N) = \frac{c}{1 - \varphi(N)} \Longrightarrow m(N) = \varphi(N).$$

As a consequence, (42) does not involve c as a parameter. This implies that a technological shock does affect the number of firms. In other words, the markup remains the same regardless of the productivity shocks, thereby implying that *under homothetic preferences the pass-through rate equal to* 1.

The impact of technological shocks on firms' size leads to ambiguous conclusions. For example, under quadratic preferences, q^* may increase and, then, decreases in response to a steadily drop in c.

The following proposition comprises a summary.

Proposition 6. *If the marginal cost of firms decreases,* (i) *the market price decreases and* (ii) *the markup and number of firms increase if and only if* (50) *holds.*

This proposition has an important implication. If the data suggest a pass-through rate smaller than 1, then it must be that $\mathcal{E}_x(\sigma) < 0$. In this case, (46) always holds while (43) is satisfied when $\mathcal{E}_N(\sigma)$ exceeds -1, thereby a bigger or richer market is more competitive and more diversified than a smaller or poorer one. However, the empirical evidence shows that the pass-through generated by a commodity tax or by trade costs need not be smaller than 1 (see, e.g., Martin, 2012, and Weyl and Fabinger, 2014). Note that (43) does not restrict the domain of admissible values of $\mathcal{E}_x(\sigma)$ for a pass-through rate smaller than 1, whereas (43) requires that $\mathcal{E}_x(\sigma)$ cannot exceed $(1 - 1/\sigma)(1 + \mathcal{E}_N(\sigma))$. Our theoretical argument thus concurs with the inconclusive empirical evidence: the pass-through rate may exceed 1, but it is more likely to be less than 1.

4.2.4 Monopolistic or oligopolistic competition

It should be clear that Propositions 4-6 have the same nature as results obtained in similar comparative analyses conducted in oligopoly theory (Vives, 1999). They may also replicate the less standard anti-competitive effects that a larger market size may trigger under some specific conditions. Therefore, we find it fair to say that *our model of monopolistic competition under non-separable preferences mimics oligopolistic competition.*

Observe that the markup (31) stems directly from preferences through the sole elasticity of substitution because we focus on monopolistic competition. However, in symmetric oligopoly models the markup emerges as the outcome of the interplay between preferences *and* strategic interactions. To illustrate, consider the case of quantity-setting firms and additive preferences over a finite-dimensional consumption set:

$$\mathcal{U}(x_1, ..., x_N) = \sum_{i=1}^{N} u(x_i)$$

where N is, for the moment, an integer. The inverse demands are given by

$$p_i = \frac{u'(x_i)}{\lambda} \qquad \lambda = \frac{1}{y}\sum_{i=1}^{N} x_i u'(x_i).$$

Unlike monopolistic competition, each firm can manipulate λ. This is captured by the first-order conditions for profit maximization:

$$\frac{p_i - c}{p_i} = r(x_i) + \mathcal{E}_{x_i}(\lambda).$$

At the symmetric outcome, this expression boils down to

$$\frac{p - c}{p} = r\left(x\right) + \frac{1}{N}(1 - r(x)) \tag{51}$$

while, under monopolistic competition with non-additive preferences, we have

$$\frac{p - c}{p} = \frac{1}{\sigma(x, N)}. \tag{52}$$

Comparing (51) and (52) shows that, when preferences are additive, the markup decreases with N under Cournot competition, as it does under monopolistic competition and non-additive preferences.

4.3 When is the SFE socially optimal?

The social planner faces the following optimization problem:

$$\max \mathcal{U}(\mathbf{x}) \qquad \text{s.t.} \quad Ly = cL\int_0^N x_i \mathrm{d}i + NF.$$

The first-order condition with respect to x_i implies that the problem may be treated using symmetry, so that the above problem may be reformulated as maximizing

$$\phi(x, N) \equiv \mathcal{U}\left(xI_{[0,N]}\right)$$

subject to $Ly = N(cLx + F)$.

The ratio of the first-order conditions with respect to x and N leads to

$$\frac{\phi_x}{\phi_N} = \frac{NcL}{cLx + F}. \tag{53}$$

It is well known that the comparison of the social optimum and market outcome leads to ambiguous conclusions for the reasons highlighted by Spence (1976). We illustrate here this difficulty in the special case of homothetic preferences. Without loss of generality, we can write $\phi(N, x)$ as follows:

$$\phi(N, x) = N\psi(N)x$$

where $\psi(N)$ is an increasing function of N. In this event, we get $\phi_x x/\phi = 1$ and $\phi_N N/\phi = 1 + N\psi'/\psi$. Therefore, (53) becomes

$$\mathcal{E}_N(\psi) = \frac{F}{cLx}$$

while the market equilibrium condition (42) is given by

$$\frac{\varphi(N)}{1 - \varphi(N)} = \frac{F}{cLx}.$$

The social optimum and the market equilibrium are identical if and only if

$$\mathcal{E}_N(\psi) = \frac{\varphi(N)}{1 - \varphi(N)}. \tag{54}$$

It should be clear that this condition is unlikely to be satisfied unless strong restrictions are imposed on the utility. To be concrete, denote by $A(N)$ the solution to

$$\mathcal{E}_N(A) + \mathcal{E}_N(\psi) = \frac{\varphi(N)}{1 - \varphi(N)}$$

which is unique up to a positive coefficient. It is then readily verified that (54) holds for all N if and only if $\phi(x, N)$ is replaced with $A(N)\phi(x, N)$. Thus, contrary to the folk wisdom, the equilibrium and the optimum may be the same for utility functions that differ from the CES (Dhingra and Morrow, 2013). This finding has an unexpected implication: *when preferences are homothetic, whatever the market outcome there exists a consumption externality such that the equilibrium is optimal regardless of the values taken by the parameters of the economy.* Hence, the choice of a particular consumption externality has subtle welfare implications, which are often disregarded in the literature. For example, if we multiply $A(N)$ by N^ν, where ν is a constant, there is growing under- (over-) provision of varieties when the difference $\nu - 1/(\sigma - 1) > 0$ rises ($\nu - 1/(\sigma - 1) < 0$ falls).

Last, in the case of additive preferences, (54) amounts to

$$m(x^*) = r(x^*) = 1 - \frac{x^* u'(x^*)}{u(x^*)}$$

which is the condition given by Kuhn and Vives (1999) for the market outcome to be optimal.

5 Extensions

In this section, we first extend our baseline model to cope with a multisector economy. We then discuss the cases of heterogeneous firms and heterogeneous consumers.

