Criminal Networks: Who is the Key Player?

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Purpose

Theoretical and empirical investigation

of the role of peers in criminal activities

using a network perspective

Policy Implications: Brute Force versus Targeting Criminals

Motivation

Growing awareness that social context matters for individual outcomes

Observation that many individual outcomes vary much more between social groups than within them

Theoretically: models of social interactions are widely used

Empirically: convincing tests of such models are still quite limited

- identification and measure of such peer effects is a quite difficult exercise

- appropriate data sets difficult to find

Policy Implications:

Brute Force (Becker) versus Targeting Criminals (Social interactions)

Concentrating efforts by targeting the “most active” criminals because of the feedback effects or “social multipliers” at work

(Sah, 1991; Kleiman, 1993, 2009; Glaeser et al., 1996; Rasmussen, 1996; Schrag and Scotchmer, 1997; Verdier and Zenou, 2004; Rogers and Zenou, 2010).

As the fraction of individuals participating in a criminal behavior increases, the impact on others is multiplied through social networks.

Thus, criminal behaviors can be magnified, and interventions can become more effective.

Our paper: Analyze the role of peer effects in juvenile crime using a network perspective (Jackson, 2008) and analyze its policy implications (Who is the Key Player)

Mechanisms

Theoretical model of individual behavior with social interactions

Identifying the Key Player

Main findings:

Peer Effects are Important in Criminal Activities.

A one standard deviation increase in the aggregate level of delinquent activity of the peers translate into a roughly 11 percent increase of a standard deviation in the individual level of activity.

Conterfactual Study to Determine the Key Player

Key players: more likely to be a male, have less educated parents, are less attached to religion and feel socially more excluded.

Feel that adults care less about them, are less attached to their school and have more troubles getting along with the teachers.

Even though some criminals are not very active in criminal activities, they can be key players because they have a crucial position in the network in terms of betweenness centrality

Theoretical model

The network $N = \{1, \ldots, n\}$ finite set of agents connected by a graph/network g.

Two individuals i and j are directly connected (i.e. best friends) in g if and only if $g_{ij} = 1$, and $g_{ij} = 0$, otherwise.

Friendship is a reciprocal relationship: $g_{ij} = g_{ji}$ and $g_{ii} = 0$.

The $n-$square adjacency matrix $\mathbf{G}$ of a network $\mathbf{g}$ keeps track of the direct connections in this network.

Example: 3 agents

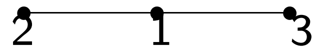


Figure 1. Three agents on a line.

Adjacency matrix :

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Preferences y_i: delinquency effort level of delinquent i,

$\mathbf{y} = (y_1, ..., y_n)$ population delinquency profile.

$$u_i(\mathbf{y}, \mathbf{g}) = \underbrace{(a_i + \eta_r + \varepsilon_i)\ y_i}_{\text{Proceeds}} - \underbrace{\frac{1}{2} y_i^2}_{\text{moral cost of crime}} - \underbrace{p\, f\, y_i}_{\text{cost of being caught}}$$

$$+ \quad \underbrace{\phi \sum_{j=1}^{n} g_{ij} y_i y_j}_{\text{positive peer effects}}$$

Utility function: standard costs/benefits structure (a la Becker) with an added element: peer effects.

Individual outcomes results from both idiosyncratic characteristics and peer effects

Payoffs are interdependent and agents choose their levels of activity simultaneously.

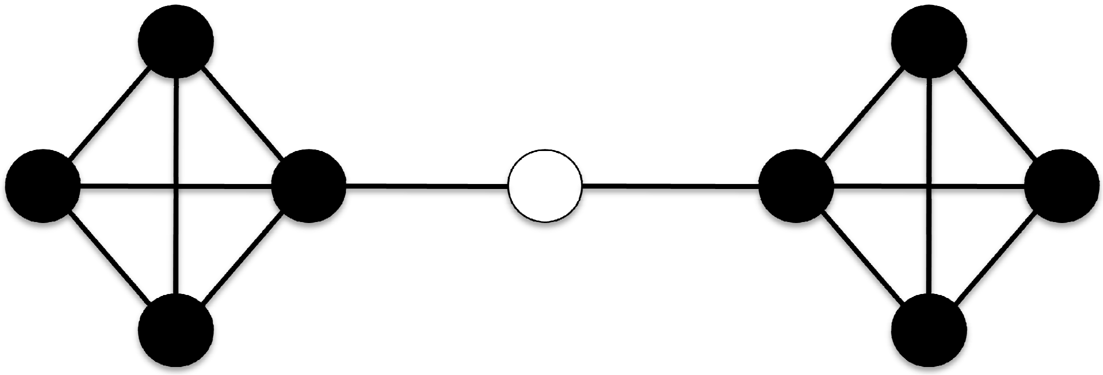
Nash equilibrium.

Different *centrality measures* to capture the prominence of actors inside a network.

Degree centrality: counts the number of connections an agent has.

Bonacich centrality: gives to any individual a particular numerical value for each of his/her direct connection. Then, give a smaller value to any connection at distance two and an even smaller value to any connection at distance three; etc. When adding up all these values, we end up with a new numerical value that is now capturing both direct and indirect connections of any order.

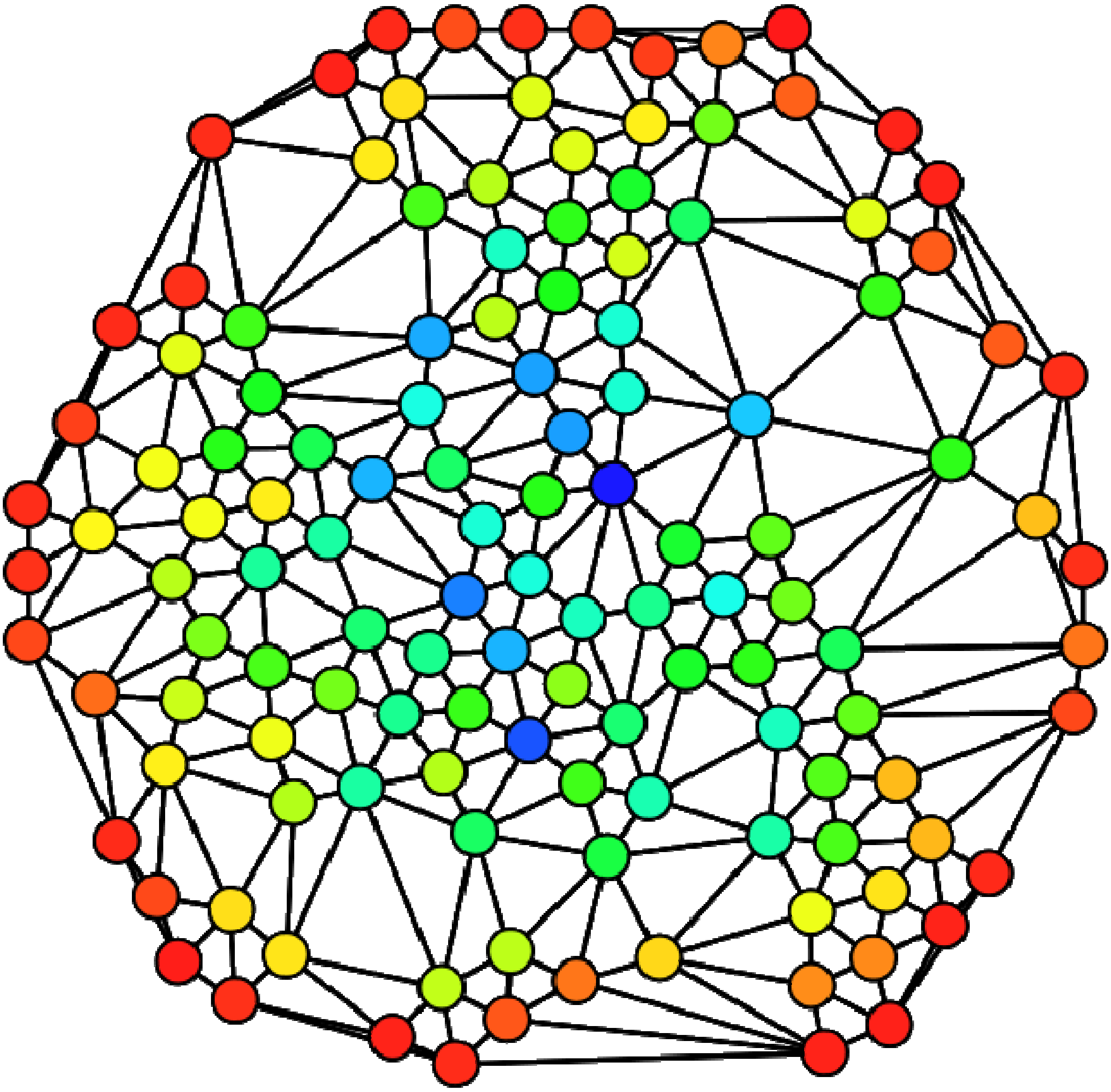
Betweenness centrality: calculates the relative number of indirect connections (or shortest paths) in which the actor into consideration is involved in with respect to the total number of paths in the network.



Agent in the middle: **Lowest degree** centrality, **highest betweenness** centrality.

Bonacich centrality: Depends on the value of the discount factor.

For small discount factors (i.e. indirect links give less benefits), this agent is the less central one while for high levels of discount (i.e. direct links are weighted less), this agent is the most central.



From red=0 to blue=max shows the node betweenness centrality

The Bonacich network centrality The kth power $\mathbf{G}^k = \mathbf{G}^{(k \overset{times}{\cdots})}\mathbf{G}$ of the adjacency matrix $\mathbf{G}$ keeps track of indirect connections in g.

The coefficient $g_{ij}^{[k]}$ in the (i, j) cell of $\mathbf{G}^k$ gives the number of paths of length k in g between i and j.

Definition 0.1 *Given a vector* $\mathbf{u} \in \mathbb{R}_{+}^{n}$*, and* $\phi \geq 0$ *a small enough scalar, we define the vector of Bonacich centralities of parameter* ϕ *in the network* $\mathbf{g}$ *as:*

$$\mathbf{b_u}(\mathbf{g}, \phi) = (\mathbf{I} - \phi\mathbf{G})^{-1}\mathbf{u} = \sum_{p=0}^{+\infty} \phi^p \mathbf{G}^p \mathbf{u}.$$

Example

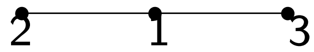


Figure 1

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

$$\mathbf{G}^{2k} = \begin{bmatrix} 2^k & 0 & 0 \\ 0 & 2^{k-1} & 2^{k-1} \\ 0 & 2^{k-1} & 2^{k-1} \end{bmatrix}, \qquad \mathbf{G}^{2k+1} = \begin{bmatrix} 0 & 2^k & 2^k \\ 2^k & 0 & 0 \\ 2^k & 0 & 0 \end{bmatrix}, k \geq 1.$$

Assume $\boldsymbol{\alpha = 1}$.

$$
\begin{aligned}
b_1(g,\phi) &= \sum_{k=0}^{+\infty} \left[\phi^{2k} 2^k + \phi^{2k+1} 2^{k+1}\right] = \frac{1+2\phi}{1-2\phi^2} \\
b_2(g,\phi) &= b_3(g,\phi) = \sum_{k=0}^{+\infty} \left[\phi^{2k} 2^k + \phi^{2k+1} 2^{k}\right] = \frac{1+\phi}{1-2\phi^2}
\end{aligned}
$$

Nash equilibrium

First-order conditions:

$$y_i = \phi \sum_{j=1}^{n} g_{ij} y_j + \sum_{m=1}^{M} \beta_m x_i^m - pf + \eta_k + \varepsilon_i$$

$\mu_1(\mathbf{G})$: largest eigenvalue of $\mathbf{G}$, $\alpha_i = a_i - pf + \eta_k + \varepsilon_i$

Proposition 0.1 *If $\phi\mu_1(\mathbf{G}) < 1$, the peer effect game with payoffs given above has a unique Nash equilibrium in pure strategies given by:*

$$\mathbf{y}^* = \mathbf{b}_{\boldsymbol{\alpha}}(\mathbf{g}, \phi)$$

Best-reply functions

$$BR_i(\mathbf{y}_{-i}) = \phi \sum_{j=1}^{n} g_{ij} y_j + \sum_{m=1}^{M} \beta_m x_i^m - pf + \eta_k + \varepsilon_i$$

$$\underbrace{Alice}_{y_A \uparrow \mathbf{\Delta}} \quad \rightarrow \quad \underbrace{Bob}_{y_B \uparrow \phi \mathbf{\Delta}} \quad \rightarrow \quad \underbrace{Charlie}_{y_C \uparrow \phi^2 \mathbf{\Delta}}$$

- Direct complementarities induce indirect complementarities of all possible order.
- There is a discount of distance ϕ^{distance}.
- This means that ϕ cannot be too large.

