The role of market size under monopolistic competition of multi-product firms: scale-scope economies versus elasticity of substitution

S.Kokovin, Ph.Ushchev, E.Zhelobodko

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Stylized facts about multi-product firms

- Multi-product firms account for the largest part of manufacturing output
- Intensive margins and extensive margins of firms are positively correlated:
  - Bernard, A.B., S.J.Redding and P.K.Schott (2010)
  - Goldberg, P., A. Khandewal, N. Pavnik and P. Topalova (2008)
  - Manova, K., and Z. Zhang (2011)
- There is positive correlation between firms' size and the efficiency of their R&D projects:
  - Henderson, R. and I. Cockburn (1996)
  - Cockburn, I. and R. Henderson (2001)

Theoretical literature on multi-product firms

- Ottaviano G.I.P. and J.F. Thisse (1999)
- Allanson, P. and C. Montagna (2005)
- Johnson, J.P. and D.P. Myatt (2003, 2006)
- Anderson, S.P. and A. de Palma (2006)
- Feenstra, R. and H. Ma (2007)
- Eckel, C. and J.P. Neary (2010)
- Nocke, V. and S. Yeaple (2006, 2008, 2012)

Questions we want to answer

- Do larger markets lead to more product diversity at the firm- and market-levels?
- How the market outcome depends on the interaction between the supply and demand sides?
- Are wider product ranges always associated with lower output (cannibalization effect)?

Why monopolistic competition

Oligopoly models:

- are difficult to handle
- neglect income effect

We propose a **more flexible** model of monopolistic competition that **mimics** oligopolistic competition.

Plan

1. Layout of the model
2. Equilibrium conditions
3. Comparative statics with respect to the market size
4. Special cases

Commodities and market structure

- There is only **one sector**
- Within this sector, a **continuum** of firms of measure N operates
- Product is assumed to be horizontally differentiated across firms as well as within firms' product lines
- Each firm j chooses:
    - its **continuous** product line of size n_j
    - its production plan $\mathbf{q}_j : [0, n_j] \to \mathbb{R}_+$
- Each variety is produced by a single firm

Consumers

- The economy is endowed by L identical consumers, each of whom
  - inelastically supplies **one unit of labour**
  - seeks to maximize her utility function

$$\mathscr{U} = \int_0^N \int_0^{n_j} u(x_{ij})\, di\, dj$$

  - faces the budget constraint

$$\int_0^N \int_0^{n_j} p_{ij} x_{ij}\, di\, dj \leq 1$$

- The function u is assumed to be increasing, concave and thrice differentiable

Inverse demand functions

- Solving the consumer's problem yields inverse demand functions:

$$p_{ij} = \frac{u'(x_{ij})}{\lambda}$$

- λ is the marginal utility of income which is interpreted as a market aggregate
- Because there is a continuum of firms, the individual influence of each firm on λ is **negligible**

Producers

- Each firm incurs:
  - a fixed cost F
  - a variable cost $V(\mathbf{q}, n)$
- The variable cost functions V is convex in $\mathbf{q}$ and satisfies the **symmetry** condition:

$$V(\mathbf{q}_1, n) = V(\mathbf{q}_2, n) \quad \forall n,$$

where $\mathbf{q}_2$ can be obtained from $\mathbf{q}_1$ by a renumbering of varieties.

Profit maximization

- Firms maximize profits

$$\Pi(\mathbf{q}, n) = \frac{1}{\lambda} \int_0^n u'(q_i/L)\, q_i di - F - V(\mathbf{q}, n)$$

- Because of symmetry, we can reformulate the firm's problem:

$$\max \pi(y, n) = \frac{1}{\lambda} u' \left(\frac{y}{nL} \right) y - F - v(y, n),$$

where $y = \int_0^n q_i di$ is firm's total output, v is the symmetrized cost function:

$$v(y, n) = V(\mathbf{q}, n)|_{\mathbf{q} \equiv y/n}$$

- We assume v to be increasing, twice continously differentiable and convex

Examples

Allanson and Montagna (2005):

$$v(y,n) = \phi n$$

Eckel and Neary (2010):

$$v(y,n) = cy + \phi n$$

Nocke and Yeaple (2006, 2008, 2012):

$$v\left(y,\, n\right) = \phi n + c(n)y,$$

where ϕ stands for fixed costs per product line, whereas $c(n)$ stands for marginal production costs, which are the same for all varieties.