5.1 Multisector economy

Following Dixit and Stiglitz (1977), we consider a two-sector economy involving a differentiated good supplied under increasing returns and monopolistic competition, and a homogeneous good - or a Hicksian composite good - supplied under constant returns and perfect competition. Both goods are normal. Labor is the only production factor and is perfectly mobile between sectors. Consumers share the same preferences given by $U(\mathcal{U}(\mathbf{x}), x_0)$ where the functional $\mathcal{U}(\mathbf{x})$ satisfies the properties stated in Section 2, while x_0 is the consumption of the homogeneous good. The upper-tier utility U is strictly quasi-concave, once continuously differentiable, strictly increasing in each argument, and such that the demand for the differentiated product is always positive.[7]

Choosing the unit of the homogeneous good for the marginal productivity of labor to be equal to 1, the equilibrium price of the homogeneous good is equal to 1. Since profits are zero at the free-entry equilibrium, the budget constraint is given by

$$\int_0^N p_i x_i \mathrm{d}i + x_0 = E + x_0 = y \tag{55}$$

where the expenditure E on the differentiated good is endogenous because competition across firms affects the relative price of this good.

Using the first-order condition for utility maximization yields

$$p_i = \frac{U_1'(\mathcal{U}(\mathbf{x}), x_0)}{U_2'(\mathcal{U}(\mathbf{x}), x_0)} D(x_i, \mathbf{x}).$$

Let p be arbitrarily given. Along the diagonal $x_i = x$, this condition becomes

$$p = S(\phi(x, N), x_0) D(x, x I_{[0,N]}) \tag{56}$$

where S is the marginal rate of substitution between the differentiated and homogeneous goods:

[7] Our results hold true if the choke price is finite but sufficiently high.

$$S(\phi, x_0) \equiv \frac{U_1'(\phi(x,N), x_0)}{U_2'(\phi(x,N), x_0)}$$

and $\phi(x, N) \equiv \mathcal{U}\left(xI_{[0,N]}\right)$.

The quasi-concavity of the upper-tier utility U implies that the marginal rate of substitution decreases with $\phi(x, N)$ and increases with x_0. Therefore, for any given (p, x, N), (56) has a unique solution $\bar{x}_0(p, x, N)$, which is the income-consumption curve. The two goods being normal, this curve is upward slopping in the plane (x, x_0).

For any given $x_i = x$, the love for variety implies that the utility level increases with the number of varieties. However, it is reasonable to suppose that the marginal utility D of an additional variety decreases. To be precise, we assume that

(B) *for all $x > 0$, the marginal utility D weakly decreases with the number of varieties.*

Observe that **(B)** holds for additive and quadratic preferences. Since $\phi(x, N)$ increases in N, S decreases. As D weakly decreases in N, it must be that x_0 increases for the condition (56) to be satisfied. In other words, $\bar{x}_0(p, x, N)$ increases in N.

We now pin down a particular value of x by using the zero-profit condition. Since by definition $m \equiv (p - c)/p$, for any given p the zero-profit and product market clearing conditions yield the per variety consumption as a function of m only:

$$x = \frac{F}{cL}\frac{1-m}{m}. \tag{57}$$

Plugging (57) and $p = c/(1 - m)$ into $\bar{x}_0$, we may rewrite $\bar{x}_0(p, x, N)$ as a function of m and N only:

$$\tilde{x}_0(m, N) \equiv \bar{x}_0\left(\frac{c}{1-m}, \frac{F}{cL}\frac{1-m}{m}, N\right).$$

Plugging (57) and $p = c/(1 - m)$ into the budget constraint (55) and solving for N, we obtain the income y such that the consumer chooses the quantity $\tilde{x}_0(m, N)$ of the homogeneous good:

$$N = \frac{Lm}{F}\left[y - \tilde{x}_0(m, N)\right]. \tag{58}$$

Since $\bar{x}_0$ and $\tilde{x}_0$ vary with N identically, $\tilde{x}_0$ also increases in N. Therefore, (58) has a unique solution $\tilde{N}(m)$ for any $m \in [0, 1]$.

Moreover, (58) implies that $\partial\tilde{N}/\partial y > 0$, while $\partial\tilde{N}/\partial L > 0$ because the income-consumption curve is upward slopping. In other words, if the price of the differentiated product is exogenously given, an increase in population size or individual income leads to a wider range of varieties.

Since $\tilde{N}(m)$ is the number of varieties in the two-sector economy, the equilibrium condition (44) must be replaced with the following expression:

$$m\sigma\left(\frac{F}{cL}\frac{1-m}{m}, \tilde{N}(m)\right) = 1. \tag{59}$$

The left-hand side $m\sigma$ of (59) equals zero for $m = 0$ and exceeds 1 when $m = 1$. Hence, by the intermediate value theorem, the set of SFEs is non-empty. Moreover, it has an infimum and a supremum, which are both SFEs because the left-hand side of (59) is continuous. In what follows, we denote the corresponding markups by $m_{\inf}$ and $m_{\sup}$; if the SFE is unique, $m_{\inf} = m_{\sup}$. Therefore, the left-hand side of (59) must increase with m in some neighborhood of $m_{\inf}$, for otherwise there would be an equilibrium to the left of $m_{\inf}$, a contradiction. Similarly, the left-hand side of (59) increases with m in some neighborhood of $m_{\sup}$.

Since $\partial\tilde{N}/\partial y > 0$, (59) implies that an increase in y shifts the locus $m\sigma$ upward if and only if $\mathcal{E}_N(\sigma) > 0$, so that the equilibrium markups $m_{\inf}$ and $m_{\sup}$ decrease in y. Consider now an increase in population size. Since $\partial\tilde{N}/\partial L > 0$, (59) implies that an increase in L shifts the locus $m\sigma$ upward if both $\mathcal{E}_x(\sigma) < 0$ and $\mathcal{E}_N(\sigma) > 0$ hold. In this event, the equilibrium markups $m_{\inf}$ and $m_{\sup}$ decrease in L.

Summarizing our results, we come to a proposition.

Proposition 8. *Assume* **(B)**. *Then, the set of SFEs is non-empty. Furthermore,* (i) *an increase in individual income leads to a lower markup and bigger firms at the infimum and supremum SFEs if and only if* $\mathcal{E}_N(\sigma) > 0$ *and* (ii) *an increase in population size yields a lower markup and bigger firms at the infimum and supremum SFEs if* $\mathcal{E}_x(\sigma) < 0$ *and* $\mathcal{E}_N(\sigma) > 0$.

This extends to a two-sector economy what Propositions 4 and 5 state in the case of a one-sector economy where the SFE is unique. Proposition 8 also shows that the elasticity of substitution keeps its relevance for studying monopolistic competition in a multisector economy. In contrast, studying how N^* changes with L or y is more problematic because the equilibrium number of varieties depends on the elasticity of substitution between the differentiated and homogeneous goods.

