Complementary Activities, Heterogeneous Firms, and Industry Structure

Peter A. Bache and Anders Laugesen

Department of Economics and Business
Aarhus University

Center for Market Studies and Spatial Economics, December 2012

Introduction

- Abundant evidence that firms are heterogeneous along many dimensions of their operations; see Bernard et al. (2012).
- A part of the heterogeneous-firms trade literature lets firms face decisions about more than one activity.
  - By the term **activity** we refer to any single variable at the discretion of the firm and we let the term **decision** denote the combined choice on all activities. Thus, an activity is simply one dimension of the firm's overall decision.

Introduction

- Abundant evidence that firms are heterogeneous along many dimensions of their operations; see Bernard et al. (2012).
- A part of the heterogeneous-firms trade literature lets firms face decisions about more than one activity.
  - By the term **activity** we refer to any single variable at the discretion of the firm and we let the term **decision** denote the combined choice on all activities. Thus, an activity is simply one dimension of the firm's overall decision.
  - Two- or multi-dimensional decisions are modelled by e.g. Antràs and Helpman (2004), Verhoogen (2008), Lileeva and Trefler (2010), Helpman and Itskhoki (2010), Helpman et al. (2010), Bustos (2011), Bernard et al. (2011), Arkolakis and Muendler (2011), Caliendo and Rossi-Hansberg (2011), Kasahara and Lapham (forthcoming), and Amiti and Davis (2011).

Introduction

- Abundant evidence that firms are heterogeneous along many dimensions of their operations; see Bernard et al. (2012).
- A part of the heterogeneous-firms trade literature lets firms face decisions about more than one activity.
  - By the term **activity** we refer to any single variable at the discretion of the firm and we let the term **decision** denote the combined choice on all activities. Thus, an activity is simply one dimension of the firm's overall decision.
  - Two- or multi-dimensional decisions are modelled by e.g. Antràs and Helpman (2004), Verhoogen (2008), Lileeva and Trefler (2010), Helpman and Itskhoki (2010), Helpman et al. (2010), Bustos (2011), Bernard et al. (2011), Arkolakis and Muendler (2011), Caliendo and Rossi-Hansberg (2011), Kasahara and Lapham (forthcoming), and Amiti and Davis (2011).
- Many of these activities are complementary in the sense that undertaking one of them increases the payoff from undertaking another.

Motivation

- Despite the frequent application of complementarities activities in general-equilibrium models, the implications of this type of interdependence are not yet fully understood.
- Most models focus on two complementary activities.
- Most models share the same assumptions, so what implications do they share?

Our model of complementary activities

- We build a heterogeneous-firms model on common assumptions...
  - ...with a general formulation of a multi-dimensional (firm-level) decision about n activities.
- We allow for firm heterogeneity in multiple dimensions.
  - Having multiple sources of firm heterogeneity is instrumental to achieving the wide variety of firm decisions observed in data.
  - In our model, the sorting of firms into different activities is not solely based on productivity.

Our model of complementary activities

- We build a heterogeneous-firms model on common assumptions...
  - ...with a general formulation of a multi-dimensional (firm-level) decision about n activities.
- We allow for firm heterogeneity in multiple dimensions.
  - Having multiple sources of firm heterogeneity is instrumental to achieving the wide variety of firm decisions observed in data.
  - In our model, the sorting of firms into different activities is not solely based on productivity.
- We assume that activities are complementary which allows us to draw clear testable predictions on the industry structure.
  - Industry structure: the endogenous distribution of active firms over the activities.

Results

- Our main contribution is to show that:

  i) the introduction of a new activity (Proposition 1); and
  ii) increased attractiveness of an existing activity (Proposition 2)

  shift the industry structure towards higher levels of *all* activities.
- Equivalently, the industry-level share of active firms undertaking (at least any given level of) any activity is nondecreasing.