Finding the key player

Planner's objective: find the key player is to generate the highest possible reduction in aggregate delinquency level by picking the appropriate delinquent.

Planner's problem:

$$\max\{y^*(g) - y^*(g_{-i}) \mid i = 1, ..., n\},$$

$$\min\{y^*(g_{-i}) \mid i = 1, ..., n\}$$

$\mathbf{M}(g, \phi) = (\mathbf{I} - \phi \mathbf{G})^{-1}$ a non-negative matrix.

Its coefficients $m_{ij}(g, \phi) = \sum_{k=0}^{+\infty} \phi^k g_{ij}^{[k]}$ count the number of walks in g starting from i and ending at j, where walks of length k are weighted by ϕ^k.

Bonacich centrality of node i: $b_{\alpha_i}(g, \phi) = \sum_{j=1}^{n} \alpha_j m_{ij}(g, \phi)$: counts the *total* number of paths in g starting from i weighted by the α_j of each linked node j

Definition 0.2 *For all networks g and for all i, the intercentrality measure of delinquent i is:*

$$d_{i^*}(g, \phi) = b_\alpha(g, \phi) - b_\alpha^{[-i]}(g, \phi) = \frac{b_{\alpha_i}(g, \phi) \sum_{j=1}^{j=n} m_{ji}(g, \phi)}{m_{ii}(g, \phi)}$$

Proposition 0.2 *A player* i^* *is the key player that solves* $\min\{y^*(g_{-i}) \mid i = 1, ..., n\}$ *if and only if* i^* *is a delinquent with the highest intercentrality in* g*, that is,* $d_{i^*}(g, \phi) \geq d_i(g, \phi)$*, for all* $i = 1, ..., n$*.*

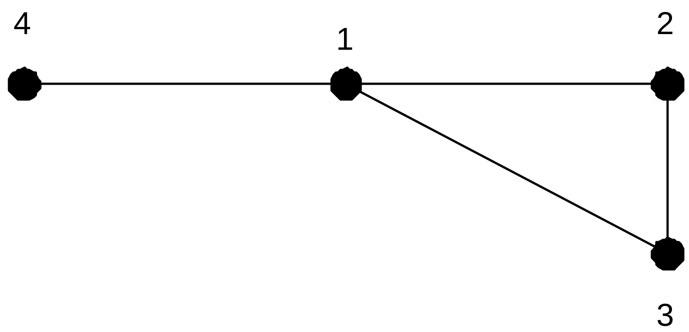
Intercentrality captures, in an meaningful way, the two dimensions of the removal of a delinquent from a network: the *direct* effect on delinquency and the *indirect* effect on others' delinquency involvement.

Example

Network of four delinquents (i.e. $n = 4$) with

$$(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (0.1, 0.2, 0.3, 0.4)$$

and



$$\mathbf{G} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Decay factor $\phi = 0.3$.

Nash equilibrium:

$$\begin{pmatrix} y_1^* \\ y_2^* \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} b_{\alpha,1}(g,\phi) \\ b_{\alpha,2}(g,\phi) \\ b_{\alpha,3}(g,\phi) \\ b_{\alpha,4}(g,\phi) \end{pmatrix} = \begin{pmatrix} 0.66521 \\ 0.60377 \\ 0.68068 \\ 0.59958 \end{pmatrix}$$

Total crime effort:

$$y^* = y_1^* + y_2^* + y_3^* = b_\alpha(g,\phi) = 2.549$$

Delinquent 3 has the highest weighted Bonacich and thus provides the highest crime effort.

Intercentrality: $d_{i^*}(g, \phi) = b_\alpha(g, \phi) - b_\alpha^{[-i]}(g, \phi)$

Remove delinquent 1.

We have now a network with three delinquents, with $(\alpha_2, \alpha_3, \alpha_4) = (0.2, 0.3, 0.4)$ and where

$$\mathbf{G} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Using the same decay factor, $\phi = 0.3$, we obtain:

$$\begin{pmatrix} y_2^* \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} b_{\alpha,2}(g^{[-1]}, \phi) \\ b_{\alpha,3}(g^{[-1]}, \phi) \\ b_{\alpha,4}(g^{[-1]}, \phi) \end{pmatrix} = \begin{pmatrix} 0.31868 \\ 0.3956 \\ 0.4 \end{pmatrix}$$

so that the total effort is now given by:

$$y^{*[-1]} = y_2^* + y_3^* + y_4^* = b_\alpha^{[-1]}(g, \phi) = 1.114$$

Thus, player 1's contribution is

$$b_\alpha(g,\phi) - b_\alpha^{[-1]}(g,\phi) = 2.549 - 1.114 = 1.435$$

Doing the similar exercise for individuals 2, 3, 4, we obtain:

$$b_\alpha(g,\phi) - b_\alpha^{[-2]}(g,\phi) = 1.244$$

$$b_\alpha(g,\phi) - b_\alpha^{[-3]}(g,\phi) = 1.146$$

$$b_\alpha(g,\phi) - b_\alpha^{[-4]}(g,\phi) = 0.988$$

Check that the key player is delinquent 1. Formula:

$$d_{1^*}(g,\phi) = \frac{b_{\alpha,1}(g,\phi)\sum_{j=1}^{j=4} m_{j1}(g,\phi)}{m_{11}(g,\phi)}$$

$$\mathbf{M} = (\mathbf{I} - \phi\mathbf{G})^{-1} = \begin{pmatrix} 1.5317 & 0.65646 & 0.65646 & 0.45952 \\ 0.65646 & 1.3802 & 0.61101 & 0.19694 \\ 0.65646 & 0.61101 & 1.3802 & 0.19694 \\ 0.45952 & 0.19694 & 0.19694 & 1.1379 \end{pmatrix}$$

$$m_{11}(g,\phi) = 1.5317$$

and

$$\begin{aligned}\sum_{j=1}^{j=4} m_{j1}(g,\phi) &= m_{11}(g,\phi)+m_{21}(g,\phi)+m_{31}(g,\phi)+m_{41}(g,\phi)\\ &= 1.5317+0.65646+0.65646+0.45952\\ &= 3.3041\end{aligned}$$

Therefore,

$$\begin{aligned}d_{1^*}(g,\phi) &= \frac{b_{\alpha,1}\sum_{j=1}^{j=3} m_{j1}(g,\phi)}{m_{11}(g,\phi)}\\ &= \frac{0.66521\times 3.3041}{1.5317}\\ &= 1.435\end{aligned}$$

$$d_{1^*}(g,\phi) = b_\alpha(g,\phi) - b_\alpha^{[-1]}(g,\phi) = 1.435$$

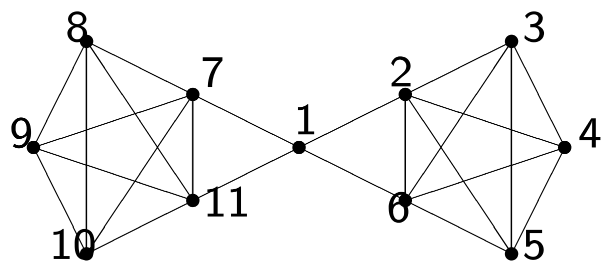
Is the key player always the more active criminal?

Holding $b_i(g, \phi)$ fixed, the intercentrality $d_i(g, \phi)$ of player i *decreases* with the proportion $m_{ii}(g, \phi)/b_i(g, \phi)$ of i's Bonacich centrality due to self-loops, and *increases* with the fraction of i's centrality amenable to out-walks.

Not always true.

Consider this network g with eleven criminals.

Figure 1: A bridge network



We distinguish three different types of equivalent actors in this network, which are the following:

Type	Criminals
1	1
2	2, 6, 7 and 11
3	3, 4, 5, 8, 9 and 10

Role of location in the network

Criminals are ex identical: $\boldsymbol{\alpha} = \mathbf{1}$

$$\mathbf{b_1}(g, \phi) = (\mathbf{I} - \phi\mathbf{G})^{-1}\mathbf{1}$$

$y_i^* = b_{1_i}(g, \phi)$ and $d_{i^*}(g, \phi) = b_1(g, \phi) - b_1^{[-i]}(g, \phi)$.

Take $\phi = 0.2$.

Table 1a: Key player versus Bonacich centrality in a bridge network

Player Type	1	2	3
$y_i = b_i$	8.33	9.17*	7.78
d_i	41.67*	40.33	32.67

Table 1b: Characteristics of criminals in a network

where the most active criminal is not the key player

Player type	1	2	3
Degree centrality	0.4	0.5	0.4
Closeness centrality	0.625	0.555	0.416
Betweenness centrality	0.555	0.2	0
Clustering coefficient	0.33	0.7	1

Table 1c: Characteristics of the network

in which the most active criminal is not the key player

Network Characteristics	
Average Distance	2.11
Average Degree	4.36
Diameter	4
Density	0.211
Asymmetry	0.125
Clustering	0.805
Degree centrality	7.78×10^{-3}
Closeness centrality	0.323
Betweenness Centrality	0.47556
Assortativity	-3.49×10^{-16}

Data

Dataset of friendship networks in the United States from the National Longitudinal Survey of Adolescent Health (AddHealth)

Richness of the information provided by the AddHealth data

Pupils were asked to identify their best friends from a school roster

Friendship information is based upon actual friends nominations.

Pupils were asked to identify their best friends from a school roster (up to five males and five females)

The limit in the number of nominations is not binding

Less than 1% of the students in our sample show a list of ten best friends

A link exists between two friends if at least one of the two individuals has identified the other as his/her best friend (undirected networks)

Information on the characteristics of nominated friends

Criminal activity

Addhealth contains an extensive set of questions on juvenile delinquency, ranging from light offenses that only signal the propensity towards a delinquent behavior to serious property and violent crime

Delinquency index

15 delinquency items:

1) paint graffiti or signs on someone else's property or in a public place

2) deliberately damage property that didn't belong to you

3)lie to your parents or guardians about where you had been or whom you were with

4)take something from a store without paying for it

5)get into a serious physical fight

6)hurt someone badly enough to need bandages or care from a doctor or nurse

7) run away from home

8) drive a car without its owner's permission

9) steal something worth more than $50

10) go into a house or building to steal something;

11) use or threaten to use a weapon to get something from someone

12) sell marijuana or other drugs

13) steal something worth less than $50

14) take part in a fight where a group of your friends was against another group

15) act loud, rowdy, or unruly in a public place.