5.2 Heterogeneous firms

It is legitimate to ask how the approach developed in this paper can cope with heterogeneous firms à la Melitz. Studying monopolistic competition as a price game appears to be more convenient when firms are heterogeneous. To describe this game, we have to define the demand functions away from the diagonal because the equilibrium is no longer symmetric. If the utility functional $\mathcal{U}$ is strictly quasi-concave, the consumer's problem has a unique solution given by the Marshallian demands $\mathcal{D}(p_i, \mathbf{p}, y)$.

5.2.1 Existence

We consider the one-period framework proposed by Melitz and Ottaviano (2008) when the mass of potential firms is still $\mathcal{N}$. Prior to entry, risk-neutral firms face uncertainty about their marginal cost and entry requires a sunk cost F_e. Once the entry cost is paid, firms observe their marginal cost drawn randomly from the continuous probability distribution $G(c)$ defined over $\mathbb{R}_+$. After observing its type c, each entrant decides to produce or not, given that an active firm incurs a fixed production cost F. Even though varieties are differentiated from the consumer's point of view, firms sharing the same marginal cost c behave in the same way. As a consequence, we may refer to any available variety by its c-type only.

The new equilibrium conditions are as follows (the subscript c refers to the marginal cost of the corresponding firms):

(i) the profit-maximization condition for firms of c-type:

$$\pi_c^*(\mathbf{p}^*) \equiv \max_{p\geq 0} \{L(p-c)\mathcal{D}(p, \mathbf{p}^*, y) - F\}$$

(ii) the product market clearing condition:

$$q_c = Lx_c \qquad c \in [0, \bar{c}]$$

(iii) the labor market clearing condition:

$$N_e F_e + \int_0^{\bar{c}} (cq_c + F)\mathrm{d}G(c) = yL$$

where N_e is the number of entrants;

(iv) the zero-profit condition for the cutoff firm $\bar{c}$:

$$(p_{\bar{c}} - \bar{c})q_{\bar{c}} = F$$

(v) firms enter the market until their expected profits net of entry costs F_e are zero:

$$\int_0^{\bar{c}} \pi_c^*(\mathbf{p}^*)\mathrm{d}G(c) = F_e. \tag{60}$$

When firms are heterogeneous, the price schedule is the fixed point of a mapping describing the above equilibrium conditions in a functional space. This turns out to be a hard task. Rather, we will use Tarski's fixed point theorem (Vives, 1999, ch.2). To do this, we need the following assumptions: assuming that the demand $\mathcal{D}(p_c, \mathbf{p}, y)$ for any variety produced by a c-type firm is differentiable with respect to p_c, its elasticity

$$\bar{\varepsilon}(p_c, \mathbf{p}, y) \equiv -\frac{\partial \mathcal{D}}{\partial p_c}\frac{p_c}{\mathcal{D}}$$

increases in p_c as in **(Abis)** and decreases when a non-zero measure set of competitors raise their prices. We also assume that $\bar{\varepsilon}(p_c, \mathbf{p}, y) > 1$ for the profit maximization conditions to have a solution.

Let us illustrate what the two conditions on $\bar{\varepsilon}(p_c, \mathbf{p}, y)$ are for our usual examples. Under additive preferences, that $\bar{\varepsilon}(p_c, \mathbf{p}, y)$ increases in p_c and decreases in $\mathbf{p}$ amounts to assuming that (i) $r(x_c)$ is an increasing function of x_c and (ii) $r(x_c) < 1$ for all $x \geq 0$. Indeed, $(u')^{-1}(\lambda p_c)$ is the demand for variety c, while its elasticity with respect to p_c is given by

$$\bar{\varepsilon}(p_c, \mathbf{p}, y) = \frac{1}{r\left[(u')^{-1}(\lambda(\mathbf{p})p_c)\right]} > 1 \tag{61}$$

where $\lambda(\mathbf{p})$ is the implicit solution to the budget constraint:

$$\int_0^{\bar{c}} (u')^{-1}(\lambda p_c)p_c \mathrm{d}G(c) = y. \tag{62}$$

Therefore, the elasticity of a c-type firm's revenue with respect to p_c is negative, thereby implying that a firm's revenue $(u')^{-1}(\lambda p_c)p_c$ decreases with p_c. It then follows from (62) that $\lambda(\mathbf{p})$ also decreases with $\mathbf{p}$. Combining this with $\sigma'(x) > 0$ or $r'(x) > 0$, we obtain from (61) that $\bar{\varepsilon}(p_c, \mathbf{p}, y)$ increases in p_c and decreases in $\mathbf{p}$.

For indirectly additive preferences, the elasticity $\bar{\varepsilon}(p_c, \mathbf{p}, y)$ is independent of $\mathbf{p}$. Existence thus follows immediately when $\bar{\varepsilon}$ increases with p_c. In the case of translog preferences, we have:

$$\mathcal{D}(p_i, \mathbf{p}, y) = \frac{y}{p_i}\left(\frac{1}{N} + \frac{\beta}{N}\int_0^N \ln p_j \mathrm{d}j - \beta \ln p_i\right)$$

whereas the elasticity is given by

$$\bar{\varepsilon}(p_i, \mathbf{p}) = 1 + \beta\frac{p_i}{\frac{1}{N} + \frac{\beta}{N}\int_0^N \ln p_j \mathrm{d}j - \beta \ln p_i}$$

which is increasing in p_i and decreasing in $\mathbf{p}$ when N is constant.

In what follows, we prove the existence of a Nash price equilibrium when the cutoff cost $\bar{c}$ and the number of entrants N_e are arbitrarily given.

Proposition 9. *Assume that firms are heterogeneous. If $\bar{c}$ and N_e are given and the demand elasticity $\bar{\varepsilon}(p_c, \mathbf{p}, y)$ increases in p_c and decreases in $\mathbf{p}$, then a Nash equilibrium of the price game exists.*

The sketch of the proof is as follows. The first-order condition for a c-type firm, conditional on the vector of prices $\mathbf{p}$ charged by the other firms, is given by

$$\frac{p_c - c}{p_c} = \frac{1}{\bar{\varepsilon}(p_c, \mathbf{p}, y)}. \tag{63}$$

Since the left- (right-)hand side of (63) continuously increases (decreases) with p_c for any given $\mathbf{p}$, there exists a unique $\hat{p}_c(\mathbf{p})$ that solves (63) for any given $\mathbf{p} \in L_2([0, \bar{c}])$ and $c \in [0, \bar{c}]$. Because $\bar{\varepsilon}$ decreases with $\mathbf{p}$, $\hat{p}_c(\mathbf{p})$ increases both in $\mathbf{p}$. Finally, since the left-hand side of (63) decreases in c, $\hat{p}_c(\mathbf{p})$ also increases in c.