Scale-scope spillovers

- Empirical work finds positive correlation between the firm's size and the efficiency of its R&D projects
- So, we assume that $v(y, n)$ exhibits positive scale-scope spillovers:

$$v_{yn} < 0$$

In words, marginal production cost decrease with respect to scope, or, equivalently, marginal scope cost decrease with respect to total output y

- Denote by v_y the *marginal production cost* (*y-marginal costs*) and by v_n the *marginal scope cost* (*n-marginal costs*)

Equilibrium

An **equilibrium** is given by

$$\left(\{n_j^*\}_{j\in[0,N]}, \{\mathbf{q}_j^*\}_{j\in[0,N]}, \{\mathbf{p}_j^*\}_{j\in[0,N]}, N^*, \lambda^*\right)$$

such that:

- $x_{ij}^* = q_{ij}^*/L$ maximizes consumer's utility under prices $p_{ij} = p_{ij}^*$;
- λ^* is the Lagrange multiplier in the consumer's problem;
- n_j^* and $\mathbf{p}_j^*$ maximize profit of firm j conditional on $\lambda = \lambda^*$ and the inverse demand functions;
- free entry condition and labour balance hold.

Symmetric equilibrium

An equilibrium is **symmetric** if:

- the equilibrium prices are the same, both across firms and varieties;
- the equilibrium quantities are the same, both across firms and varieties;
- the equilidrium scopes are the same across firms.

Relative love for variety

The key-factor for the market outcome is the **relative love for variety** given by

$$r_u \equiv -\frac{xu''}{u'}$$

Alternative interpretations of r_u are:

- inverse demand elasticity
- the reciprocal of the elasticity of substitution (under symmetric consumption pattern)

Equilibrium conditions

Pricing:

$$p = \frac{v_y}{1 - r_u}.$$

Free entry:

$$py = F + v(y, n).$$

Labour balance:

$$L = N\left(F + v(y, n)\right).$$

The "unit elasticity" condition (follows from zero profit and producer's FOC):

$$\frac{v_y\, y}{F + v(y, n)} + \frac{v_n\, n}{F + v(y, n)} = 1.$$

Reduced equilibrium conditions

Proposition 1. *The system of equilibrium conditions can be reduced to the following system of two equations in terms of total output* y *and the scope* n*:*

$$\frac{v_y\, y}{F + v(y, n)} + \frac{v_n\, n}{F + v(y, n)} = 1,$$

$$\frac{v_n\, n}{F + v(y, n)} = r_u \left(\frac{y}{nL}\right).$$

Once y *and* n *are found, the equilibrium values of price* p *and the mass of firms* N *are uniquely determined from free entry and labour balance.*

Elasticities of marginal costs

- The key-factor for the market outcome is the elasticity of the inverse demand r_u
- By analogy, we consider elasticities of marginal costs
- We have **four** marginal costs elasticities:
  - the y-elasticity of y-marginal costs $y\,\dfrac{v_{yy}}{v_y}$
  - the n-elasticity of y-marginal costs $n\,\dfrac{v_{yn}}{v_y}$
  - the y-elasticity of n-marginal costs $y\,\dfrac{v_{ny}}{v_n}$
  - the n-elasticity of n-marginal costs $n\,\dfrac{v_{nn}}{v_n}$

Market price p^* and product diversity $n^* N^*$

Proposition 2.

