# Multi-plant firms and firms investing in productivity as limiting cases of multi-product firms

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# Consider once more a familiar situation...

- There is **one sector**
- Within this sector, a **continuum** of firms of measure $N$ operates
- Product is assumed to be horizontally differentiated across firms as well as within firms' product lines
- Each firm $j$ chooses:
  - its **continuous** product line of size $n_j$
  - its production plan $\mathbf{q}_j : [0, n_j] \to \mathbb{R}_+$
- Each variety is produced by a single firm
- Question of interest: comparative statics of product ranges and the number of firms w.r.t the market size

# Plan

# Consumers

- The economy is endowed with $L$ identical consumers, each of whom
  - inelastically supplies **one unit of labour**
  - maximizes her utility function

$$\mathscr{U} = \int_0^N U\left(\int_0^{n_j} u(x_{ij})\, di\right) dj$$

  - and faces the budget constraint

$$\int_0^N \int_0^{n_j} p_{ij} x_{ij}\, di\, dj \leq 1$$

# Utilities

- We call $U$ the **upper-tier utility function**, whereas $u$ is the **lower-tier utility function**
- The functions $u, \ U$ are assumed to be:
  - increasing
  - thrice differentiable
  - such that $\mathscr{U}$ is convex
- Also, $u$ is convex, whereas $U$ **can be concave**

# Product differentiation

The two-tier utility function accounts for **two levels of** product differentiation:

- the **inter-brand** differentiation;
- the **intra-brand** differentiation.

# Two-tier utility functions in the literature

- Nested CES: Alanson, Montagna (2005), Arkolakis, Muendler (2011), Shimomura, Thisse (2012);
- Nested logit: Anderson, de Palma (2006);
- Nested linear-quadratic utility: Eckel, Neary (2010);
- Not so many examples, actually...

# What does the approach yield?

- Allows to rigorously define cannibalization effect and to find exact conditions when it takes place;
- Allows to obtain models underlying different stories as limiting cases of the general model.

# Inverse demand functions

- Solving the consumer's problem yields inverse demand functions:

$$p_{ij} = \frac{u'(x_{ij})}{\lambda} U' \left( \int_0^{n_j} u(x_{kj})\, dk \right)$$

- $\lambda$ is the marginal utility of income
- Because there is a continuum of firms, the individual influence of each firm on $\lambda$ is **negligible**

# Useful notation

Elasticity of utility w.r.t. the individual consumption level:

$$\varepsilon_u(x) \equiv \frac{xu'(x)}{u(x)}$$

The **intra-brand relative love for variety** (the curvature of the lower-tier utility function):

$$r_u(x) \equiv -\frac{xu''(x)}{u'(x)}$$

The **inter-brand relative love for variety** (the curvature of the upper-tier utility function):

$$R_U(X) \equiv -\frac{X\,U''(X)}{U'(X)}.$$

# Cannibalization effect

Assume that firm $j$ charges the same price $p_j$ for all varieties it produces.

Inverse demands for all varieties supplied by firm $j$ then become:

$$p_j = \frac{U'\left(n_j u(x_j)\right)}{\lambda} u'(x_j).$$

**Definition**. We say that **weak cannibalization effect** takes place if

$$\frac{\partial x_j}{\partial n_j} < 0.$$

**Intuition**

If a firm expands the product range holding all prices fixed and the same, then, other things equal, sales of each incumbent variety fall.

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# When does WCE occur?

**Proposition 1**. *If the upper-tier utility $U$ is concave, per-variety cannibalization effect always takes place*.

**Proof**. Direct calculation yields:

$$\frac{\partial x_j}{\partial n_j}\frac{n_j}{x_j} = \frac{-R_U}{r_u + R_U\,\varepsilon_u}$$

**Remark**. Further on, we will see, that in equilibrium $r_u + \varepsilon_u R_U > 0$ should always hold. Thus, if we consider only firms' behavior in the neighbourhood of equilibrium, Proposition 1 yields necessary and sufficient condition for cannibalization.

# Producers

- Each firm incurs:
  - a fixed cost $F$
  - a variable cost $V(\mathbf{q}, n)$
- The variable cost functions $V$ is convex in $\mathbf{q}$ and satisfies the **symmetry** condition:

$$V(\mathbf{q}_1, n) = V(\mathbf{q}_2, n) \quad \forall n,$$

where $\mathbf{q}_2$ can be obtained from $\mathbf{q}_1$ by a renumbering of varieties.