Let us define the best-reply mapping $\mathcal{P}$ from $L_2([0, \bar{c}])$ into itself:

$$\mathcal{P}(\mathbf{p}; c) \equiv \hat{p}_c(\mathbf{p}).$$

The proposition holds true because $\mathcal{P}$ has a fixed point $\mathbf{p}^*(\bar{c}, N_e)$ as shown in Appendix 5. Plugging $\mathbf{p}^*(\bar{c}, N_e)$ into $\pi^*(c, \mathbf{p}^*)$ yields $\pi^*(c, \bar{c}, N_e)$, which is a strictly decreasing function of c. Note that $\bar{c}$ and N_e may be viewed as two market aggregates.

We now come to the cutoff cost. Consider two firms with marginal costs c and c' such that $c > c'$. Evidently, we have

$$(p_c - c)\mathcal{D}(p_c, \mathbf{p}, y) < (p_c - c')\mathcal{D}(p_c, \mathbf{p}, y) \qquad (p_c, \mathbf{p}) \in \mathbb{R}_+ \times L_2([0, \mathcal{N}])$$

which implies the perfect sorting of firms along their cost type. As a consequence, if there exists a solution $\bar{c}(\mathbf{p})$ to the equation

$$\pi_c^*(\mathbf{p}) = 0$$

this solution is unique. Furthermore, when the Inada conditions (20) hold, the above equation has a solution. Without imposing more structure on preferences and the cost distribution, the cutoff cost need not be monotone in the price vector $\mathbf{p}$. Therefore, competitive shocks generate complex effects in the selection of firms whose study is beyond the scope of this paper.

5.2.2 Examples

At the above level of generality, it is hard to determine what Propositions 3 to 6 become. Nevertheless, this can be accomplished for the special classes of preferences discussed in the case of homogeneous firms.

Under *additive* preferences and Melitz-like heterogeneous firms, Zhelobodko et al. (2012) have shown that the cutoff cost decreases (increases) while the equilibrium price distribution is shifted downward (upward) when σ decreases (increases) with x, thus generalizing Proposition 4 (4bis), whereas the per capita income has no impact. By contrast, when preferences are *indirectly additive*, Bertoletti and Etro (2013) demonstrated that the population size has no impact on the cutoff cost and the equilibrium price and firm size distributions. All these results hold regardless of the cost distribution.

Let us now turn to *homothetic* preferences. First, we show that our result on complete pass-through under homothetic preferences still holds when firms are heterogeneous. To this end, consider a proportionate drop in marginal costs by a factor $\mu > 1$, so that the distribution of marginal costs is now given by $G(\mu c)$. We start by investigating the impact of μ on firms' operating profits when the cutoff $\bar{c}$ is unchanged. The cutoff firms now have a marginal cost equal to $\bar{c}/\mu$. Furthermore, under homothetic preferences, $\bar{\varepsilon}(p_i, \mathbf{p})$ does not depend on the income y and is positive homogeneous of degree 0. Therefore, (63) is invariant to the same proportionate reduction in c, p_c and $\mathbf{p}$. As a consequence, the new price equilibrium profile over $[0, \bar{c}]$ is obtained by dividing all prices by μ. To put it differently, regardless of the cost distribution, *under homothetic preferences the equilibrium price distribution changes in proportion with the cost distribution*, thereby leaving unchanged the distribution of equilibrium markups, as in Proposition 6.

We now show that the profits of the $\bar{c}$-type firms do not change in response to the cost drop. Indeed, both marginal costs and prices are divided by μ, while homothetic preferences imply that demands are shifted upwards by the same factor μ. Therefore, the operating profit of the $\bar{c}$-type firms is unchanged because

$$\left(\frac{p_{\bar{c}}}{\mu} - \frac{\bar{c}}{\mu}\right) L\mu x_{\bar{c}} = \left(p_{\bar{c}} - \bar{c}\right) L x_{\bar{c}} = F$$

which also shows that *the new cutoff is given by* $\bar{c}/\mu$.

Last, we discuss the existence of a free-entry equilibrium under homothetic preferences. The indirect utility is given by

$$V(\mathbf{p}, y) = \frac{y}{P(\mathbf{p})}$$

where $P(\mathbf{p})$ is an increasing, linear homogeneous and concave function of $\mathbf{p}$, which is assumed to be continuously differentiable. The function $P(\mathbf{p})$ has the nature of a "price index," and its value expresses the competitiveness of the economy.

(i) Consider for the moment the case where the number of varieties is finite, $i = 1, ..., n$. Then, using Roy's identity, we obtain the demand for variety i:

$$\mathcal{D}(p_i, \mathbf{p}, y) = \frac{y}{p_i}\frac{\partial \ln P(\mathbf{p})}{\partial \ln p_i}. \tag{64}$$

In the spirit of what we did in the case of homogeneous firms, we find it legitimate to ask the following question: can we reduce the dimensionality of the problem with heterogeneous firms? This can achieved by using a statistic that has the nature of a price index, but which need not be P. To be precise, we reformulate our question as follows: under which conditions are the Marshallian demands (64) given by

$$\mathcal{D}(p_i, \mathbf{p}, y) = \frac{y}{p_i}\bar{\theta}\left(p_i, \mathbb{P}(\mathbf{p})\right) \tag{65}$$

where $\mathbb{P}(\mathbf{p})$ is a price statistic. Since demands are homogeneous of degree zero in prices and income, $\bar{\theta}$ must be a homogeneous function of degree zero in $\mathbf{p}$. Furthermore, for the elasticity of $\mathcal{D}$ to exceed 1, $\bar{\theta}$ must decrease in p_i.

We show in Appendix 6 that, without loss of generality, we can restrict ourselves to price statistics $\mathbb{P}(\mathbf{p})$ that are linear homogeneous in $\mathbf{p}$. Therefore, (65) can be rewritten as follows:

$$\mathcal{D}(p_i, \mathbf{p}, y) = \frac{y}{p_i}\theta\left(\frac{p_i}{\mathbb{P}(\mathbf{p})}\right). \tag{66}$$

Since θ is decreasing in p_i, (66) implies that the demand $\mathcal{D}$ is shifted upward when $\mathbb{P}$ takes on a higher value. In other words, $\mathbb{P}$ has the nature of an inverse measure of the competitiveness of the economy, very much like a price index P. However, we will show that $\mathbb{P} = P$ if and only if preferences are CES.