Each response is coded using an ordinal scale ranging from 0 (i.e. never participate) to 1 (i.e. participate 1 or 2 times), 2 (participate 3 or 4 times) up to 3 (i.e. participate 5 or more times)

The delinquency index is a composite score: It ranges between 0.09 and 9.63.

Because of the theoretical model, we focus only on networks of delinquents

thus excluding the individuals who report never participating in any delinquent activity (roughly 40% of the total).

Final sample: 1,297 criminals distributed over 150 networks.

Minimum number of individuals in a delinquent network: 4, maximum: 77.

Mean and the standard deviation of network size: roughly 9 and 12 pupils.

On average, delinquents declare having 2.26 friends with a standard deviation of 1.52.

Table 1: List of controls

Female
Black or African American
Other races
Age
Religion practice
Health status
School attendance
Student grade
Organized social participation
Motivation in education
Relationship with teachers
Social exclusion
School attachment
Parental care
Household size
Two married parent family
Single parent family
Public assistance
Mother working

Parental education
Parent age
Parent occupation manager
Parent occupation professional or technical
Parent occupation office or sales worker
Parent occupation manual
Parent occupation military or security
Parent occupation farm or fishery
Parent occupation retired
Parent occupation other

Neighborhood quality
Residential building quality
Neighborhood safety
Residential area suburban
Residential area urban - residential only
Residential area commercial properties - retail
Residential area commercial properties - industrial
Residential area type other
Friend attachment
Friend involvement
Friend contacts
Physical development
Self esteem

Empirical model

First-order conditions:

$$y_i = \phi \sum_{j=1}^{n} g_{ij} y_j + \sum_{m=1}^{M} \beta_m x_i^m + \frac{1}{g_i} \sum_{m=1}^{M} \sum_{j=1}^{n} \gamma_m g_{ij} \, x_j^m - pf + \eta_k + \varepsilon_i$$

Econometric equivalent:

$$y_{i,r} = \phi \sum_{j=1}^{n_r} g_{ij,r} y_{j,r} + x'_{i,r}\beta + \frac{1}{g_{i,r}} \sum_{j=1}^{n_r} g_{ij,r} x'_{j,r}\gamma + \eta_r^* + \epsilon_{i,r},$$

$\bar{r}$: total number of networks in the sample (150 in our dataset),

n_r: number of individuals in the rth network

$n = \sum_{r=1}^{\bar{r}} n_r$ total number of sample observations.

$x_{i,r} = (x_{i,r}^1, \cdots, x_{i,r}^m)'$, $\eta_r^* = \eta_r - pf$, and $\epsilon_{i,r}$'s are i.i.d. innovations with zero mean and variance σ^2 for all i and r.

Matrix form:

$$Y_r = \phi G_r Y_r + X_r \delta_1 + G_r^* X_r \delta + \eta_r^* l_{n_r} + \epsilon_r,$$

G_r^* row-normalized of G_r

Estimation issues

Are we really capturing peer effects?

or

Are we only capturing the effects of

- exogenous peer characteristics
- correlation in tastes of people that sort in the same group

Situations: individuals in the same group tend to behave similarly.

Example: Effects of neighborhood average crime rate on criminal activities of an individual.

Three consistent explanations

Endogenous Effects Neighbors' decisions directly affects her own decision.

Contextual Effects Distribution of background characteristics lead similar behavior.

Correlated Effects Similar environment/characteristics (neighborhood quality) lead similar behavior.

Desirable policy may be totally different, depending on the source of seemingly related behavior.

Policy implications. Identifying peer effects: Why is it important? Manski (1993, 2000) and Moffitt (2001): it is important to *separately* identify **peer or endogenous effects** from **contextual or exogenous effects**.

Because endogenous effects generate a *social multiplier* while contextual effect don't.

Example of crime: A special program targeting some individuals will have multiplier effects: the individual affected by the program will reduce its criminal activities and will influence the criminal activities of his/her peers, which, in turn, will affect the criminal activities of his/her peers, and so on.

If only contextual effects are present, then there will be no social multiplier effects from any policy affecting only the "context" (for example, improving the quality of the teachers at school).

It is important to *separately* identify **peer or endogenous effects** from **confounding or correlated effects**.

The formation of peer group is not random and individuals *do select* into groups of friends.

The same criminal activities may be due to common unobservable variables (such as, for example, the fact that individuals from the same network like bowling together) faced by individuals belonging to the same network rather than peer effects.

If the high-crime rates are due to the fact that teenagers like to bowling together, then obviously the implications are very different than if it is due to peer effects.

1) **The reflection problem (Manski, 1993)**

Is it possible to disentangle the endogenous effects, i.e. the influence of peer outcomes, from the (contextual) exogenous effects, i.e. the influence of exogenous peer characteristics?

It arises because in the standard approach individuals interact in groups, that is individuals are affected by all others in their group and by none outside the group

In social networks groups overlaps

Consider our model without network fixed effects:

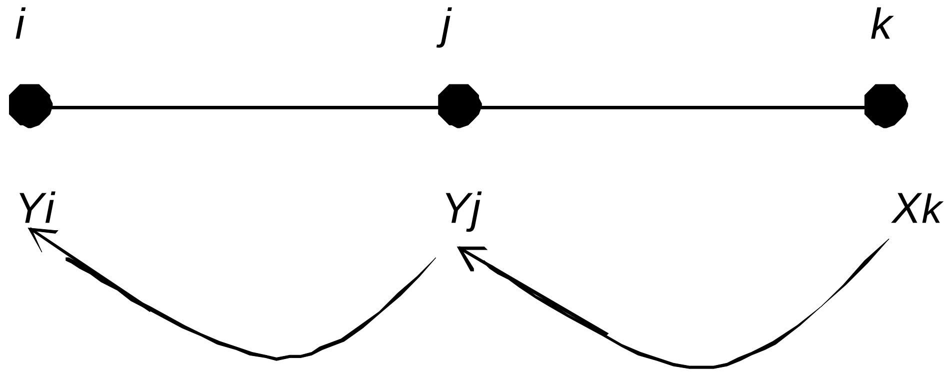
$$Y_r = \phi G_r Y_r + X_r \delta_1 + G_r^* X_r \delta + \epsilon_r,$$

This model is identified if and only if $E\left(G_r Y_r \mid X_r\right)$ is not perfectly collinear with the regressors (X_r, $G_r^* X_r$) so that instruments can be found for the endogenous vector $G_r Y_r$.

Bramoulle et al (2009): This condition is equivalent to I_r, G_r and G_r^2 are linearly independent.

This is true as long as the networks are partially overlapping: some individuals may not be friends with his/her friends' friends (i is friend to j and j is friend to k but k is not friend with i).

For individual i, the characteristics of peers of peers $G_r^2 X_r$ (i.e. $x_{k,r}$) is a valid instrument for peers' behavior $G_r^2 Y_r$ (i.e. $y_{j,r}$) since $x_{k,r}$ affects $y_{i,r}$ only indirectly through its effect on $y_{j,r}$ (distance 2)



The natural exclusion restrictions induced by the network structure (existence of an intransitive triad) guarantee identification of the model.

2) **Correlated effects/selection**

Is it possible to disentangle "endogenous effects" from "correlated effects", i.e. those due to the fact that individuals in the same group tend to behave similarly because they face a common environment?

Correlated effects might originate from the possible sorting of agents into "groups"

If the variables that drive this process of selection are not fully observable, potential correlations between (unobserved) group-specific factors and the target regressors are major sources of bias.

Selection on observables

Our particularly large information on individual, parental, school, neighborhood variables should reasonably explain the process of selection into groups

Selection on unobservables

Assume agents self-select into different networks in a first step, and that link formation takes place within groups in a second step.

Bramoullé *et al.* (2009): if link formation is uncorrelated with the observable variables, this two-step model of link formation generates network fixed effects.

Assuming additively separable network heterogeneity, a within group specification is able to control for selection issues

Bramoullé et al. (2009): by subtracting from the individual-level variables the network average, social effects are again identified and one can disentangle endogenous effects from correlated effects

Consider our model with network fixed effects:

$$Y_r = \phi G_r Y_r + X_r \delta_1 + G_r^* X_r \delta + \eta_r^* l_{n_r} + \epsilon_r,$$

We can eliminate the network fixed effect by the network-mean transformation, that is by multiplying this equation by the matrix: $J_r = I_{m_r} - \frac{1}{m_r} l_r l_r'$ (I_{m_r} identity matrix, l_r vector of 1).

Model becomes:

$$J_r Y_r = \phi J_r G_r Y_r + J_r X_r \delta_1 + J_r G_r^* X_r \delta + J_r \epsilon_r$$

Model can be written as:

$$\widehat{Y_r} = \phi G_r \widehat{Y}_r + \widehat{X_r}\delta_1 + G_r^* \widehat{X_r}\delta + \widehat{\epsilon}_r$$

where $\widehat{Y_r} = J_r Y_r$, $\widehat{X_r} = J_r X_r$, $\widehat{\epsilon}_r = J_r \epsilon_r$.

The model can be identified if and only if $E\left(G_r \widehat{Y}_r \mid \widehat{X_r}\right)$ is not perfectly collinear with the regressors $\left(\widehat{X_r},\, G_r^* \widehat{X_r}\right)$.

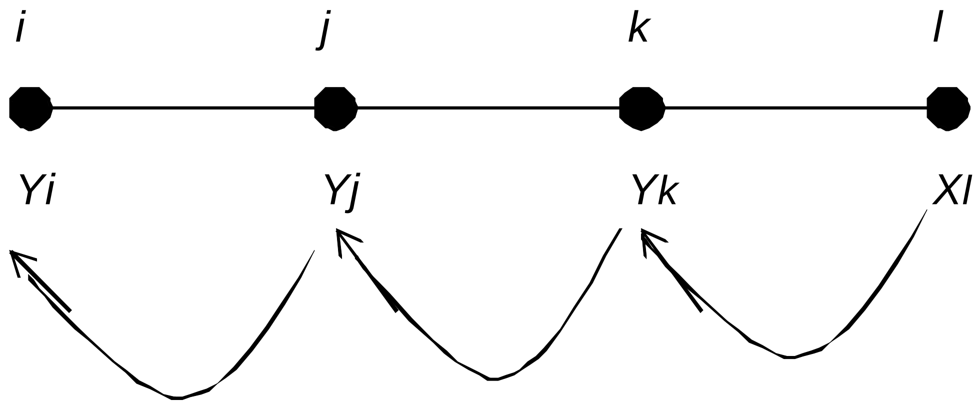
This condition is equivalent to I_r, G_r, G_r^2 and G_r^3 are linearly independent.

The condition is more demanding because some information has been used to deal with the fixed effects.

Bramoulle et al (2009) show that if two agents i and j in a network are separated by a link of distance 3, then I_r, G_r, G_r^2 and G_r^3 are linearly independent. Model is identified.

Consider four individuals: ij, jk, kl, but l is not friend with i.

$x_{l,r}$ can serve as an instrument for $y_{j,r}$ in individual i's equation since $x_{l,r}$ affects $y_{i,r}$ but only indirectly through its effect on $y_{k,r}$.