Results

- Our main contribution is to show that:

  i) the introduction of a new activity (Proposition 1); and
  ii) increased attractiveness of an existing activity (Proposition 2)

  shift the industry structure towards higher levels of *all* activities.
- Equivalently, the industry-level share of active firms undertaking (at least any given level of) any activity is nondecreasing.
- Meanwhile, at the firm level, the effects of i) and ii) are more subtle (Proposition 3).
  - General-equilibrium effects imply that an individual firm may reduce its level of all activities when faced with new or improved opportunities.
  - It is in fact possible that all firms strictly reduce their levels of a given activity even though the industry structure shifts towards weakly higher values of this particular activity.

Results

- Our main contribution is to show that:

  i) the introduction of a new activity (Proposition 1); and
  ii) increased attractiveness of an existing activity (Proposition 2)

  shift the industry structure towards higher levels of *all* activities.
- Equivalently, the industry-level share of active firms undertaking (at least any given level of) any activity is nondecreasing.
- Meanwhile, at the firm level, the effects of i) and ii) are more subtle (Proposition 3).
  - General-equilibrium effects imply that an individual firm may reduce its level of all activities when faced with new or improved opportunities.
  - It is in fact possible that all firms strictly reduce their levels of a given activity even though the industry structure shifts towards weakly higher values of this particular activity.
  - Hence, the firm-level results in Milgrom and Roberts (1990, 1995) and Topkis (1995) do not necessarily hold in general equilibrium.

Why is this important?

- Our industry-level results provide new and clear testable predictions from a very large group of models conforming to our setup. Prominent examples:
  - Helpman et al. (2004), Helpman and Itskhoki (2010), Helpman et al. (2010), Arkolakis (2010), Bernard et al. (2011), Bustos (2011), Caliendo and Rossi-Hansberg (2011). If you assume Pareto-distributed productivities, then also Melitz (2003) and Kasahara and Lapham (forthcoming). Antràs and Helpman (2004) under a particular parameterisation.
- It will be both useful og interesting to confront these theoretical models with industry-level data and our propositions which ought to hold for these models .

Why is this important?

- Our industry-level results provide new and clear testable predictions from a very large group of models conforming to our setup. Prominent examples:
  - Helpman et al. (2004), Helpman and Itskhoki (2010), Helpman et al. (2010), Arkolakis (2010), Bernard et al. (2011), Bustos (2011), Caliendo and Rossi-Hansberg (2011). If you assume Pareto-distributed productivities, then also Melitz (2003) and Kasahara and Lapham (forthcoming). Antràs and Helpman (2004) under a particular parameterisation.
- It will be both useful og interesting to confront these theoretical models with industry-level data and our propositions which ought to hold for these models .
- Firm-level complementarities assert themselves more clearly at the industry level than at the firm level of analysis. This may be useful if you want to test the presence of complementaries.

Why is this important?

- Our industry-level results provide new and clear testable predictions from a very large group of models conforming to our setup. Prominent examples:
  - Helpman et al. (2004), Helpman and Itskhoki (2010), Helpman et al. (2010), Arkolakis (2010), Bernard et al. (2011), Bustos (2011), Caliendo and Rossi-Hansberg (2011). If you assume Pareto-distributed productivities, then also Melitz (2003) and Kasahara and Lapham (forthcoming). Antràs and Helpman (2004) under a particular parameterisation.
- It will be both useful og interesting to confront these theoretical models with industry-level data and our propositions which ought to hold for these models .
- Firm-level complementarities assert themselves more clearly at the industry level than at the firm level of analysis. This may be useful if you want to test the presence of complementaries.
- Our paper accentuates the importance of looking at complementary activities when one has to explain industry-level trends in a given activity.
  - Example: if R&D and offshoring are complementary activities, then better R&D opportunities may explain some of the rise in offshoring.

Related literature

- Firm-level analysis of complementary activities: Milgrom and Roberts (1990,1995) and Topkis (1995).
- In contrast to those studies, we analyse a general equilibrium with monopolistic competition among heterogeneous firms and derive comparative statics on the industry structure.