# Profit maximization

Because of symmetry, we can pose the firm's problem as follows:

$$\max \pi(y, n) = \frac{1}{\lambda} u' \left( \frac{y}{nL} \right) U' \left( n\, u(x) \right) y - F - v\left( y, n \right),$$

where

- $y = \int_0^n q_i di$ is firm's total output
- $v$ is the symmetrized cost function:

$$v(y, n) = V(\mathbf{q}, n)|_{\mathbf{q} \equiv y/n}$$

We assume $v$ to be increasing, twice continously differentiable and convex

# Two examples of cost functions

- Additively separable production costs + product line-specific fixed costs:

$$V(\mathbf{q}, n) \equiv \int_0^n v(q_i)di + \phi n$$

- Constant MPC, decreasing with respect to the scope + + product line-specific fixed costs:

$$V(\mathbf{q}, n) = c(n) \int_0^n q_i di + \phi n$$

# Equilibrium conditions

Pricing:

$$p = \frac{v_y}{1-(r_u + R_U \varepsilon_u)} \quad (\Rightarrow r_u + R_U \varepsilon_u > 0 \text{ in equilibrium})$$

Free entry:

$$py = F + v(y, n)$$

Labour balance:

$$L = N\left(F + v(y, n)\right)$$

The "unit elasticity" condition (follows from zero profit and producer's FOC):

$$\frac{v_y\, y}{F + v(y, n)} + \frac{v_n\, n}{F + v(y, n)} = 1 - R_U$$

# Reduced equilibrium conditions

The system of equilibrium conditions can be reduced to the following system of two equations in terms of total output $y$ and the scope $n$:

$$\frac{v_y\, y}{F + v(y, n)} + \frac{v_n\, n}{F + v(y, n)} = 1 - R_U$$

$$\frac{y v_y}{n v_n} = \frac{1 - (r_u + R_U \varepsilon_u)}{r_u - R_U \left(1 - \varepsilon_u\right)}$$

Once $y$ and $n$ are found, the equilibrium values of price $p$ and the mass of firms $N$ are uniquely determined from free entry and labour balance.

A problem with comparative statics w.r.t. the market size $L$: an increase in $L$ now shifts **both curves**.

# No brand effects

- If $U(X) \equiv X$, which means no inter-brand differentiation (Kokovin, Ushchev, Zhelobodko, 2012), then utility function becomes additively separable across varieties:

$$\mathscr{U} = \int_0^N \int_0^{n_j} u(x_{ij})\, di\, dj$$

- In this case:
  - no cannibalization effect
  - full characterization of comparative statics w.r.t. the market size $L$

# No intra-brand differentiation

- If $u(x) \equiv x$, which means that varieties supplied by the same firm are perfect substitutes, then

$$\mathscr{U} \equiv \int_0^N U(X_j)\, dj,$$

where $X_j \equiv \int_0^{n_j} x_{ij} di$ is total consumption of firm $j$'s products

- Thus, firms are virtually no longer multi-product!
- However, are there useful interpretations for this case?

# Interpretation 1: investments in productivity

Assume that variable costs are of the form

$$v(y, n) = c(n)y + \phi n,$$

where $c'(n) < 0$.

Then the model is **formally equivalent** to the one proposed in (Bykadorov, Kokovin, Zhelobodko, 2012)

**How to use it?**

If $u$ is almost linear, then comparative statics w.r.t. $L$ should be **almost the same** as when $u(x) \equiv x$. However, due to the formal equivalence of two models, **we know** comparative statics w.r.t. the market size $L$ for the limiting case.

# Interpretation 2: multi-plant firms

Assume now that variable costs are of the form

$$V(\mathbf{q}, n) \equiv \int_0^n v(q_i)di + \phi n$$

This case can be treated as the case of **multi-plant** firms, where:

- $n$ is the number of plants
- $v$ are variable production costs of a separate plant
- $\phi$ is the cost of building a new plant

# Multi-plant producer's problem

- Symmetrized variable costs are

$$v(y, n) = n v\left(\frac{y}{n}\right) + \phi n$$

- Then the producer's problem is:

$$\max_{n,y} \pi(n, y) \equiv y \frac{U'(y/L)}{\lambda} - n v\left(\frac{y}{n}\right) - \phi n - F$$

- **Note**: total revenue does not depend on $n$, $\Rightarrow$the optimal number of plants $n^*(y)$ under a given output $y$ is exactly the one solving

$$\min_{n} \left[ n v\left(\frac{y}{n}\right) + \phi n \right]$$

# Optimal plant size

**Proposition 2**. *The optimal plant size* $q^*(y) \equiv y/n^*(y)$ *is independent on* $y$.

**Proof**. The FOC for the VC minimization sub-problem is:

$$\phi = qv'(q) - v(q)$$

Thus, $q^*$ is a (unique) solution of an equation which does not contain $y$.
QED

Corollary. The optimal number of plants is given by

$$n^*(y) = \frac{1}{q^*}y$$

# Producer's second step problem

After $n^*(y)$ is chosen optimally, the producer seeks to

$$\max_y \pi^*(y) = y\,\frac{U'(y/L)}{\lambda} - (F + cy)$$

where $\pi^*(y)$ stands for the profit already optimized w.r.t. $n$, and where $c \equiv [v(q^*) + \phi]/q^*$.

Thus, the model is **formally equivalent** to the one in (ZKPT, 2012), for which we know comparative statics.

# Applications

- Offshoring?
- Export vs FDI dilemma?
- Any other suggestions?

# Thank you for your attention!