To sum-up:

We estimate:

$$y_{i,r} = \phi \sum_{j=1}^{n_r} g_{ij,r} y_{j,r} + x'_{i,r}\beta + \frac{1}{g_{i,r}} \sum_{j=1}^{n_r} g_{ij,r} x'_{j,r}\gamma + \eta^*_r + \epsilon_{i,r},$$

with 6 different methods (best one: bias-corrected many-IV GMM estimator).

Results

The estimated effect of ϕ*,which measures the intensity of peer effects is positive and highly statistically significant*

The impact is not negligible in magnitude

A one-standard deviation increase in the aggregate level of delinquent activity of the peers translate into a roughly 11 percent increase of a standard deviation in the individual level of activity.

Stronger peer effects for directed networks.

Different types of crime

The literature on local interactions has uncovered some interesting differences between different types of crime

For instance, Ludwig et al. (2000) find that neighborhood effects are large and negative for violent crime but have a mild positive effect on property crime

In contrast, Glaeser et al. (1996) find instead that social interactions seem to have a large effect on petty crime, a moderate effect on more serious crime and a negligible effect on very violent crime

Split the reported offences between *petty crimes* and *more serious crimes*.

The first group (*type-1 crimes or petty crimes*) encompasses the following offences: (i) paint graffiti or sign on someone else's property or in a public place; (ii) lie to the parents or guardians about where or with whom having been; (iii) run away from home; (iv) act loud, rowdy, or unruly in a public place; (v) take part in a group fight; (vi) damage properties that do not belong to you; (vii) steal something worth more than \$50.

The second group (*type-2 crimes or more serious crimes*) consists of (i): taking something from a store without paying for it; (ii) hurting someone badly enough to need bandages or care from a doctor or nurse; (iii) driving a car without its owner's permission; (iv) stealing something worth more than \$50; (v) going into a house or building to steal something; (iv) using or threatening to use a weapon to get something from someone; (vii) selling marijuana or other drugs; ($viii$) getting into a serious physical fight.

We obtain a sample of 1099 petty criminal distributed over 132 networks and a sample of 545 more serious criminals distributed over 75 networks.

Petty crime networks have a minimum of 4 individuals and a maximum of 73 (with mean equals to 8.33 and standard deviation equals to 10.74),

whereas the range for more serious crime networks is between 4 and 38 (with mean equals to 7.27 and standard deviation equals to 6.64).

We estimate the following modified version of our empirical model

$$y_{i,r,l} = \phi_l \sum_{j=1}^{n_r} g_{ij,r} y_{j,r,l} + x'_{i,r,l}\beta_l + \frac{1}{g_{i,r,l}} \sum_{j=1}^{n_r} g_{ij,r,l} x'_{j,r,l}\gamma_l + \eta_r^* + \epsilon_{i,r,l}$$

where l denotes the type of crime committed by individual i in network r $(l = 1, 2)$

Estimation of ϕ

The impact of peer effects on crime are much higher for more serious crimes than for petty crimes.

A standard deviation increase in the aggregate level of delinquent activity of the peers translate into a roughly 8 percent and 14.5 increase of a standard deviation in the individual level of activity for petty crimes and more serious crimes.

Dynamic network formation models

The model

So far: network fixed.

When the key player was removed, no new links were formed.

Invariant assumption on the reduced network $g^{[-i]}$, i.e. we assume that, when the key player is removed, the other criminals in the network do not form new links.

Now: *dynamic model* where both network formation and effort decisions take place.

Crime decision (participation) is taken by each individual before $t = 0$ and that this decision will not change afterwards.

At each period of time t, a person is chosen at random among the n criminal in the network g_t, and has to decide whether or not to create a link and, in case of link formation, with whom she wants to create this link.

Morning-afternoon game:

At each period of time t, the timing of the game is as follows.

In the *morning* of day t, an agent (say, agent i) is chosen with *equal probability* and makes a link-formation decision under uncertainty as she does not know the realization of a random shock ϵ_i.

The shock is individual specific such that $\mathsf{E}[\epsilon_i] = 0$ for all i, $\mathsf{Var}[\epsilon_i] = \sigma_\epsilon^2 > 0$, and all shocks are $i.i.d$ across individuals.

We assume that the agent i who initiates a link formation with agent j pays the cost c_i of the link ij and that she is *myopic*, i.e. agent i only takes into account the *current benefit* in terms of utility.

Directed networks so that, in terms of the adjacency matrix $\mathbf{G} = [g_{ij}]$, agent i is only allowed to change a non-diagonal element of the ith row of $\mathbf{G}$ to one if that element was zero at the beginning of day t.

At *noon* of day t, a new network is formed, which is denoted by $\mathbf{G}_t = \mathbf{G}_t(i,j)$ (each cell being $g_{ij,t}(i,j)$) where a new link pointing from j to i is created. If $i = j$, then $\mathbf{G}_t$ is the same as $\mathbf{G}_{t-1}$.

In the *afternoon* of day t, the random shock ϵ_i is realized and its value becomes complete information for all agents.

All agents in the (new) network simultaneously choose their effort level y to maximize their utility at time t after the link ij has been added or not.

This utility is given by

$$\begin{aligned} u_{i,t}(i,j) &= [a_i + \eta + \epsilon_i]\, y_{i,t}(i,j) - \frac{1}{2}\left[y_{i,t}(i,j)\right]^2 \\ &\quad + \phi \sum_{k=1}^{n} g_{ik,t}(i,j) y_{i,t}(i,j) y_{k,t}(i,j) - c_i g_{i,t}(i,j) \end{aligned}$$

Unique Nash equilibrium of the "afternoon game"

(assuming that $\phi\mu_1(\mathbf{G}_t(i,j)) < 1$)

is such that

$$y_{i,t}^*(i,j) = \phi \sum_{k=1}^{n} g_{ik,t}(i,j) y_{j,t}^*(i,j) + a_i + \eta + \epsilon_i$$

or in vector form

$$\mathbf{y}_t^*(i,j) = [\mathbf{I} - \phi \mathbf{G}_t(i,j)]^{-1} (\mathbf{a} + \eta.\mathbf{l_n} + \boldsymbol{\epsilon})$$

The expected utility model

Consider again the "morning game": the chosen agent i makes her link formation decision by maximizing her expected utility $\max_j \mathsf{E}\left[u_i^*(i,j)\right]$ where

$$\begin{aligned}
\mathsf{E}\left[u_{i,t}^*(i,j)\right] &= (a_i+\eta)\,\mathsf{E}\left[y_{i,t}^*(i,j)\right]+\mathsf{E}\left[\epsilon_i\, y_{i,t}^*(i,j)\right] \\
&\quad -\frac{1}{2}\mathsf{E}\left\{\left[y_{i,t}^*(i,j)\right]^2\right\} \\
&\quad +\phi\sum_{k=1}^{n} g_{ik,t}(i,j)\mathsf{E}\left[y_{i,t}^*(i,j)y_{k,,t}^*(i,j)\right]-c_i g_{i,t}(i,j)
\end{aligned}$$

Denote: $\mathbf{S}_t(i,j) = [\mathbf{I} - \phi \mathbf{G}_t(i,j)]^{-1}$ and $\mathbf{S}_{i,t}(i,j)$ be the ith row of the matrix $\mathbf{S}_t(i,j)$ and $S_{ii,t}(i,j)$ be the ith element of the vector $\mathbf{S}_{i,t}(i,j)$.

$$y^*_{i,t}(i,j) = \mathbf{S}_{i,t}(i,j)\left(\mathbf{a} + \eta.\mathbf{l_n} + \boldsymbol{\epsilon}\right)$$

$$\mathsf{E}\left[\epsilon_i \, y^*_{i,t}(i,j)\right] = \sigma^2_\epsilon S_{ii,t}(i,j)$$

$$\mathsf{E}\left\{\left[y^*_{i,t}(i,j)\right]^2\right\} = \left\{\mathsf{E}\left[y^*_{i,t}(i,j)\right]\right\}^2 + \sigma^2_\epsilon \mathbf{S}_{i,t}(i,j)\mathbf{S}'_{i,t}(i,j)$$

and

$$\mathsf{E}\left[y^*_{i,t}(i,j) y^*_{k,,t}(i,j)\right] = \mathsf{E}\left[y^*_{i,t}(i,j)\right] \, \mathsf{E}\left[y^*_{k,,t}(i,j)\right] + \sigma^2_\epsilon \mathbf{S}_{k,t}(i,j)\mathbf{S}'_{i,t}(i,j)$$

Determine a lower bound of c_i:

$$\underline{c}_{i,t}^{EO} = \max_j \frac{1}{2} \left\{ \mathsf{E}\left[y_{i,t}^*(i,j) \right] \right\}^2 - \frac{1}{2} \max_j \left\{ \mathsf{E}\left[y_{i,t}^*(i,i) \right] \right\}^2$$

If $c_i \geq \underline{c}_{i,t}^{EO}$ for all i, then no link will be ever created at period t.

Convergence and equilibrium

Let time be measured at countable dates $t = 0, 1, 2, ...$ and consider a *discrete time Markov chain* for the network formation process $(\mathbf{G}_t)_{t=0}^{\infty}$ with $\mathbf{G}_t = (N, L_t)$ comprising the set of delinquents $N = \{1, ..., n\}$ together with a set of links L_t at time t between them.

Definition 0.5 *Consider a discrete time Markov chain* $(\mathbf{G}_t)_{t=0}^{\infty}$ *on the probability space* $(\Omega, \mathcal{F}, \mathbb{P})$*. Consider a network* $\mathbf{G}_t = (N, L_t)$ *at time* t *with delinquents* $N = \{1, ..., n\}$ *and links* L_t*. Let* $\mathbf{G}_t(i,j)$ *be the graph obtained from* $\mathbf{G}_t$ *by the addition of the edge* $ij \notin L_t$ *between agents* $i, j \in N$*. Let* $\mathbf{u}_t^*(i,j) = \left(u_{1,t}^*(i,j), ..., u_{n,t}^*(i,j)\right)$ *denote the profile of Nash equilibrium payoffs of the delinquents in* $\mathbf{G}_t(i,j)$ *following from the above payoff function with parameter* $\phi < 1/\mu_1(\mathbf{G}_t)$*. Then delinquent* j *is a best response of delinquent* i *if* $u_{i,t}^*(i,j) \geq u_{i,t}^*(i,k)$ *for all* $j, k \in N \backslash \mathcal{N}_{i,t} \cup \{i\}$*, where* $\mathcal{N}_{i,t} = \{j \in N : ij \in L_t\}$ *is the neighborhood of individual* $i \in N$*. The set of delinquent* i*'s best responses is denoted by* $BR_{i,t}$*.*

Definition 0.6 *We define the network formation process* $(\mathbf{G}_t)_{t=0}^{\infty}$ *with* $\mathbf{G}_t = (N, L_t)$*, as a sequence of networks* $\mathbf{G}_0, \mathbf{G}_1, ...$ *in which, at every time* $t = 0, 1, 2, ...$*, a delinquent* $i \in N$ *is uniformly selected at random. This delinquent* i *initiates a link to a best response delinquent* $j \in BR_{i,t}$*. The link is created if* $BR_{i,t} \neq \emptyset$ *and* $u_{i,t}^{*}(i, j) \geq u_{i,t}^{*}(i, i)$*. No link will be created otherwise. If* $BR_{i,t}$ *is not unique, then* i *randomly selects one delinquent in* $BR_{i,t}$*.*

$(\mathbf{G}_t)_{t=0}^{\infty}$ is a finite state, discrete time, homogeneous Markov chain. Moreover, the transition matrix $\mathbf{P}$ is defined by

$$(\mathbf{P})_{ij} = \mathbb{P}\left(\mathbf{G}_{t+1} = \mathbf{G}_j \mid \mathbf{G}_t = \mathbf{G}_i\right) \text{ for any } \mathbf{G}_i, \mathbf{G}_j \in \boldsymbol{\Omega}$$

Definition 0.7 *Consider the network formation process* $(\mathbf{G}_t)_{t=0}^{\infty}$ *with* $\mathbf{G}_t = (N, L_t)$ *described above, where, at each period of time* t*, the morning-afternoon game is played. We say that the network* $\mathbf{G}_0$ *at time* $t = 0$ *converges to an equilibrium network* $\mathbf{G}_T$ *at time* $t = T$ *when each of the* n *delinquents in the network* $\mathbf{G}_T$ *has no incentive to create a new link. That is, for all* $i = 1, ..., n$*, when* $\mathsf{E}\left[u_{i,T}^{*}(i,i)\right] > \max_k \mathsf{E}\left[u_{i,T}^{*}(i,k)\right]$.