Related literature

- Firm-level analysis of complementary activities: Milgrom and Roberts (1990,1995) and Topkis (1995).
- In contrast to those studies, we analyse a general equilibrium with monopolistic competition among heterogeneous firms and derive comparative statics on the industry structure.
- We assume that profits are supermodular in the activities and draw upon results developed and discussed in Topkis (1978, 1995), Milgrom and Roberts (1990), and Milgrom and Shannon (1994).
- Mrazova and Neary (2011) emphasise the role of supermodularity in the selection of heterogeneous firms into a single activity (foreign market access). They also investigate the conditions under which supermodularity may arise.
  - We consider not only a given equilibrium but derive comparative statics on the industry structure across equilibria and contrast these results with the firm-level responses. The cost of this approach is additional assumptions.

Related literature

- Firm-level analysis of complementary activities: Milgrom and Roberts (1990,1995) and Topkis (1995).
- In contrast to those studies, we analyse a general equilibrium with monopolistic competition among heterogeneous firms and derive comparative statics on the industry structure.
- We assume that profits are supermodular in the activities and draw upon results developed and discussed in Topkis (1978, 1995), Milgrom and Roberts (1990), and Milgrom and Shannon (1994).
- Mrazova and Neary (2011) emphasise the role of supermodularity in the selection of heterogeneous firms into a single activity (foreign market access). They also investigate the conditions under which supermodularity may arise.
  - We consider not only a given equilibrium but derive comparative statics on the industry structure across equilibria and contrast these results with the firm-level responses. The cost of this approach is additional assumptions.
- Thankfully, it is exactly those assumptions that allow us to come up with the testable predictions for a vast number of existing models. Therefore, we see our assumptions as standard in the literature. Nevertheless we now work in the direction of providing necessary conditions for our industry-level results to hold.

The model

- Monopolistic competition in an industry characterised by a vector of some industry parameters, β, and the endogenous demand level, A. Free entry requires an entry cost.
- Upon entry, a firm realises its productivity level, $\theta \in [\theta_0, \infty)$, and its vector of other firm characteristics, $\gamma \in \Gamma$.

The model

- Monopolistic competition in an industry characterised by a vector of some industry parameters, β, and the endogenous demand level, A. Free entry requires an entry cost.
- Upon entry, a firm realises its productivity level, $\theta \in [\theta_0, \infty)$, and its vector of other firm characteristics, $\gamma \in \Gamma$.
- If a firm chooses to produce, then the firm has to make the decision $x = (x_1, \ldots, x_n)$ where x_i denotes the chosen level of activity i.
- We let $x \in S$ where S is the set of all admissible decisions. The set of all conceivable decisions, X, is assumed to be a lattice which, loosely speaking, means that undertaking a higher level of any activity may require, but importantly, cannot prevent, undertaking a higher level of another activity. We assume that $S \subseteq X$. This formulation allows us to analyse changes in the set of admissible decisions, S.

The model

- Monopolistic competition in an industry characterised by a vector of some industry parameters, β, and the endogenous demand level, A. Free entry requires an entry cost.
- Upon entry, a firm realises its productivity level, $\theta \in [\theta_0, \infty)$, and its vector of other firm characteristics, $\gamma \in \Gamma$.
- If a firm chooses to produce, then the firm has to make the decision $x = (x_1, \ldots, x_n)$ where x_i denotes the chosen level of activity i.
- We let $x \in S$ where S is the set of all admissible decisions. The set of all conceivable decisions, X, is assumed to be a lattice which, loosely speaking, means that undertaking a higher level of any activity may require, but importantly, cannot prevent, undertaking a higher level of another activity. We assume that $S \subseteq X$. This formulation allows us to analyse changes in the set of admissible decisions, S.
  - In Melitz (2003), a firm's decision comprises two activities: exporting and the amount of labour input. Hence, we have $x = (\mathbb{1}_{ex}, l)$. In an open economy, $S = \{0, 1\} \times \mathbb{R}_+$. In autarky, we have that $S = \{0\} \times \mathbb{R}_+$.