Plugging (66) into the budget constraint shows that $\mathbb{P}$ is implicitly defined as the solution to the equation:

$$\sum_{i=1}^{n}\theta\left(\frac{p_i}{\mathbb{P}(\mathbf{p})}\right) = 1 \tag{67}$$

while comparing (64) and (66) shows that the price index $P(\mathbf{p})$ is related to the price statistic $\mathbb{P}(\mathbf{p})$ as follows:

$$\frac{\partial \ln P(\mathbf{p})}{\partial \ln p_i} = \theta\left(\frac{p_i}{\mathbb{P}(\mathbf{p})}\right) \qquad i = 1, ...n. \tag{68}$$

As a result, (67) and (68) characterize the class of homothetic preferences whose demands depends on a single price statistic. This class contains empirically relevant preferences. For example, when preferences are CES, we have

$$\frac{\partial \ln P(\mathbf{p})}{\partial \ln p_i} = \frac{\exp\left((1-\sigma)\ln p_i\right)}{\sum_{j=1}^{n}\exp\left((1-\sigma)\ln p_j\right)} = \left(\frac{p_i}{\mathbb{P}(\mathbf{p})}\right)^{1-\sigma}$$

hence $\theta(z) = z^{1-\sigma}$ and $\mathbb{P} = P$.[8] Under translog preferences, we have:

$$\ln P(\mathbf{p}) = \frac{1}{n}\sum_{i=1}^{n}\ln p_i - \frac{\beta}{2n}\left[\sum_{i=1}^{n}(\ln p_i)^2 - \frac{1}{n}\left(\sum_{i=1}^{n}\ln p_i\right)^2\right].$$

Therefore,

$$\frac{\partial \ln P(\mathbf{p})}{\partial \ln p_i} = \frac{1}{n}\left[1 - \beta\ln\frac{p_i}{\left(\Pi_{j=1}^{n}p_j\right)^{1/n}}\right].$$

[8] Conversely, it can be shown that replacing $\mathbb{P}$ by P in (68) implies that θ is a power function, which means that preferences are CES.

Thus, we have $\theta(z) = (1 - \beta \ln z)/n$ and $\mathbb{P}(\mathbf{p}) = \left(\Pi_{j=1}^{n} p_j\right)^{1/n}$, that is, the geometric mean of prices.

(ii) Consider now a monopolistically competitive environment with a continuum of heterogeneous firms. The price statistic $\mathbb{P}(\mathbf{p})$ must now solve the following equation:

$$N_e \int_0^{\bar{c}} \theta\left(\frac{p_c}{\mathbb{P}(\mathbf{p})}\right) \mathrm{d}G(c) = 1. \tag{69}$$

The price elasticity of the demand faced by a c-firm is given by

$$\mathcal{E}_{p_c}(\mathcal{D}) = 1 - \frac{p_c}{\mathbb{P}} \frac{\theta'\left(\frac{p_c}{\mathbb{P}}\right)}{\theta\left(\frac{p_c}{\mathbb{P}}\right)} = \sigma\left(\frac{p_c}{\mathbb{P}}\right)$$

where $\sigma(p_c/\mathbb{P})$ is the common elasticity of substitution between any pair of varieties supplied by firms sharing the marginal cost c, which charge the same price p_c. Through the common per capita consumption x_c^* of a variety produced by these firms, σ *depends only upon the price statistic* $\mathbb{P}$, which is common to all firms, *and the price selected by the c-type firms.*[9] Furthermore, σ decreases (increases) in $\mathbb{P}$ if σ increases (decreases) in p_c, which agrees with the assumption that varieties are strong gross substitutes.

We are now equipped to prove the existence and uniqueness of a free-entry equilibrium and to determine the impact of market size Ly on the equilibrium outcome. The profit-maximization condition of a c-type firm may be written as follows:

$$H\left(\frac{p_c}{\mathbb{P}}\right) \equiv \frac{p_c}{\mathbb{P}} \frac{\sigma\left(\frac{p_c}{\mathbb{P}}\right) - 1}{\sigma\left(\frac{p_c}{\mathbb{P}}\right)} = \frac{c}{\mathbb{P}}. \tag{70}$$

This equation has a unique solution in $p_c/\mathbb{P}$ for any $c/\mathbb{P}$ if and only if its left-hand side increases in $p_c/\mathbb{P}$, that is,

$$\mathcal{E}_{p_c}(\sigma) > 1 - \sigma. \tag{71}$$

Since σ exceeds 1, (71) means that the elasticity does not decrease "too fast" in p_c. In particular, (71) holds when σ increases in p_c (see **Abis**), while $\mathcal{E}_{p_c}(\sigma) = 0$ under CES preferences.

Under (71), $p_c^*/\mathbb{P} = h(c/\mathbb{P}) \equiv H^{-1}(c/\mathbb{P})$ is the unique solution to (70). Plugging $p_c^*/\mathbb{P}$ into σ yields

$$\sigma_c^*(\mathbb{P}) \equiv \sigma(h(c/\mathbb{P}))$$

which depends only upon firms' type c and $\mathbb{P}$. In other words, *the elasticity of substitution is now c-specific and varies with* $\mathbb{P}$.

Note that $\mathcal{E}_{p_c}(\sigma) > 0$ ($1 - \sigma < \mathcal{E}_{p_c}(\sigma) < 0$) implies $0 < \mathcal{E}_{c/\mathbb{P}}(h) < 1$ ($\mathcal{E}_{c/\mathbb{P}}(h) > 1$), that is, h increases less (more) than proportionally with $c/\mathbb{P}$. Since the best reply of a c-type firm is given

[9] Because $\mathbb{P}$ is homogeneous linear, at any symmetric outcome, $p_c/\mathbb{P}$, whence σ, depends only upon the mass N of active firms, as in 3.2.

by

$$p_c^*(\mathbb{P}) = \frac{c\sigma_c^*(\mathbb{P})}{\sigma_c^*(\mathbb{P}) - 1} = \mathbb{P}h(c/\mathbb{P})$$

such a firm reacts to a hike in $\mathbb{P}$ by a hike (drop) in its own price if σ increases (decreases) in p_c. In other words, the best reply is upward- (downward-) sloping in $\mathbb{P}$, while the CES is again the borderline case since the corresponding best reply is horizontal.

The profit of a c-type firm gross of entry cost is given by

$$\pi_c^*(\mathbb{P}) = Ly\left[1 - \frac{c/\mathbb{P}}{h(c/\mathbb{P})}\right]\theta\left[h\left(\frac{c}{\mathbb{P}}\right)\right] - F. \tag{72}$$

If $\mathcal{E}_{p_c}(\sigma) > 0$, it is clear that $\pi_c^*(\mathbb{P})$ decreases with c for any given $\mathbb{P}$, that is, *less productive firms earn lower profits.* Moreover, $\pi_c^*(\mathbb{P})$ increases with $\mathbb{P}$, which means that *a more competitive environment leads to lower profits for all active firms.* Although $\mathbb{P}$ differs from P under non-CES preferences, this result shows that $\mathbb{P}$ plays the same role as the price index P under CES preferences.