In terms of Markov chain, this means that the equilibrium network $\mathbf{G}_T$ is an *absorbing state*.

Finding the key player

Key-player policy in the dynamic-network formation model.

At $t = 0$, before the dynamic-network formation game described above starts, the planner will choose the key player i^* in the following way.

The planner will compare the (expected) total crime that will emerge in equilibrium when she does not remove a delinquent and when she does.

The key player i^* will be the delinquent who reduces the most the total (expected) crime.

The expected equilibrium effort outcome is equal to:

$$\mathsf{E}\left(\mathbf{y}_T^*\right) = \left[\mathbf{I} - \phi\mathbf{G}_T\right]^{-1}\left(\mathbf{a} + \eta.\mathbf{l_n}\right)$$

After removing a delinquent at $t = 0$, the expected equilibrium effort outcome is equal to:

$$\mathsf{E}\left(\mathbf{y}_T^{[-i]*}\right) = \left[\mathbf{I} - \phi\mathbf{G}_T^{[-i]}\right]^{-1}(\mathbf{a} + \eta.\mathbf{l_n})$$

Planner's objective:

$$\max_i \left\{ \sum_{j=1}^{n} \mathsf{E}\left(\mathbf{y}_{j,T}^{*}\right) - \sum_{j=1, j\neq i}^{n} \mathsf{E}\left(\mathbf{y}_{j,T}^{[-i]*}\right) \mid i = 1, ..., n \right\}$$

This is equivalent to:

$$\min_i \left\{ \sum_{j=1, j\neq i}^{n} \mathsf{E}\left(\mathbf{y}_{j,T}^{[-i]*}\right) \mid i = 1, ..., n \right\}$$

Who is the key player? Econometric issues

Determining the key player without endogenous participation

We need first to estimate the total expected crime in equilibrium when a delinquent is removed, i.e. $\sum_{j=1,j\neq i}^{n} \mathsf{E}\left(\mathbf{y}_{j,T}^{[-i]*}\right)$.

Expected equilibrium crime effort to be determined:

$$\mathsf{E}\widehat{\left(\mathbf{y}_{T}^{[-i]*}\right)} = \left[\mathbf{I} - \widehat{\phi}\mathbf{G}_{T}^{[-i]}\right]^{-1}(\widehat{\mathbf{a}} + \widehat{\eta}.\mathbf{l_n})$$

Bias-corrected many-IV GMM estimation procedure: ϕ and a_i can be estimated by $\widehat{\phi}$ and $\widehat{a}_i$ while η can be estimated by the average of the estimation residuals.

The only parameter undetermined from the bias-corrected many-IV GMM estimation is c_i.

Assume that the network observed in the AddHealth data is *stable*, that is, no one has an incentive to create a new link.

Let the observed network in the AddHealth data be denoted by $\mathbf{G}_0$. We estimate c_i by $\underline{c}_{i,0}^{EU}$ (lower bound that guarantees $\mathbf{G}_0$.

We will thus estimate c_i by

$$\begin{aligned}
\widehat{\underline{c}}^{EU}_{i,t} &= \max_j \frac{1}{2}\left\{\mathsf{E}\left[\widehat{y^*_{i,t}(i,j)}\right]\right\}^2 - \frac{1}{2}\max_j \left\{\mathsf{E}\left[\widehat{y^*_{i,t}(i,i)}\right]\right\}^2 \\
&+\frac{\widehat{\sigma}^2_\epsilon}{2}\left[\widehat{\mathbf{S}}_{i,t}(i,i)\widehat{\mathbf{S}}'_{i,t}(i,i) - \widehat{\mathbf{S}}_{i,t}(i,j)\widehat{\mathbf{S}}'_{i,t}(i,j)\right] \\
&+\widehat{\sigma}^2_\epsilon\left[\widehat{S}_{ii,t}(i,j) - \widehat{S}_{ii,t}(i,i)\right] \\
&+\widehat{\sigma}^2_\epsilon\left[\widehat{\phi}\,\widehat{\mathbf{G}}_{i,t}\widehat{\mathbf{S}}_{k,t}(i,j)\widehat{\mathbf{S}}'_{i,t}(i,j) - \widehat{\phi}\,\widehat{\mathbf{G}}_{i,t}(i,i)\widehat{\mathbf{S}}_{k,t}(i,i)\widehat{\mathbf{S}}'_{i,t}(i,i)\right]
\end{aligned}$$

at each period of time t.

Simulation results

Only 5 networks do not satisfy this condition and thus we end up with 1,038 criminals distributed over 145 networks.

The average network is size 7 with a minimum of 4 and a maximum of 64 delinquents.

Figure 1 displayed the distribution of these 145 networks by their size.

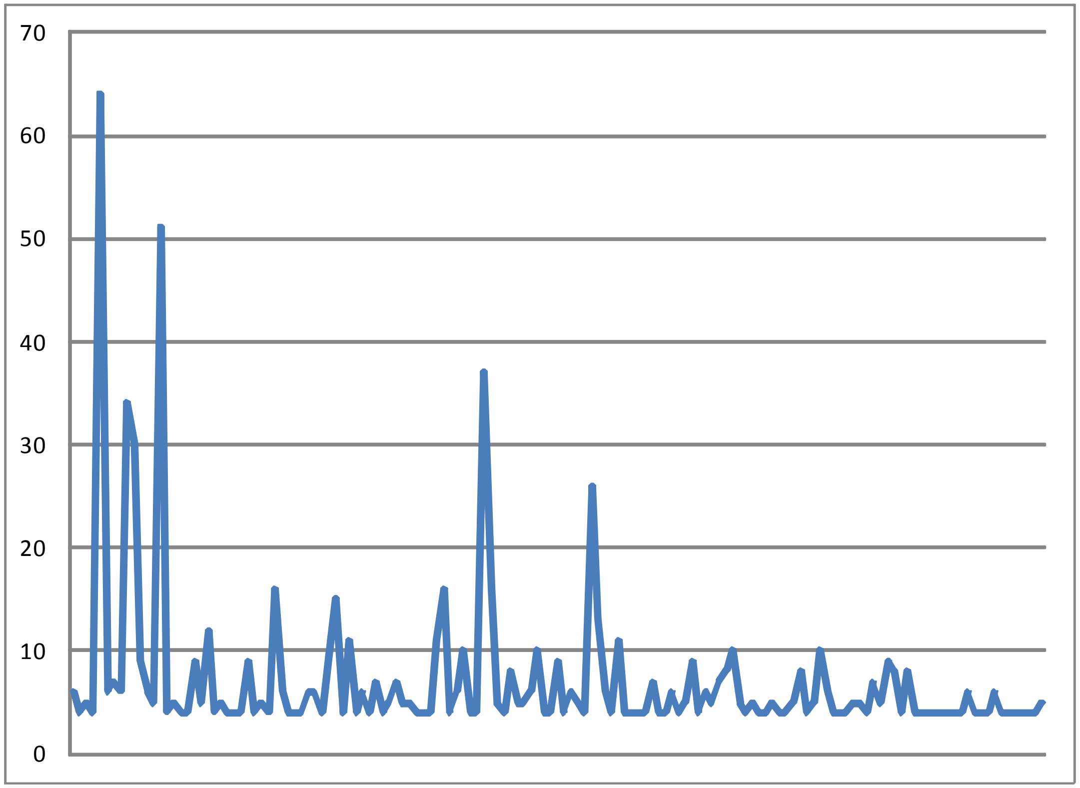


Figure 1: Distribution of networks by size for all crimes

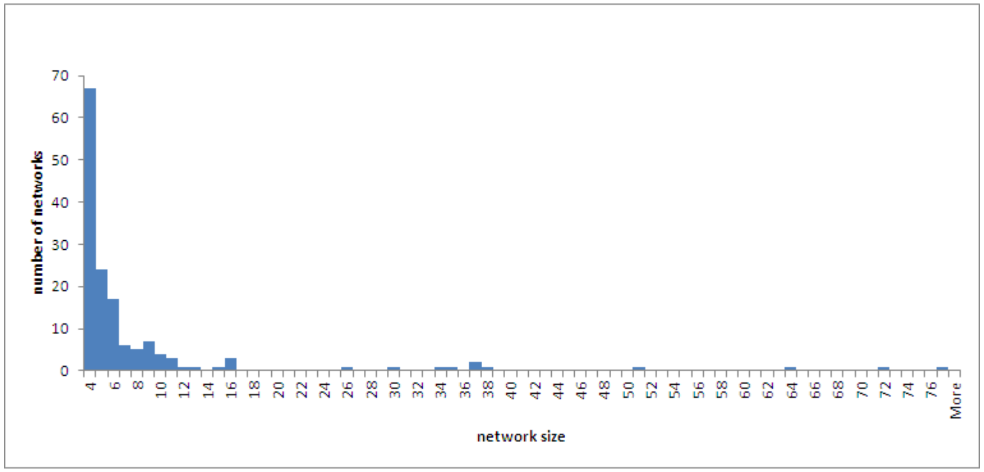


Figure 1: Distribution of networks by size for all crimes

Table 4a: Dynamic network formation and key players for all crimes when $c_i = \underline{c}_i$