	$r'_u > 0$	CES	$r'_u < 0$
$\mathcal{E}_{q/L}$	$0 < \mathcal{E}_{q/L} < 1$	$\mathcal{E}_{q/L} = 0$	$\mathcal{E}_{q/L} < 0$
$\mathcal{E}_{p/L}$	$-1 < \mathcal{E}_{p/L} < 0$	$\mathcal{E}_{p/L} = 0$	$\mathcal{E}_{p/L} > 0$
$\mathcal{E}_{nN/L}$	$0 < \mathcal{E}_{nN/L} < 1$	$\mathcal{E}_{nN/L} = 1$	$\mathcal{E}_{nN/L} > 1$

Firm's scope n^*

Proposition 4.

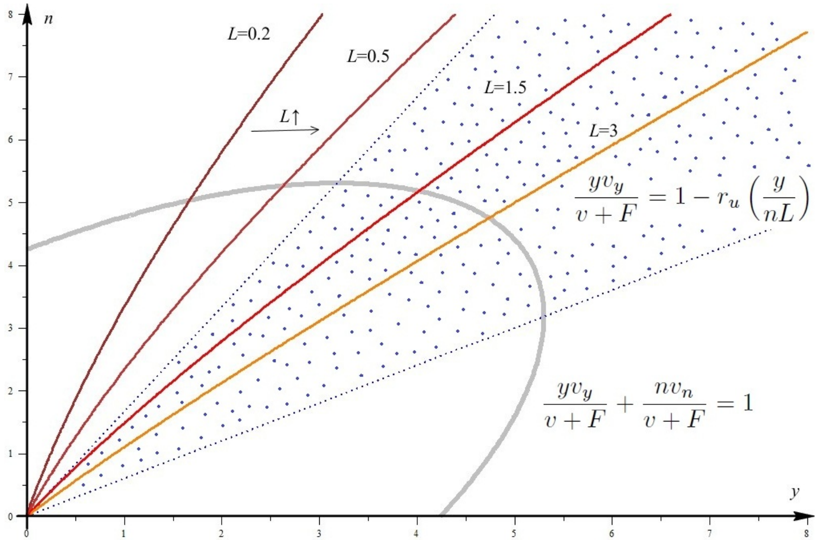
Cost	RLV		
	$r_u' > 0$	CES	$r_u' < 0$
$y\dfrac{v_{yy}}{v_y} > -n\dfrac{v_{yn}}{v_y}$	$\mathcal{E}_{n/L} < 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} > 0$
$y\dfrac{v_{yy}}{v_y} = -n\dfrac{v_{yn}}{v_y}$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} = 0$
$y\dfrac{v_{yy}}{v_y} < -n\dfrac{v_{yn}}{v_y}$	$\mathcal{E}_{n/L} > 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} < 0$

Is there always cannibalization?

$$v(y,n) = \frac{y^2}{2} + \frac{n^2}{2} - \gamma yn + \alpha y + \beta n$$

- Here $\alpha, \beta \geq 0,\ 0 < \gamma < 1$
- Take $\alpha = 15.89,\ \beta = 4,\ \gamma = 0.6,\ F = 9$

Cannibalization or no cannibalization?



Firm's total output y^*

Proposition 3.

Cost	RLV		
	$r_u' > 0$	CES	$r_u' < 0$
$n\frac{v_{nn}}{v_n} > -y\frac{v_{ny}}{v_n}$	$\mathcal{E}_{y/L} > 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} < 0$
$n\frac{v_{nn}}{v_n} = -y\frac{v_{ny}}{v_n}$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} = 0$
$n\frac{v_{nn}}{v_n} < -y\frac{v_{ny}}{v_n}$	$\mathcal{E}_{y/L} < 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} > 0$

The mass of firms N^*

Proposition 5.

Cost	RLV		
	$r'_u > 0$	CES	$r'_u < 0$
$y\frac{v_{yy}}{v_y} + n\frac{v_{yn}}{v_y} > n\frac{v_{nn}}{v_n} + y\frac{v_{ny}}{v_n}$	$\mathcal{E}_{N/L} > 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} < 1$
$y\frac{v_{yy}}{v_y} + n\frac{v_{yn}}{v_y} = n\frac{v_{nn}}{v_n} + y\frac{v_{ny}}{v_n}$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$
$y\frac{v_{yy}}{v_y} + n\frac{v_{yn}}{v_y} < n\frac{v_{nn}}{v_n} + y\frac{v_{ny}}{v_n}$	$\mathcal{E}_{N/L} < 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} > 1$

Case 1: no scale-scope spillover

Each firm incurs:

- a fixed cost F
- an R&D cost (monitoring cost) $S(n)$
- a variable production cost $v(y)$

Total cost is $F + V(y) + S(y)$.