The least efficient type of active firms is determined from the cutoff condition:

$$\pi_{\bar{c}}^*(\mathbb{P}) = 0. \tag{73}$$

Since π_c^* decreases with c, (73) has a unique solution $\bar{c}(\mathbb{P})$. Furthermore, $\bar{c}(\mathbb{P})$ increases in $\mathbb{P}$, for an increase in $\mathbb{P}$ shifts upwards the π_c^*-locus. Note that the nature of function $\bar{c}(\mathbb{P})$ is independent of the productivity distribution G.

Finally, the free entry condition is given by

$$\int_0^{\bar{c}(\mathbb{P})} \pi_c^*(\mathbb{P})\mathrm{d}G(c) = F_e. \tag{74}$$

Plugging $\bar{c}(\mathbb{P})$ into (74), we find that the left-hand side of (74) increases with $\mathbb{P}$. Hence, (74) pins down the unique equilibrium value $\mathbb{P}^*$ of the price statistic.

Having determined $\mathbb{P}^*$, we obtain the equilibrium value of the cutoff cost $\bar{c}(\mathbb{P}^*)$. The mass N_e of entrants is then uniquely determined by (69). Indeed,

$$N_e(\mathbb{P}) \equiv \frac{1}{\int_0^{\bar{c}(\mathbb{P})} \theta\left[h\left(\frac{c}{\mathbb{P}}\right)\right]\mathrm{d}G(c)} \tag{75}$$

is a decreasing function of $\mathbb{P}$, which is in accordance with the fact that $\mathbb{P}$ is an inverse measure of competitiveness. It follows from (75) that $N_e^* = N_e(\mathbb{P}^*)$. As a consequence, *under homothetic preferences satisfying* (67), (68) *and* $\mathcal{E}_{p_c}(\sigma) > 0$, *there exists a unique free-entry equilibrium.*

As shown by (72), a hike in total income Ly results in an increase in profits for all firms. Thus, we get:

$$\frac{\partial \mathbb{P}^*}{\partial (Ly)} < 0 \quad \frac{\partial \bar{c}}{\partial (Ly)} < 0. \tag{76}$$

Combining (75) with (76), we come to

$$\frac{\partial N_e^*}{\partial (Ly)} > 0.$$

In sum, *the mass of entrants increases while the cutoff cost decreases with the population size, L, and the per capita income, y.* Since $N = N_e$ when firms are homogeneous, these results extend those obtained in 4.2.

When firms are heterogeneous, the mass of active firms $N = N_e G(\bar{c})$ depends on the productivity distribution G because N_e rises, whereas $G(\bar{c})$ falls with L or y.

5.3 Heterogeneous consumers

Accounting for consumer heterogeneity in models of monopolist competition is not easy but doable. Let $\mathcal{D}(p_i, \mathbf{p}; y, \theta)$ be the Marshallian demand for variety i of a (y, θ)-type consumer where θ is the taste parameter. The aggregate demand faced by firm i is then given by

$$\Delta(p_i, \mathbf{p}) \equiv L \int_{\mathbb{R}_+ \times \Theta} \mathcal{D}(p_i, \mathbf{p}; y, \theta) \mathrm{d}G(y, \theta) \tag{77}$$

where G is a continuous joint probability distribution of income y and taste θ.

Ever since the Sonnenschein-Mantel-Debreu theorem (Mas-Colell et al., 1995, ch.17), it is well known that the aggregate demand (77) need not inherit the properties of the individual demand functions. By contrast, as in Section 2, for each variety i, the aggregate demand $\Delta(p_i, \mathbf{p})$ for variety i is decreasing in p_i regardless of the income-taste distribution G. A comparison with Hildenbrand (1983) and Grandmont (1987), who derived specific conditions for the Law of demand to hold when the number of goods is finite, shows how working with a continuum of goods, which need not be the varieties of a differentiated product, vastly simplifies the analysis.

The properties of $\mathcal{D}$ crucially depend on the relationship between income and taste. Indeed, since firm i's profit is given by $\pi(p_i, \mathbf{p}) = (p_i - c)\Delta(p_i, \mathbf{p}) - F$, the first-order condition for a symmetric equilibrium becomes

$$p \left[1 - \frac{1}{\varepsilon(p, N)} \right] = c \tag{78}$$

where $\varepsilon(p, N)$ is the elasticity of $\Delta(p, \mathbf{p})$ evaluated at the symmetric outcome. If $\varepsilon(p, N)$ is an increasing function of p and N, most of the results derived above hold true. Indeed, integrating consumers' budget constraints across $\mathbb{R}_+ \times \Theta$ and applying the zero-profit condition yields the markup.

$$m(N)=\frac{NF}{LY} \qquad \text{where} \quad Y\equiv\int_{\mathbb{R}_+\times\Theta} y\mathrm{d}G(y,\theta). \tag{79}$$

Note that (79) differs from (42) only in one respect: the individual income y is replaced with the mean income Y, which is independent of L. Consequently, if $\varepsilon(p,N)$ decreases both with p and N, a population hike or a productivity shock affects the SFE as in the baseline model (see Propositions 4 and 6). By contrast, the impact of an increase in Y is ambiguous because it depends on how θ and y are related.

There is no reason to expect the aggregate demand to exhibit an increasing price elasticity even when the individual demands satisfy this property. To highlight the nature of this difficulty, we show in Appendix 7 that

$$\begin{gathered}\frac{\partial\varepsilon(p,N)}{\partial p}=\int_{\mathbb{R}_+\times\Theta}\frac{\partial\varepsilon(p,N;\,y,\theta)}{\partial p}s(p,N;\,y,\theta)\mathrm{d}G(y,\theta)-\\ -\frac{1}{p}\int_{\mathbb{R}_+\times\Theta}\left[\varepsilon(p,N;\,y,\theta)-\varepsilon(p,N)\right]^2 s(p,N;\,y,\theta)\mathrm{d}G(y,\theta)\end{gathered} \tag{80}$$

where $\varepsilon(p,N;\,y,\theta)$ is the elasticity of the individual demand $\mathcal{D}(p_i,\mathbf{p};y,\theta)$ evaluated at a symmetric outcome ($p_i=p_j=p$), while $s(p,N;\,y,\theta)$ stands for the share of demand of (y,θ)-type consumers in the aggregate demand, evaluated at the same symmetric outcome:

$$s(p,N;y,\theta)\equiv\left.\frac{\mathcal{D}(p,\mathbf{p};\,y,\theta)}{\Delta(p,\mathbf{p})}\right|_{\mathbf{p}=pI_{[0,N]}}. \tag{81}$$

Because the second term of (80) is negative, the market demand may exhibit decreasing price elasticity even when individual demands display increasing price elasticities. Nevertheless, (80) has an important implication.