Network (# nodes)	Highest Between	Highest Bonacich	KP Invariant	KP Dynamics	T crime Initial	ET crime Dynamics	ET crime Invariant KP	ET crime Dynamics KP	Density Initial (Diameter)	Density Dynamics (Diameter)	Density Invariant KP (Diameter)	Density Dynamics KP (Diameter)	# days Before (after KP)
1 (6)	3	1	3	3	11.235	11.216	8.718	8.783	0.167 (1)	0.167 (1)	0.200 (1)	0.250 (1)	0 (1)
2 (4)	2	4	3	3	7.732	7.762	5.489	5.489	0.333 (3)	0.333 (3)	0.500 (2)	0.500 (2)	0 (0)
3 (5)	2	2	2	2	12.631	12.616	9.222	9.222	0.300 (3)	0.300 (3)	0.333 (1)	0.333 (1)	0 (0)
4 (4)	1	1	1	1	8.117	8.138	5.532	5.663	0.250 (2)	0.250 (2)	0.167 (1)	0.500 (1)	0 (2)
5 (64)	28	57	35	62	148.653	148.204	144.184	144.858	0.0303 (10)	0.0306 (10)	0.0294 (8)	0.030 (10)	0 (0)
6 (6)	6	5	6	6	13.137	13.077	10.379	10.379	0.167 (1)	0.167 (1)	0.200 (1)	0.200 (1)	0 (0)
7 (7)	1	3	5	5	15.191	15.223	12.468	12.756	0.143 (2)	0.143 (2)	0.167 (1)	0.333 (2)	0 (5)
8 (6)	6	4	5	5	11.378	11.297	8.797	8.797	0.400 (4)	0.400 (4)	0.400 (3)	0.400 (3)	0 (0)
9 (34)	8	30	24	30	74.768	75.0138	69.923	72.055	0.0615 (7)	0.0615 (7)	0.057 (7)	0.063 (7)	0 (0)
10 (30)	30	13	26	26	79.651	79.665	75.546	75.546	0.0402 (3)	0.0402 (3)	0.037 (3)	0.037 (3)	0 (0)
11 (9)	4	5	4	3	38.188	37.939	32.819	33.208	0.125 (2)	0.125 (2)	0.054 (1)	0.125 (2)	0 (0)
12 (6)	5	5	5	5	12.962	13.023	10.156	10.156	0.167 (3)	0.167 (3)	0.150 (1)	0.150 (1)	0 (0)
13 (5)	2	1	1	1	11.657	11.593	8.524	8.524	0.250 (4)	0.250 (4)	0.333 (3)	0.333 (3)	0 (0)
14 (51)	6	4	39	13	142.113	141.823	137.131	137.523	0.039 (10)	0.039 (10)	0.038 (10)	0.040 (10)	0 (0)
15 (4)	1	1	1	1	14.524	14.467	10.493	10.493	0.250 (1)	0.250 (1)	0.167 (1)	0.167 (1)	0 (0)
16 (5)	3	4	5	5	9.556	9.5605	7.233	7.547	0.300 (3)	0.300 (3)	0.333 (2)	0.417 (2)	0 (1)
17 (4)	2	3	4	4	7.366	7.367	5.119	5.217	0.417 (3)	0.417 (3)	0.50 (2)	0.667 (2)	0 (1)
18 (4)	3	1	3	3	10.884	10.847	7.226	8.106	0.333 (2)	0.333 (2)	0.167 (1)	0.667 (2)	0 (3)
19 (9)	9	6	6	1	23.911	23.736	20.528	20.945	0.139 (5)	0.139 (5)	0.107 (5)	0.161 (4)	0 (1)
20 (5)	3	1	2	5	14.123	14.109	10.787	10.868	0.350 (3)	0.350 (3)	0.417 (3)	0.333 (1)	0 (0)

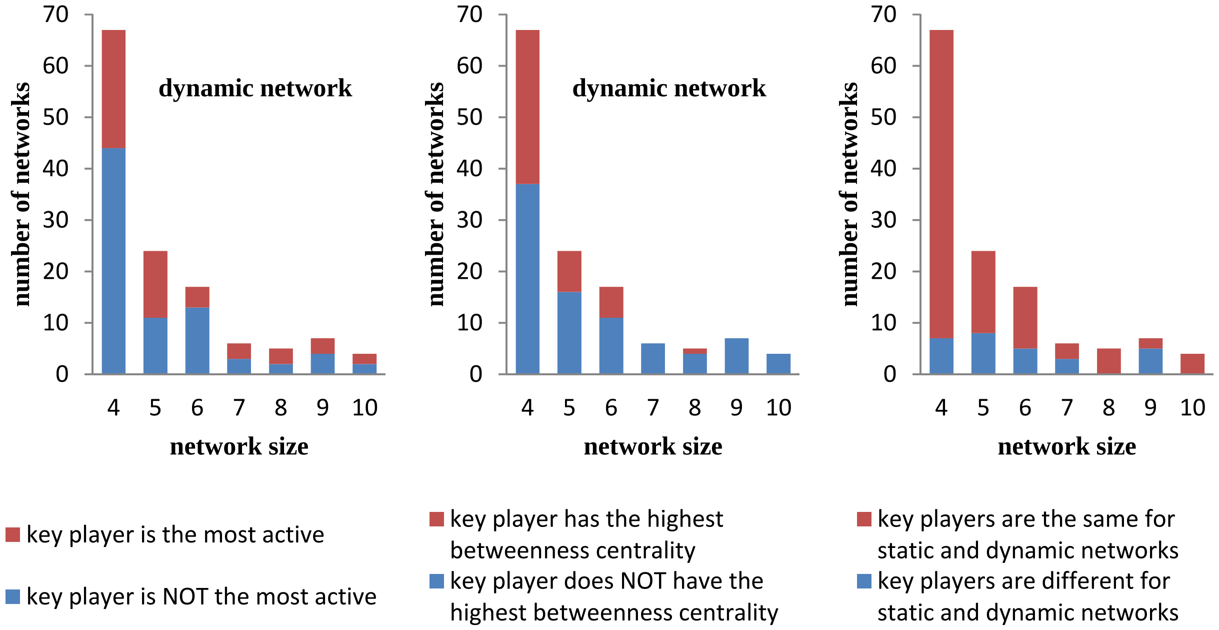
In Figure 3, we investigate the three following questions:

Is the KP the most active delinquent in the network (left panel)?

Does the KP has the highest betweenness centrality (middle panel)?

Is the KP in the DM also the key player in the static network model?

Figure 3: Who is the key player?



Individual characteristics of key players

Table 5: Who is the Key Player?
-Significant Differences-
All crimes

	All Criminals		Key Player Criminals		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.53	0.50	0.23	0.42	0.0000
Religion practice	3.65	1.41	3.28	1.57	0.0078
Parent education	3.23	1.06	3.01	1.14	0.0279
Mathematics score	2.18	1.00	2.53	1.05	0.0003
Parental care	0.93	0.26	0.80	0.40	0.0002
School attachment	4.12	0.87	3.71	1.07	0.0000
Relationship with teachers	0.99	0.92	1.79	1.22	0.0000
Social inclusion	4.47	0.74	4.23	0.86	0.0018
Residential building quality	1.51	0.79	1.70	0.96	0.0226
Two married parent families	0.74	0.44	0.61	0.49	0.0020
Single parent family	0.22	0.42	0.30	0.46	0.0706
Parent occupation manager	0.11	0.31	0.17	0.38	0.0704
Parent occupation military or security	0.02	0.14	0.00	0.00	0.0000
Parent occupation other	0.16	0.37	0.11	0.31	0.0673
Friends' characteristics					
Religious practice	2.52	1.98	3.02	1.80	0.0025
Student grade	6.42	4.33	7.64	3.85	0.0006
Parental education	2.30	1.66	2.61	1.54	0.0279
Mathematics score	1.54	1.24	1.87	1.24	0.0033
Self esteem	2.84	1.99	3.28	1.76	0.0066
Physical development	2.44	1.76	2.69	1.52	0.0810
Parental care	0.65	0.46	0.75	0.42	0.0152
School attachment	2.90	1.99	3.35	1.74	0.0055
Social inclusion	3.12	2.09	3.65	1.83	0.0019
Residential building quality	1.05	0.89	1.19	0.83	0.0621
Residential area urban	0.43	0.48	0.55	0.48	0.0033
Household size	3.13	2.22	3.48	1.97	0.0474
Single parent families	0.14	0.31	0.23	0.39	0.0105
N.obs.	893		145		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 6: Key Player versus Bonacich centrality
-Significant Differences-
All crimes

	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.12	0.33	0.30	0.46	0.0080
Social inclusion	3.98	0.86	4.39	0.82	0.0053
Residential building quality	1.91	1.03	1.57	0.89	0.0459
Friends' characteristics					
Residential area urban	0.67	0.46	0.48	0.48	0.0231
N.obs.	56		89		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 7: Who is the Key Player?

-Significant Differences-

Petty crimes

	All Criminals		Key Player Criminals		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.54	0.50	0.24	0.43	0.0000
Mathematics score	2.17	1.00	2.44	1.01	0.0049
Physical development	3.33	1.09	3.55	1.06	0.0325
Parental care	0.93	0.25	0.74	0.44	0.0000
School attachment	4.11	0.88	3.69	1.09	0.0001
Relationship with teachers	0.99	0.94	1.62	1.16	0.0000
Social inclusion	4.48	0.73	4.14	0.88	0.0001
Residential area urban	0.56	0.50	0.65	0.48	0.0523
Parent occupation manager	0.11	0.31	0.18	0.38	0.0463
Parent occupation manual	0.33	0.47	0.22	0.41	0.0065
Friends' characteristics					
Student grade	6.53	4.39	7.66	3.95	0.0034
Religion practice	2.29	1.65	2.69	1.57	0.0086
Mathematics score	1.52	1.21	1.81	1.17	0.0108
Self esteem	2.85	2.00	3.24	1.76	0.0224
Parental care	0.65	0.46	0.75	0.42	0.0170
School attachment	2.90	1.99	3.29	1.76	0.0251
Social inclusion	3.12	2.09	3.62	1.86	0.0063
Residential area urban	0.41	0.47	0.54	0.47	0.0047
Single parent family	0.15	0.31	0.23	0.38	0.0181
Parent occupation professional/technical	0.14	0.31	0.20	0.36	0.0646
N.obs.	807		128		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 8: Who is the Key Player?

-Significant Differences-

More serious crimes

	All Criminals		Key Player Criminals		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.44	0.50	0.23	0.42	0.0004
Physical development	3.25	1.11	3.69	1.04	0.0023
School attachment	3.98	0.95	3.68	1.05	0.0271
Relationship with teachers	1.16	1.04	1.97	1.35	0.0000
Parent occupation manager	0.11	0.31	0.03	0.17	0.0022
Parent occupation military or security	0.01	0.09	0.00	0.00	0.0833
Friends' characteristics					
School attachment	2.74	1.95	3.17	1.78	0.0721
Social inclusion	3.07	2.11	3.53	1.96	0.0828
Parent occupation military or security	0.01	0.07	0.00	0.00	0.0718
Parent occupation farm or fishery	0.02	0.13	0.00	0.00	0.0115
N.obs.	334		70		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 9: Key Player for Petty and Serious Crimes
-Significant Differences-

	Key Player Petty Crime		Key Player More Serious Crime		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Black or African American	0.17	0.38	0.31	0.47	0.0308
Self esteem	4.04	1.08	3.73	1.11	0.0542
Parental care	0.74	0.44	0.90	0.30	0.0033
Relationship with teachers	1.62	1.16	1.97	1.35	0.0723
Social inclusion	4.14	0.88	4.47	0.70	0.0041
Parent occupation manager	0.18	0.38	0.03	0.17	0.0002
Parent occupation military or security	0.03	0.17	0.00	0.00	0.0451
Friends' characteristics					
Female	0.40	0.43	0.26	0.40	0.0315
Black or African American	0.13	0.32	0.24	0.43	0.0539
Relationship with teachers	0.70	0.71	1.05	1.03	0.0132
Parent occupation manager	0.11	0.29	0.05	0.19	0.0825
Parent occupation military or security	0.02	0.14	0.00	0.00	0.0575
Parental occupation farm or fishery	0.02	0.11	0.00	0.00	0.1027
N.obs.	128		70		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 10: Key Player versus Bonacich centrality
-Significant Differences-
Petty crimes