Firm behaviour

- Firms make their decisions, x, to maximise profits, π, under the constraint that $x \in S$ while taking θ, γ, β, and A as given:

$$x^*(\theta, A, \gamma, \beta, S) \in \arg\max_{x \in S} \pi(x, \theta, A, \gamma, \beta)$$

- In case a firm's maximum obtainable profits, $\pi^*(x^*(\theta, A, \gamma, \beta, S), \theta, A, \gamma, \beta)$, are negative, the firm exits the industry and forfeits the fixed cost of entry.
- For all $\gamma \in \Gamma$, the least productive firms choose not to produce.

Firm behaviour

- Firms make their decisions, x, to maximise profits, π, under the constraint that $x \in S$ while taking θ, γ, β, and A as given:

$$x^*(\theta, A, \gamma, \beta, S) \in \arg\max_{x \in S} \pi(x, \theta, A, \gamma, \beta)$$

- In case a firm's maximum obtainable profits, $\pi^*(x^*(\theta, A, \gamma, \beta, S), \theta, A, \gamma, \beta)$, are negative, the firm exits the industry and forfeits the fixed cost of entry.
- For all $\gamma \in \Gamma$, the least productive firms choose not to produce.
- These optimal firm-level decisions give rise to an endogenous multivariate distribution of active firms over the activities which we refer to as the industry structure.
- We let the marginal c.d.f., $G_i(x_i; \beta, S)$, denote the endogenous distribution of active firms over activity i.
  - Hence, $G_i(x_i; \beta, S)$ simply describes one dimension of the industry structure, namely the ith.

Firm behaviour

- Firms make their decisions, x, to maximise profits, π, under the constraint that $x \in S$ while taking θ, γ, β, and A as given:

$$x^*(\theta, A, \gamma, \beta, S) \in \arg\max_{x \in S} \pi(x, \theta, A, \gamma, \beta)$$

- In case a firm's maximum obtainable profits, $\pi^*(x^*(\theta, A, \gamma, \beta, S), \theta, A, \gamma, \beta)$, are negative, the firm exits the industry and forfeits the fixed cost of entry.
- For all $\gamma \in \Gamma$, the least productive firms choose not to produce.
- These optimal firm-level decisions give rise to an endogenous multivariate distribution of active firms over the activities which we refer to as the industry structure.
- We let the marginal c.d.f., $G_i(x_i; \beta, S)$, denote the endogenous distribution of active firms over activity i.
  - Hence, $G_i(x_i; \beta, S)$ simply describes one dimension of the industry structure, namely the ith.
- Our aim is to derive comparative statics on the industry structure w.r.t. (β, S).

Assumption 1

- Let an equilibrium exist and let the following be true:
  - i) Profits, π, only depend on θ and A through $\Theta = A\theta$, i.e.,

$$\pi = \pi(x, \Theta, \gamma, \beta).$$

  - ii) Conditional on γ, the distribution of productivities, θ, is Pareto.

Assumption 1

- Let an equilibrium exist and let the following be true:
  - i) Profits, π, only depend on θ and A through $\Theta = A\theta$, i.e.,

$$\pi = \pi(x, \Theta, \gamma, \beta).$$

  - ii) Conditional on γ, the distribution of productivities, θ, is Pareto.
- No assumptions are necessary about the distribution of the "other firm characteristics" in γ.

Assumption 2

- The profit function, $\pi(x, \Theta, \gamma, \beta)$, is supermodular in (x, Θ).

Interpretation

- Activities are complementary with each other and with (demand-adjusted) productivity.

Assumption 2

- The profit function, $\pi(x, \Theta, \gamma, \beta)$, is supermodular in (x, Θ).

Interpretation

- Activities are complementary with each other and with (demand-adjusted) productivity.

Lemma 1

- Under Assumption 2, x^* is monotone nondecreasing in Θ.

Proof

- Follows from Theorem 5 of Milgrom and Shannon (1994).

Equilibrium

- Let us assume that an equilibrium exists. In this case, Lemma 1 implies the following.
- If a firm with productivity θ' undertakes the level x_i of activity i, then all firms with the same γ and productivities $\theta > \theta'$ undertake at least the level x_i of activity i.
  - All else equal, higher productivity firms sort into weakly higher levels of all activities.