In this case: **no 3s**.

Firm-variables

Proposition 7.

	$r'_u > 0$	CES	$r'_u < 0$
$\mathscr{E}_{p/L}$	$-1 < \mathscr{E}_{p/L} < 0$	$\mathscr{E}_{p/L} = 0$	$\mathscr{E}_{p/L} > 0$
$\mathscr{E}_{y/L}$	$0 < \mathscr{E}_{y/L} < 1$	$\mathscr{E}_{y/L} = 0$	$\mathscr{E}_{y/L} < 0$
$\mathscr{E}_{q/L}$	$0 < \mathscr{E}_{q/L} < 1$	$\mathscr{E}_{q/L} = 0$	$\mathscr{E}_{q/L} < 0$
$\mathscr{E}_{n/L}$	$-1 < \mathscr{E}_{n/L} < 0$	$\mathscr{E}_{n/L} = 0$	$\mathscr{E}_{n/L} > 0$

Negative spillovers

Proposition 6. *If there is no positive spillover, there is always cannibalization.*

Industry-variables

Proposition 8.

	$r'_u > 0$	CES	$r'_u < 0$
$\mathscr{E}_{yN/L}$	$\mathscr{E}_{yN/L} > 1$	$\mathscr{E}_{yN/L} = 1$	$\mathscr{E}_{yN/L} < 1$
$\mathscr{E}_{nN/L}$	$0 < \mathscr{E}_{nN} < 1$	$\mathscr{E}_{nN} = 1$	$\mathscr{E}_{nN} > 1$

Mass of firms N^*

Proposition 9.

Cost	RLV		
	$r'_u > 0$	CES	$r'_u < 0$
$y\dfrac{v''(y)}{v'(y)} < n\dfrac{S''(n)}{S'(n)}$	$0 < \mathscr{E}_{N/L} < 1$	$\mathscr{E}_{N/L} = 1$	$\mathscr{E}_{N/L} > 1$
$y\dfrac{v''(y)}{v'(y)} = n\dfrac{S''(n)}{S'(n)}$	$\mathscr{E}_{N/L} = 1$	$\mathscr{E}_{N/L} = 1$	$\mathscr{E}_{N/L} = 1$
$y\dfrac{v''(y)}{v'(y)} > n\dfrac{S''(n)}{S'(n)}$	$\mathscr{E}_{N/L} > 1$	$\mathscr{E}_{N/L} = 1$	$\mathscr{E}_{N/L} < 1$

Case 2: per-variety additive costs

Consider *per-variety additive* variable costs:

$$V(\mathbf{q}, n) = \int_0^n v(q_i)di + S(n).$$

where v is variable costs of a separate plant, S is monitoring costs; both are increasing and convex.

Scope and market size

Proposition 10.

	$r_u' > 0$	$r_u' = 0$	$r_u' < 0$
$\mathscr{E}_{n/L}$	$\mathscr{E}_{n/L} = 0$	$\mathscr{E}_{n/L} = 0$	$\mathscr{E}_{n/L} = 0$
$\mathscr{E}_{y/L}$	$0 < \mathscr{E}_{y/L} < 1$	$\mathscr{E}_{y/L} = 0$	$\mathscr{E}_{y/L} > 0$
$\mathscr{E}_{N/L}$	$0 < \mathscr{E}_{N/L} < 1$	$\mathscr{E}_{N/L} = 1$	$\mathscr{E}_{N/L} > 1$

Scope is independent of market size.

Future work

- Heterogeneous firms
- Open economy
- Firms' endogenous choice between being single- or multi-product

Thank you for your attention!