	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Residential building quality	1.94	1.11	1.43	0.76	0.0073
Friends' characteristics					
Mathematics score	1.57	1.11	1.95	1.20	0.0690
Relationship with teachers	0.57	0.58	0.78	0.76	0.0789
N.obs.	47		81		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Table 11: Key Player versus Bonacich centrality
-Significant Differences-
More Serious crimes

	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Religion practice	3.96	1.26	3.40	1.41	0.0884
Parental care	0.96	0.19	0.86	0.35	0.1056
School attachment	3.92	1.01	3.52	0.99	0.1049
Relationship with teachers	2.43	1.45	1.67	1.20	0.0256
Residential area urban	0.61	0.50	0.81	0.40	0.0775
Friends' characteristics					
Other races	0.00	0.00	0.05	0.18	0.1031
Relationship with teachers	1.31	1.15	0.87	0.91	0.0951
Parental occupation professional/technical	0.23	0.42	0.07	0.20	0.0610
N.obs.	28		42		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported

Key players and network topology

Table 12: Key Players and network topology
Individual centrality measures

	All crimes			
	Betweenness	Clustering	Closeness	Bonacich
percentiles				
p50	0	0	0.33	2.49
p75	0.02	0	0.50	3.51
p90	0.20	0.17	0.75	5.20
p95	0.33	0.50	0.83	5.83
p99	0.42	1	1	6.99
min	0	0	0	1.51
max	0.5	1	1	9.39
	Key Players Not the Most Active Criminals			
>p90	71%	58%	27%	21%
	Petty crimes			
percentiles	Betweenness	Clustering	Closeness	Bonacich
p50	0	0	0.33	2.78
p75	0.05	0	0.57	3.63
p90	0.33	0.17	0.75	5.51
p95	0.33	0.5	0.83	6.23
p99	0.5	1	1	7.31
min	0	0	0	1.61
max	0.5	1	1	7.60
	Key Players Not the Most Active Criminals			
>p90	40%	72%	54%	8%
	More serious crimes			
percentiles	Betweenness	Clustering	Closeness	Bonacich
p50	0	0	0.40	2.50
p75	0	0	0.67	4.21
p90	0.09	0.21	0.67	6.70
p95	0.17	1	0.75	7.62
p99	0.5	1	1	11.37
min	0	0	0	1.69
max	0.5	1	1	11.37
	Key Players Not the Most Active Criminals			
>p90	71%	71%	50%	14%

Table 13: Key Players and network topology
Network centrality measures

All crimes					
	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
Network characteristics	Mean	St. dev	Mean	St. dev	t-test
Diameter	2.52	1.43	2.59	1.47	0.7534
Average distance	1.43	0.44	1.44	0.41	0.8879
Average degree	1.02	0.32	1.03	0.29	0.8402
Density	0.23	0.10	0.24	0.10	0.6293
Asymmetry	0.68	0.24	0.67	0.23	0.9666
Network clustering	0.07	0.15	0.05	0.12	0.4527
Network degree	0.12	0.12	0.13	0.12	0.7331
Network closeness	0.30	0.12	0.30	0.14	0.5119
Assortativity	1.60×10^{-17}	1.81×10^{-16}	5.27×10^{-18}	1.06×10^{-16}	0.6892
Network betweeness-	0.13	0.12	0.15	0.12	0.3584
N.obs.	56		89		

Petty crimes					
	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
Network characteristics	Mean	St. dev	Mean	St. dev	t-test
Diameter	2.53	1.40	2.62	1.53	0.7482
Average distance	1.44	0.44	1.45	0.42	0.8793
Average degree	1.04	0.33	1.03	0.29	0.8316
Density	0.25	0.10	0.23	0.10	0.3710
Asymmetry	0.68	0.22	0.66	0.22	0.6797
Network clustering	0.05	0.14	0.06	0.12	0.7413
Network degree	0.13	0.12	0.12	0.10	0.4249
Network closeness	0.31	0.13	0.29	0.13	0.3919
Assortativity	2.94×10^{-17}	8.27×10^{-17}	7.00×10^{-18}	1.26×10^{-16}	0.2275
Network betweeness	0.16	0.13	0.14	0.12	0.3715
N.obs.	47		81		

More serious crimes					
	Key Player Most Active Criminal		Key Player Not the Most Active Criminal		
Network characteristics	Mean	St. dev	Mean	St. dev	t-test (p-value)
Diameter	2.36	1.03	2.21	1.16	0.5900
Average distance	1.41	0.31	1.34	0.35	0.4370
Average degree	1.04	0.33	0.98	0.23	0.4221
Density	0.25	0.10	0.25	0.09	0.8746
Asymmetry	0.73	0.20	0.76	0.20	0.1606
Network clustering	0.06	0.14	0.05	0.11	0.7377
Network degree	0.13	0.13	0.16	0.13	0.4338
Network closeness	0.32	0.12	0.30	0.12	0.5638
Assortativity	2.15×10^{-17}	9.09×10^{-17}	1.17×10^{-17}	8.32×10^{-17}	0.6481
Network betweeness	0.18	0.15	0.13	0.13	0.1589
N.obs.	28		42		

Policy implications

Punishment should not be random but targeted to individuals that generate the highest multiplier effects.

The way a key player is calculated is precisely using the multiplier effects due to endogenous peer effects.

Key player removal policy: When a delinquent is removed from network r, the intercentrality measures of all the delinquents that remain active are reduced, which triggers a decrease in delinquency involvement for all of them.

Policy implications

1) We develop further the qualitative approach (simulations) to evaluate the benefits versus costs of a key-player policy.

2) Use our analysis of key-player policies to address general policies against crime.

3) KP methodology can be used to address policies in other activities and other networks such as financial networks, R&D networks, networks in developing economies, etc.

For each of the 145 networks (all crimes), we have calculated the reduction in total crime, following the removal of the key player.

Figure 2 displays the results.

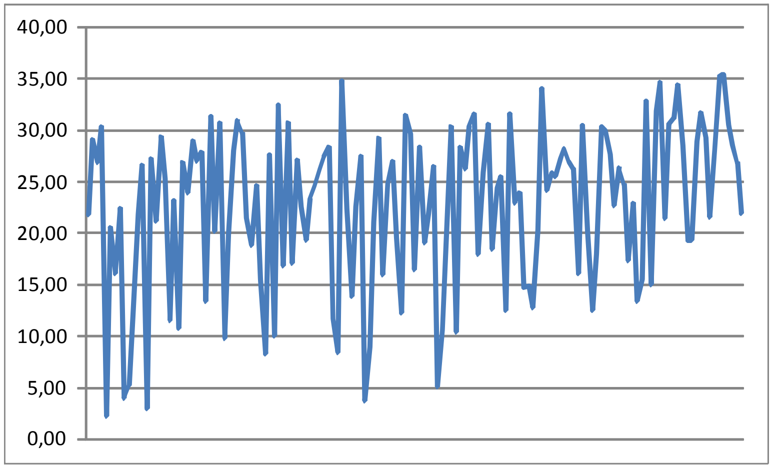


Figure 2: Distribution of networks by reduction in crime after the removal of the key player

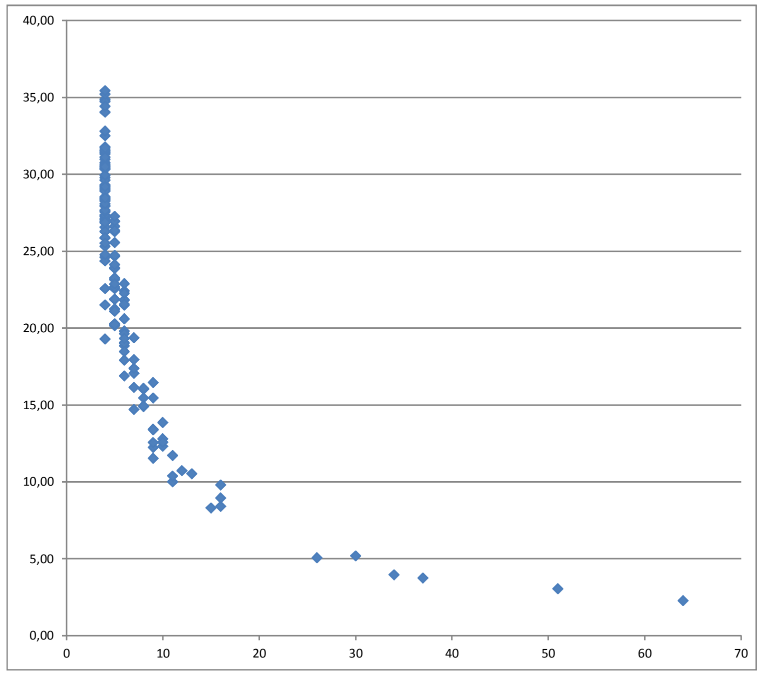


Figure 3: Crime reduction and network size

Crime reduction is much more important in small networks than in large networks.

A way to capture the size effect is to fix an objective in terms of crime reduction (say 10 percent) and analyze how many key players need to be removed in order to reach this objective.

For the small networks (less than 10 delinquents), one key player is often enough (see Figure 2) while for large networks (more than 40 delinquents), more than three key players can be necessary to reach the objective of 10 percent reduction in crime.

Table 14: Crime reduction when different policies are implemented (for $c_i = \underline{c}_i$)

Network (# nodes)	KP Dynamics	T crime Initial	ET crime Dynamics	ET crime Dynamics KP	Reduction Crime KP (%)	Reduction Crime MIN (%)	Reduction Crime AVERAGE (%)	Reduction Crime RANDOM (%)
1 (6)	3	11.235	11.236	8.783	21.80	10.78	16.71	10.78
2 (4)	3	7.732	7.740	5.489	29.08	13.88	21.78	29.09
3 (5)	2	12.631	12.621	9.222	26.93	13.06	17.612	15.02
4 (4)	1	8.117	8.130	5.663	30.35	19.74	24.64	27.37
5 (64)	62	148.653	148.204	144.858	2.26	-19.72	-3.27	-19.35
6 (6)	6	13.137	13.069	10.379	20.58	9.86	14.46	9.86
7 (7)	5	15.191	15.208	12.756	16.13	2.41	11.84	15.20
8 (6)	5	11.378	11.337	8.797	22.40	5.88	13.98	14.52
9 (34)	30	74.768	75.014	72.055	3.94	-26.35	-9.82	-26.35
10 (30)	26	79.651	79.665	75.546	5.17	-10.10	1.53	-10.10
11 (9)	3	38.188	37.831	33.208	12.22	-2.70	7.12	-2.70
12 (6)	5	12.962	12.990	10.156	21.81	6.039	12.88	6.04
13 (5)	1	11.657	11.612	8.524	26.59	10.30	17.76	13.16
14 (51)	13	142.113	141.823	137.523	3.032	-22.73	-3.49	0.45
15 (4)	1	14.524	14.420	10.493	27.23	24.64	25.55	24.64
16 (5)	5	9.556	9.581	7.547	21.23	6.22	13.36	6.22
17 (4)	4	7.366	7.3782	5.217	29.30	18.75	23.25	18.75
18 (4)	3	10.884	10.850	8.106	25.29	14.80	22.319	24.52
19 (9)	1	23.911	23.670	20.945	11.51	0.54	4.848	1.80
20 (5)	5	14.123	14.138	10.868	23.13	15.69	18.92	16.59