Proposition 10. *If individual demand elasticities are increasing and their variance is not too large, then the elasticity of the aggregate demand is increasing, and thus there exists a unique symmetric free-entry equilibrium.*

In this case, all the properties of Section 4 hold true. Yet, when consumers are very dissimilar, like in the Sonnenschein-Mantel-Debreu theorem, the aggregate demand may exhibit undesirable properties.

The equation (80) shows that the effect of heterogeneity in tastes and income generally differ. In particular, consumers with different incomes and identical tastes have different willingness-to-pay for the same variety, which increases the second term in (80). By contrast, if consumers have the same income and only differ in their ideal variety, one may expect the second term in (80) to be close to zero when the market provides these varieties.

The main issue regarding consumer heterogeneity is to study how different types of consumer heterogeneity affect the variance of the distribution of individual elasticities. A first step in this direction has been taken by Di Comite et. al. (2014) who show how the main ingredients of

Hotelling's approach to product differentiation - i.e. taste heterogeneity across consumers who have each a different ideal variety - can be embedded into the quadratic utility while preserving the properties of this model.

6 Concluding remarks

By using a noncooperative game with non-atomic players, we have shown that monopolistic competition can be modeled in a much more general way than what is typically expected. In particular, our approach displays enough versatility to obviate the main pitfalls of the CES model. Furthermore, our framework also mimics a wide range of strategic effects usually captured by oligopoly theory, and it does so without encountering several of the difficulties met in general equilibrium under oligopolistic competition.

We would be the last to say that monopolistic competition is able to replicate the rich array of findings obtained in industrial organization. However, it is our contention that models similar to those presented in this paper may help avoiding several of the limitations imposed by the partial equilibrium analyses of oligopoly theory. Although we acknowledge that monopolistic competition is the limit of oligopolistic equilibria, we want to stress that monopolistic competition may be used in different settings as a substitute for oligopoly models when these ones appear to be unworkable.

Last, some industries are dominated by a few large firms that operate strategically together with a monopolistically competitive fringe in which firms adjust to the big firms' actions. Combining these two types of firms whose market behavior is very different open the door to new issues that we plan to explore in the future.

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Appendix

Appendix 1. Proof of Proposition 1.

(i) We first show (1) for the case where N/k is a positive integer. Note that

$$1_{[0,N]} = \sum_{i=1}^{N/k} 1_{[(i-1)k,\, ik]} \tag{A.1}$$

while symmetry implies

$$\mathcal{U}\left(\frac{X}{k} 1_{[(i-1)k,\, ik]}\right) = \mathcal{U}\left(\frac{X}{k} 1_{[0,k]}\right) \quad \text{for all} \quad i \in \{2, ..., N/k\}. \tag{A.2}$$

Together with quasi-concavity, (A.1) – (A.2) imply

$$\mathcal{U}\left(\frac{X}{N} 1_{[0,N]}\right) = \mathcal{U}\left(\frac{k}{N}\sum_{i=1}^{N/k} \frac{X}{k} 1_{[(i-1)k,\, ik]}\right) > \min_i \mathcal{U}\left(\frac{X}{k} 1_{[(i-1)k,\, ik]}\right) = \mathcal{U}\left(\frac{X}{k} 1_{[0,k]}\right).$$

Thus, (1) holds when N/k is a positive integer.

(ii) We now extend this argument to the case where N/k is a rational number. Let r/s, where both r and s are positive integers and $r \geq s$, be the irredundant representation of N/k. It is then readily verified that

$$s 1_{[0,N]} = \sum_{i=1}^{r} 1_{[N\{(i-1)k/N\},\, N\{ik/N\}]} \tag{A.3}$$

and

$$\mathcal{U}\left(\frac{X}{k}1_{[N\{(i-1)k/N\},\,N\{ik/N\}]}\right)=\mathcal{U}\left(\frac{X}{k}1_{[0,k]}\right)\ \text{ for all }\ i\in\{2,...,r\} \tag{A.4}$$

where the fractional part of the real number a is denoted by $\{a\}$.

Using (A.3) – (A.4) instead of (A.1) – (A.4) in the above argument, we obtain

$$\mathcal{U}\left(\frac{X}{N}1_{[0,N]}\right)=\mathcal{U}\left(\frac{1}{r}\sum_{i=1}^{r}\frac{X}{k}1_{[N\{(i-1)k/N\},\,N\{ik/N\}]}\right)>\mathcal{U}\left(\frac{X}{k}1_{[0,k]}\right).$$

Thus, (1) holds when N/k is rational.

(iii) Finally, since $\mathcal{U}$ is continuous while the rational numbers are dense in $\mathbb{R}_+$, (1) holds for any real number $N/k>1$. Q.E.D.

Appendix 2. Proof of Proposition 2.

It is readily verified that the inverse demands generated by preferences (8) are given by $D(x_i,\mathbf{x})=u'(x_i)$. The uniqueness of the Frechet derivative implies that preferences are additive. This proves part (i).

Assume now that $\mathcal{U}$ is homothetic. Since a utility is defined up to a monotonic transformation, we may assume without loss of generality that $\mathcal{U}$ is homogenous of degree 1. This, in turn, signifies that $D(x_i,\mathbf{x})$ is homogenous of degree 0 with respect to $(x_i,\,\mathbf{x})$. Indeed, because $t\mathcal{U}(\mathbf{x}/t)=\mathcal{U}(\mathbf{x})$ holds for all $t>0$, (2) can be rewritten as follows:

$$\mathcal{U}(\mathbf{x}+\mathbf{h})=\mathcal{U}(\mathbf{x})+\int_0^N D\left(\frac{x_i}{t},\frac{\mathbf{x}}{t}\right)\,h_i\,di+\circ\left(||\mathbf{h}||_2\right). \tag{A.5}$$

Uniqueness of the Frechet derivative together with (A.5) implies

$$D\left(\frac{x_i}{t},\frac{\mathbf{x}}{t}\right)=D(x_i,\mathbf{x})\ \text{ for all }\ t>0$$

which shows that D is homogenous of degree 0. As a result, there exists a functional Φ belonging to $L_2([0,\mathcal{N}])$ such that $D(x_i,\mathbf{x})=\Phi\left(\mathbf{x}/x_i\right)$. Q.E.D.