Equilibrium

- Let us assume that an equilibrium exists. In this case, Lemma 1 implies the following.
- If a firm with productivity θ' undertakes the level x_i of activity i, then all firms with the same γ and productivities $\theta > \theta'$ undertake at least the level x_i of activity i.
  - All else equal, higher productivity firms sort into weakly higher levels of all activities.
- This pattern of firm sorting into various activities can be seen as an extension of the insight of Mrazova and Neary (2011) to our case of multidimensional firm decisions and multidimensional firm heterogeneity.
- Such a sorting pattern is tremendously helpful in characterising industry equilibria.

A short digression on the role of γ

- The result that higher productivity firms sort into higher levels of all activitities does not seem to resonate with reality.
- Nevertheless, this result appears under Assumption 2 when firms are only heterogeneous in the productivity dimension.
- Further, the number of different decisions that can be observed is greatly reduced. For example, if we have two binary activities, there are four possible decisions, $X = \{(0,0),(1,0),(0,1),(1,1)\}$. If $x' = (1,0)$ is the optimal decision for a firm with productivity θ, then the different decision, $x'' = (0,1)$ cannot be the optimal decision for some other firm as this would be a contradiction with Lemma 1. That is, one would not be able to observe both $x' = (1,0)$ and $x'' = (0,1)$.

A short digression on the role of γ

- The result that higher productivity firms sort into higher levels of all activitities does not seem to resonate with reality.
- Nevertheless, this result appears under Assumption 2 when firms are only heterogeneous in the productivity dimension.
- Further, the number of different decisions that can be observed is greatly reduced. For example, if we have two binary activities, there are four possible decisions, $X = \{(0,0),(1,0),(0,1),(1,1)\}$. If $x' = (1,0)$ is the optimal decision for a firm with productivity θ, then the different decision, $x'' = (0,1)$ cannot be the optimal decision for some other firm as this would be a contradiction with Lemma 1. That is, one would not be able to observe both $x' = (1,0)$ and $x'' = (0,1)$.
- Allowing for additional sources of firm heterogeneity through γ, alleviates these undesirable features. While the strict sorting pattern based on productivity holds for a given vector of characteristics, γ, it does not necessarily hold across firms with different characteristics.
- Given that many sources of firm heterogeneity are unobservable in data, we later integrate γ out to get our empirical predictions.

Industry-level comparative statics

- Assumption 2 not only gives us a simple and convenient pattern of firm sorting, it also implies monotone comparative statics for the industry structure.
- We now consider an increase in the size of S. This includes the possibility of introducing a new activity.

Definition

- An introduction of new activities is a change from $S = S'$ to $S = S''$ such that $S' \subseteq S''$ and $S' \leq_s S''$ where $\leq_s$ is the strong set order. Further, we only consider cases where the decisions of the least productive active firms (given γ) under S'' were admissible under S'.

Industry-level comparative statics

- Assumption 2 not only gives us a simple and convenient pattern of firm sorting, it also implies monotone comparative statics for the industry structure.
- We now consider an increase in the size of S. This includes the possibility of introducing a new activity.

Definition

- An introduction of new activities is a change from $S = S'$ to $S = S''$ such that $S' \subseteq S''$ and $S' \leq_s S''$ where $\leq_s$ is the strong set order. Further, we only consider cases where the decisions of the least productive active firms (given γ) under S'' were admissible under S'.

Proposition 1

Consider an increase in S from S' to S''. Under Assumption 1 and 2, we have that $G_i(x_i; \beta, S') \geq G_i(x_i; \beta, S'')$ for all levels, x_i, of all activities, $i = 1, 2, ..., n$. Consequently, $G_i(\cdot\,; \beta, S'')$ first-order stochastically dominates $G_i(\cdot\,; \beta, S')$.

- The industry structure shifts towards higher levels of activities in all dimensions.
- Equivalently, the industry-level share of active firms undertaking at least any given level of any activity is nondecreasing.