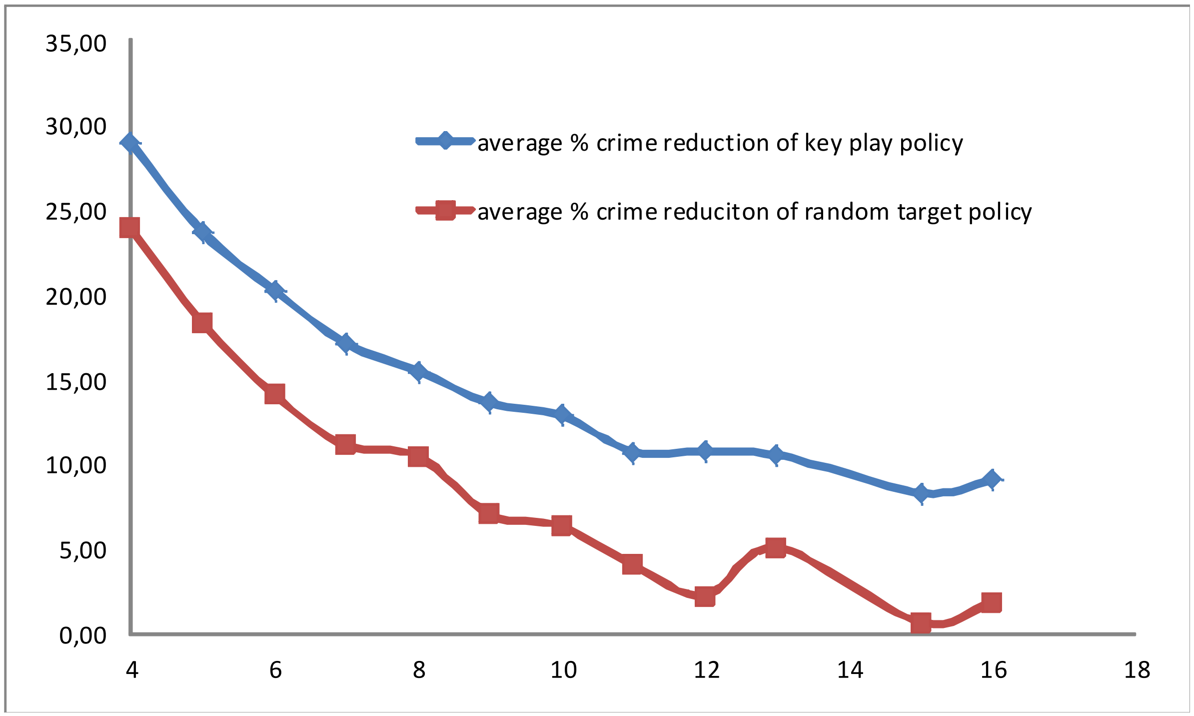
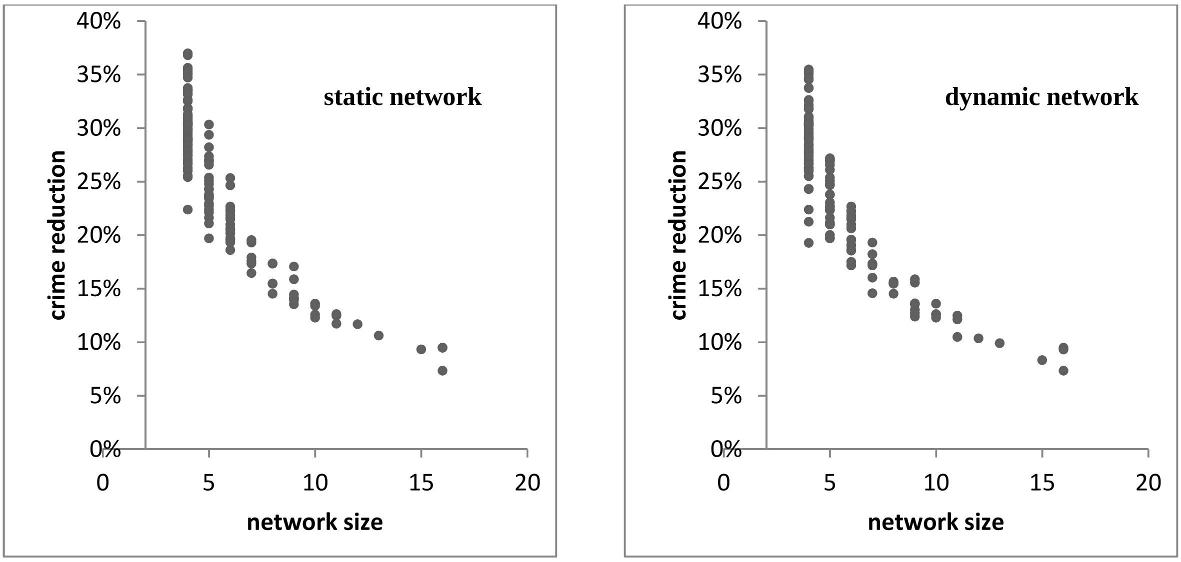


Figure 4: Difference between a key-player and a random-target policy

Table 14b: Average crime reduction when different policies are implemented by network size (for $c_i = \underline{c}_i$)

Network size	Average % crime reduction of a key play policy	Average % crime reduction of a random target policy
4	28,94	23,86
5	23,67	18,29
6	20,21	14,05
7	17,08	11,07
8	15,46	10,44
9	13,56	7,00
10	12,87	6,29
11	10,68	3,99
12	10,72	2,18
13	10,51	5,02
15	8,28	0,60
16	9,04	1,79
26	5,05	0,96
30	5,17	1,53
34	3,94	-9,82
37	3,73	0,23
51	3,03	-3,49
64	2,26	-3,27

Figure 5: Crime reduction and network size



How a key-player policy can be useful in fighting crime?

Data about criminal networks

(i) *Juvenile deliquency in schools (survey data):* There are data similar to the AddHealth data for students in schools.

Weerman (2011) uses data from the Netherlands Institute for the Study of Crime and Law Enforcement (NSCR) “School Study,” a study that focused on social networks and the role of peers in delinquency with two waves, conducted in the spring of 2002 and 2003.

(ii) *Adult crime (police data)*: the police has in fact quite a lot of information on criminal networks.

Sarnecki (2001) was able to construct the network of all criminals in Stockholm.

Each time two (or more) persons are suspected about a crime (co-offenders), the police in Sweden registers this information.

A link in a network is then created between individuals i and j, i.e. $g_{ij} = 1$, whenever individuals i and j are suspected of a crime together.

This type of information can be obtained from the police in many countries.

In the United States, there is also similar data.

For example, Coplink was one of the first large scale research projects in crime data mining, and an excellent work in criminal network analysis.

Colink has information about the perpetrators' habits and close associations in crime to capture the *connections* between people, places, events, and vehicles, based on past crimes.

(iii) *Gang networks*

McGloin (2005) use data from the Newark portion of the North Jersey Gang Task Force, a regional problem analysis project that sought to define the local gang landscape in Northern New Jersey.

Another example of available gang network data: Mastrobuoni and Patacchini (2012).

They use a data set provided by the Federal Bureau of Narcotics on criminal profiles of 800 US Mafia members active in the 1950s and 1960s and on their connections within the Cosa Nostra network.

How to implement a key-player policy?

Once we have identified a key player in a network, one cannot put him/her in prison if he/she hasn't committed any crime.

However, different policies can be implemented to reduce crime using a key-player approach.

Implementation of KP policy in crime

Criminal networks: Identify the KP and offer him/her incentives to leave the gang or the criminal network.

Examples: Police can offer them a job or conditional transfers (by asking them to move to another city) or monitor them more.

In Canada, some gang members of criminal networks were persuaded to abandon gang life in return for needed employment training, educational training, and skills training (Tremblay et al., 1996).

Implementation of KP policy in crime

Police can target key players in a meaningful way.

Recent innovation in policing: "pulling-levers policing" (Kennedy, 1998, 2008).

Policy called the "Ceasefire approach": combines a strong law enforcement response with a "pulling levers" deterrence effort aimed at chronic gang offenders.

The key to the success is to use a "levers/lever pulling" approach, which is a crime deterrence strategy that attempts to prevent violent behavior by using a *targeted individual* or *group's vulnerability* to law enforcement as a means of gaining their compliance.

Locate gang members who had *outstanding arrest warrants* or *had violated probation* or *parole regulations*.

Also gang members who had *violated public housing rules*, *failed to pay child support*, or were similarly vulnerable.

Operation Ceasefire. first launched in Boston and youth homicide fell by two-thirds after the Ceasefire strategy was put in place in 1996.

It was then implemented in Los Angeles in 2000: Strong effects.

Implementation of KP policy in crime

A key-player policy can also help for related issues.

Composition of groups. how to allocate under-age kids who have committed an offence into juvenile facilities (detention centers).

Young criminals are sent to juvenile detention centers.

Learning is an important aspect in prison (Bayer et al., QJE 2009).

Rank these kids by key player centralities and avoid to put together high KPC with low KPC criminals.

Using our methodology to find key players for other types of networks and activities

Financial networks

Huge information available on financial networks where links are usual bank loans.

Cohen-Cole, Patacchini and Zenou (2011) use transaction level data on interbank lending from an electronic interbank market, the e-MID SPA (or e-MID)

Boss et al. (2004) analyze the network of Austrian banks in the year 2008 where links in the network represent exposures between Austrian-domiciled banks on a non-consolidated basis (i.e. no exposures to foreign subsidiaries are included).

Key player policy: Which bank should we bail out in order to reduce systemic risk or maximize total activity?

"Too big to fail" versus "too interconnected to fail"

Which bank to monitor

R&D networks

Lot of information on R&D networks.

García-Canal et al. (2008) use alliance data steming from the Thomson SDC Platinum data base.

Our model is easily adapted to R&D networks where efforts correspond to quantity produced and a link (i.e. an alliance) decreases the cost of producing goods.

Key-player policy: Identifying the key firms that are the most critical for industry productivity.

Which firm should we subsidy in order to generate maximum total activity?

Networks in developing economies

Many network data for developing countries (mainly surveys)

Fafchamps and Lund (2003): conducted a survey in four villages in the Cordillera mountains of northern Philippines between July 1994 and March 1995

Krishnan and Sciubba (2009): used the second round of the Ethiopian Rural Household Survey, conducted in 1994).

Banerjee et al. (2012): Survey on 75 rural villages in Karnataka, India.

Look at the diffusion of a microfinance program in these villages.

Show: if the bank in charge of this program, had targeted individuals in the village with the highest eigenvector centrality, the diffusion of the microfinance program (i.e. take-up rates) would have been much higher.

Key player policy: Adoption of a new technology since there is strong evidence of social learning (Conley and Udry, 2010) and take-up rates in microfinance programs.

Political networks

Evidence on lobbying to persuade public opinion when members of the public influence each other's opinion.

When people are deciding how to vote or which product to buy, they discuss their decision with people in their social environment.

Competitions to persuade public opinion are the essence of political campaigns, but also occur in marketing between rival firms or in lobbying with interests groups on opposite sides of a legislation.

Our key-player policy suggests that resources should be spent on *key voters* who have an influential position in the social network.

Tax evasion

Strong evidence that there are strong social interaction effects in tax evasion (Fortin et al., 2007).

Galbiati and Zanella (2012) estimate social externalities of tax evasion in a model where congestion of the auditing resources of local tax authorities generates a social multiplier.

Relevant question would be: Which person(s) should we target to reduce total tax evasion in a country?