Appendix 3. We use an infinite-dimensional version of the definition proposed by Nadiri (1982). Setting $D_i=\partial D(x_i,\mathbf{x})/\partial x_i$, the elasticity of substitution between varieties i and j is given by

$$\bar{\sigma}=-\frac{D_iD_j(x_iD_j+x_jD_i)}{x_ix_j\left[\frac{\partial D_i}{\partial x_i}D_j^2-\left(\frac{\partial D_i}{\partial x_j}+\frac{\partial D_j}{\partial x_i}\right)D_iD_j+\frac{\partial D_j}{\partial x_j}D_i^2\right]}.$$

Since $\mathbf{x}$ is defined up to a zero measure set, it must be that

$$\frac{\partial D_i(x_i,\mathbf{x})}{\partial x_j}=\frac{\partial D_j(x_j,\mathbf{x})}{\partial x_i}=0$$

for all $j \neq i$. Therefore, we obtain

$$\bar{\sigma} = -\frac{D_i D_j (x_i D_j + x_j D_i)}{x_i x_j \left(\frac{\partial D_i}{\partial x_i} D_j^2 + \frac{\partial D_j}{\partial x_j} D_i^2 \right)}.$$

Setting $x_i = x_j = x$ implies $D_i = D_j$, and thus we come to $\bar{\sigma} = 1/\bar{\eta}(x, \mathbf{x})$. Q.E.D.

Appendix 4. Let

$$\bar{\varepsilon}(p_i, \mathbf{p}, y) \equiv -\frac{\partial \mathcal{D}(p_i, \mathbf{p}, y)}{\partial p_i} \frac{p_i}{\mathcal{D}(p_i, \mathbf{p}, y)}$$

be the elasticity of the Marshallian demand (6). At any symmetric outcome, we have

$$\varepsilon(p, N) \equiv \bar{\varepsilon}(p, pI_{[0,N]}).$$

Using the budget constraint $p = y/Nx$ and (30) yields

$$\varepsilon(y/Nx, N) = \eta(x, N) = \frac{1}{\sigma(x, N)}. \tag{A.6}$$

When preferences are indirectly additive, it follows from (11) that $\varepsilon(y/Nx, N) = 1 - \theta(y/p)$ where θ is given by (34). Combining this with (A.6), we get $\sigma(x, N) = 1/\theta(Nx)$. Q.E.D.

Appendix 5. $\mathcal{P}$ has a fixed point.

Since $\hat{p}_c(\mathbf{p})$ increases in $\mathbf{p}$, $\mathcal{P}$ is an increasing operator. To check the assumption of Tarski's fixed-point theorem, it suffices to construct a set $S \subset L_2([0, \bar{c}])$ such that (i) $\mathcal{P}S \subseteq S$, i.e. $\mathcal{P}$ maps the lattice S into itself and (ii) S is a complete lattice.

Denote by $\bar{p}$ the unique symmetric equilibrium price when all firms share the marginal cost $\bar{c}$ (which exists as shown in Section 4) and observe that $\bar{p} = \hat{p}_{\bar{c}}(\bar{\mathbf{p}})$, where $\bar{\mathbf{p}} \equiv \bar{p}1_{[0,\bar{c}]}$. Since $\hat{p}_c(\mathbf{p})$ increases in c, we have $\bar{p} \geq \hat{p}_c(\bar{\mathbf{p}})$ for all $c \in [0, \bar{c}]$ or, equivalently, $\bar{\mathbf{p}} \geq \mathcal{P}\bar{\mathbf{p}}$. Furthermore, because $\mathcal{P}$ is an increasing operator, $\mathbf{p} \leq \bar{\mathbf{p}}$ implies $\mathcal{P}\mathbf{p} \leq \mathcal{P}\bar{\mathbf{p}} \leq \bar{\mathbf{p}}$. In addition, $\mathcal{P}\mathbf{p}$ is an increasing function of c because $\hat{p}_c(\mathbf{p})$ increases in c. In other words, $\mathcal{P}$ maps the set S of all non-negative weakly increasing functions bounded above by $\bar{p}$ into itself. It remains to show that S is a complete lattice, i.e. any non-empty subset of S has a supremum and an infimum that belong S. This is so because pointwise supremum and pointwise infimum of a family of increasing functions are also increasing.

To sum-up, since S is a complete lattice and $\mathcal{P}S \subseteq S$, Tarski's theorem implies that $\mathcal{P}$ has a fixed point. Q.E.D.

Appendix 6. Since Marshallian demands are homogeneous of degree zero in $\mathbf{p}$ and y, $\bar{\theta}(p_i, \mathbb{P}(\mathbf{p}))$ is homogeneous of degree zero in $\mathbf{p}$. Applying Euler's theorem yields

$$\frac{\partial \bar{\theta}}{\partial p_i} p_i + \frac{\partial \bar{\theta}}{\partial \mathbb{P}} \sum_{j=1}^{n} \frac{\partial \mathbb{P}}{\partial p_j} p_j = 0 \qquad i = 1, ..., n.$$

This implies that

$$\sum_{j=1}^{n} \frac{\partial \mathbb{P}}{\partial p_j} p_j = \frac{\frac{\partial \bar{\theta}}{\partial p_i}}{\frac{\partial \bar{\theta}}{\partial \mathbb{P}}} p_i \qquad i = 1, ..., n$$

which holds if and only if the left-hand side of the expression is equal to a constant K. In other words, $\mathbb{P}$ is homogeneous of degree K in $\mathbf{p}$. Choosing $\mathbb{P}^{1/K}$ yields the price statistic we need. Q.E.D.

Appendix 7. At a symmetric outcome the aggregate demand elasticity is given by

$$\varepsilon(p, N) = \int_{\mathbb{R}_+ \times \Theta} \varepsilon(p, N;\, y, \theta) s(p, N;\, y, \theta) \mathrm{d}G(y, \theta) \tag{A.7}$$

where $s(p, N;\, y, \theta)$ is the share of the (y, θ)-type consumer's individual demand in the aggregate demand.

Differentiating (A.7) with respect to p yields

$$\frac{\partial \varepsilon(p, N)}{\partial p} = \int_{\mathbb{R}_+ \times \Theta} \left(\frac{\partial \varepsilon(p, N;\, y, \theta)}{\partial p} s + \varepsilon(p, N;\, y, \theta) \frac{\partial s}{\partial p} \right) \mathrm{d}G(y, \theta). \tag{A.8}$$

Using (81), we obtain

$$\mathcal{E}_p(s) = \varepsilon(p, N) - \varepsilon(p, N;\, y, \theta).$$

Hence,

$$\frac{\partial s}{\partial p} = \frac{s}{p} \left[\varepsilon(p, N) - \varepsilon(p, N;\, y, \theta) \right].$$

Note that

$$\int_{\mathbb{R}_+ \times \Theta} s(p, N;\, y, \theta) \mathrm{d}G(y, \theta) = 1 \;\Rightarrow\; \int_{\mathbb{R}_+ \times \Theta} \frac{\partial s}{\partial p} \mathrm{d}G(y, \theta) = 0. \tag{A.9}$$

Therefore, plugging (A.9) into (A.8) and subtracting $(\varepsilon(p, N)/p) \int_{\mathbb{R}_+ \times \Theta} (\partial s / \partial p) \mathrm{d}G(y, \theta) = 0$ from both sides of (A.9), we obtain the desired expression (80). Q.E.D.