Moving towards Proposition 2

Assumption 3

- The profit function, $\pi(x, \Theta, \gamma, \beta)$, is supermodular in (x, β).
- That is, an increase in β makes it more profitable to increase the levels of the activities given Θ, i.e., given the demand level, A.
- An increase in the attractiveness of existing activities is a change from $\beta = \beta'$ to $\beta = \beta''$ where $\beta' \leq \beta''$. Further, we restrict attention to changes in β that neither affect the decision, nor directly affect the profits, of the least productive active firms conditional on γ.

Moving towards Proposition 2

Assumption 3

- The profit function, $\pi(x, \Theta, \gamma, \beta)$, is supermodular in (x, β).
- That is, an increase in β makes it more profitable to increase the levels of the activities given Θ, i.e., given the demand level, A.
- An increase in the attractiveness of existing activities is a change from $\beta = \beta'$ to $\beta = \beta''$ where $\beta' \leq \beta''$. Further, we restrict attention to changes in β that neither affect the decision, nor directly affect the profits, of the least productive active firms conditional on γ.

Proposition 2

Consider increasing the attractiveness of existing activities by moving from β' to β''. Under Assumption 1–3, we have that $G_i(x_i; \beta', S) \geq G_i(x_i, \beta'', S)$ for all levels, x_i, of all activities, $i = 1, 2, ..., n$. Consequently, $G_i(\cdot\,; \beta'', S)$ first-order stochastically dominates $G_i(\cdot\,; \beta', S)$.

- Again, the industry structure shifts towards higher levels of activities in all dimensions.
- Equivalently, the industry-level share of active firms undertaking at least any given level of any activity is nondecreasing.

Monotone comparative statics at the industry level

- Introducing a new activity, or improving the attractiveness of an existing activity, not only affects the industry structure in that dimension, it systematically shifts the industry structure to the right in all dimensions.
- Importantly, these clear and testable results appear despite...
- ... non-monotone comparative statics at the firm level.
- Therefore, the industry-level monotone comparative statics do not arise due to simple aggregation of monotone comparative statics at the firm-level.

Firm-level comparative statics

Lemma 2

- Under Assumption 2, an introduction of new activities by moving from S' to S'' implies that $A(\beta, S') \geq A(\beta, S'')$. Under Assumption 2 and 3, an increase in the attractiveness of existing activities by moving from β' to β'' implies that $A(\beta', S) \geq A(\beta'', S)$.

Firm-level comparative statics

Lemma 2

- Under Assumption 2, an introduction of new activities by moving from S' to S'' implies that $A(\beta, S') \geq A(\beta, S'')$. Under Assumption 2 and 3, an increase in the attractiveness of existing activities by moving from β' to β'' implies that $A(\beta', S) \geq A(\beta'', S)$.

Intuition

- The intuition underlying Lemma 2 is straight forward. When presenting firms with new or better opportunities, then all else equal, all firms are able to earn weakly higher profits. In order to maintain an expected value of entry of zero, the response must be a decline in the endogenous demand level, $A = A(\beta, S)$, faced by a given firm.
- Note that Lemma 2 does not depend on Pareto-distributed productivities or the assumption that the profit function only depends on θ and A through $\Theta = A\theta$. To obtain Lemma 2, you can therefore drop Assumption 1. Moreover, you can replace Assumption 2 with the weaker assumption that $\pi = \pi(x, A, \theta, \gamma, \beta)$ is strictly increasing in (A, θ) and supermodular in (x, A, θ).
- This decline in A complicates the comparative statics at the firm level.

Proposition 3

Following an introduction of new activities, or an increase in the attractiveness of existing activities, a given firm may increase, maintain, or reduce its level of any given activity. Further, the direction of adjustment may very well vary across activities within a firm and also across firms.

Proposition 3

Following an introduction of new activities, or an increase in the attractiveness of existing activities, a given firm may increase, maintain, or reduce its level of any given activity. Further, the direction of adjustment may very well vary across activities within a firm and also across firms.

Intuition

- Under the complementarities imposed by Assumption 2 and 3, we have that x^* is nondecreasing in β and S for a given A.
- This is reminiscent of the firm-level monotone comparative statics obtained by Milgrom and Roberts (1990, 1995) and Topkis (1995). In our setup, this is not the whole story. Since all firms have access to the new or more attractive activities, the result is a decline in the demand level, A, as stated by Lemma 2.

Proposition 3

Following an introduction of new activities, or an increase in the attractiveness of existing activities, a given firm may increase, maintain, or reduce its level of any given activity. Further, the direction of adjustment may very well vary across activities within a firm and also across firms.

Intuition

- Under the complementarities imposed by Assumption 2 and 3, we have that x^* is nondecreasing in β and S for a given A.
- This is reminiscent of the firm-level monotone comparative statics obtained by Milgrom and Roberts (1990, 1995) and Topkis (1995). In our setup, this is not the whole story. Since all firms have access to the new or more attractive activities, the result is a decline in the demand level, A, as stated by Lemma 2.
- This decline in the demand level induces firms to weakly decrease their levels of all activities. Therefore, at the firm level, the incentives to undertake higher levels of the activities following an introduction of new activities, or an increase in the attractiveness of existing activities, are opposed by the decrease in the demand level. Depending on the characteristics of a given firm, either of these two effects may dominate the other for a given activitity.

This is noteworthy for a number of reasons

- In their analysis of complementary activities, Milgrom and Roberts (1990, 1995) and Topkis (1995) assume that all variables that affect profits, but are outside the control of the firm, are exogeneous. Hence, they do not capture the feedback from general equilibrium.
- It is exactly the feedback from the endogenous demand level which implies that the decisions of individual firms do not exhibit monotone comparative statics in (β, S), in spite of the complementarities imposed by Assumption 2 and 3.
- Therefore, Milgrom and Roberts (1990, 1995) and Topkis (1995) cannot explain why a firm faced with new or better opportunities may ultimately undertake lower levels of activities that are complementary to these opportunities.

This is noteworthy for a number of reasons

- In their analysis of complementary activities, Milgrom and Roberts (1990, 1995) and Topkis (1995) assume that all variables that affect profits, but are outside the control of the firm, are exogeneous. Hence, they do not capture the feedback from general equilibrium.
- It is exactly the feedback from the endogenous demand level which implies that the decisions of individual firms do not exhibit monotone comparative statics in (β, S), in spite of the complementarities imposed by Assumption 2 and 3.
- Therefore, Milgrom and Roberts (1990, 1995) and Topkis (1995) cannot explain why a firm faced with new or better opportunities may ultimately undertake lower levels of activities that are complementary to these opportunities.
- Second, we may observe that all firms reduce their levels of a given activity even though the industry structure shifts weakly towards higher values of this particular activity. This may seem paradoxical at first glance.
- The explanation lies in endogenous exit. Even if the number of firms undertaking at least a given level of of a given activity declines, the share of active firms undertaking at least that level of the activity may increase if sufficiently many low productivity firms shut down.

This is noteworthy for a number of reasons (cont.)

- Third, our results emphasise that, following an introduction of new activities or increased attractiveness of existing activities, the firm-level complementarities assert themselves more clearly in the industry structure than at the firm level.
- This is not only theoretically interesting but is also important to take into account when testing models with firm-level complementarities that conform to our assumptions.
- Even though a given activity is complementary to e.g. exporting, one may observe a lot of firms scaling down on that activity following a trade liberalisation.

Summing up

- In a common heterogeneous-firms model, complementarities imply clear predictions for the industry structure.
- On the other hand, firm-level responses are ambiguous.
- Important for empirical purposes. Testing models that comply with our setup may best be done at the industry level.
- Our model is a flexible tool for modelling explanations of trends in the industry structure based on complementary activities.

Future research

- Given that monotone comparative statics at the industry level are desirable properties when you want to test of model, it makes sence to explore what are the necessary assumptions for monotone comparative statics.
- In other words, how much structure must we provide in Assumption 1?
- Our strategy is to take a partial approach. So, given Pareto-distributed productivities, what are needed when it comes to consumer preferences? Given the preferences, what are needed when it comes to productivities?