Oligopolistic competition with varying competitive aggressiveness

Rodolphe Dos Santos Ferreira

HSE at St. Petersburg
September 2012

- d’Aspremont C., Dos Santos Ferreira R., Gérard-Varet L.-A. (2007), "Competition for market share or for market size: Oligopolistic equilibria with varying competitive toughness", *International Economic Review* **48**, 761-784.
- d’Aspremont C., Dos Santos Ferreira R. (2009), "Price-quantity competition with varying toughness", *Games and Economic Behavior* 65, 62-82.
- d’Aspremont C., Dos Santos Ferreira R. (2010), "Oligopolistic competition as a common agency game", *Games and Economic Behavior* 70, 21-33.
- d’Aspremont, Dos Santos Ferreira R., Thépot J. (2012), "Hawks and doves in segmented markets: A formal approach to competitive aggressiveness“, CORE D.P. 2007/78, revised.

1. Introduction

Motivation

- Dissatisfaction with the standard dichotomy between the so-called Cournot and Bertrand regimes of oligopolistic competition.
  - It does not fit either Cournot's or Bertrand's texts.
  - It reduces the characterization of the two main regimes of competition to the alternative choice of one specific strategic variable. Why should the players renounce to the use of **two** strategic variables?
  - It leaves empty the space between Cournot and Bertrand. This space has been filled up by the *conjectural variations* approach, which is however unsatisfactory from the point of view of game theory.

Motivation

- Dissatisfaction with the standard dichotomy between the so-called Cournot and Bertrand regimes of oligopolistic competition.
  - It does not fit either Cournot's or Bertrand's texts.
  - It reduces the characterization of the two main regimes of competition to the alternative choice of one specific strategic variable. Why should the players renounce to the use of **two** strategic variables?
  - It leaves empty the space between Cournot and Bertrand. This space has been filled up by the *conjectural variations* approach, which is however unsatisfactory from the point of view of game theory.

Motivation

- Dissatisfaction with the standard dichotomy between the so-called Cournot and Bertrand regimes of oligopolistic competition.
  - It does not fit either Cournot's or Bertrand's texts.
  - It reduces the characterization of the two main regimes of competition to the alternative choice of one specific strategic variable. Why should the players renounce to the use of two strategic variables?
  - It leaves empty the space between Cournot and Bertrand. This space has been filled up by the *conjectural variations* approach, which is however unsatisfactory from the point of view of game theory.

Motivation

- Dissatisfaction with the standard dichotomy between the so-called Cournot and Bertrand regimes of oligopolistic competition.
  - It does not fit either Cournot's or Bertrand's texts.
  - It reduces the characterization of the two main regimes of competition to the alternative choice of one specific strategic variable. Why should the players renounce to the use of two strategic variables?
  - It leaves empty the space between Cournot and Bertrand. This space has been filled up by the *conjectural variations* approach, which is however unsatisfactory from the point of view of game theory.

Objective

- Our objective: to find a convincing characterization and a convenient parameterization of a whole spectrum of regimes of oligopolistic competition between Cournot and Bertrand or, more generally, between tacit collusion and perfect competition.
- Also, to keep the approach simple enough to make it suitable to be inserted in a general equilibrium model as an alternative to the usual monopolistic competition approach.
- We further want to keep the approach static so as not to interfere with the dynamics of macroeconomic interactions.

Objective

- Our objective: to find a convincing characterization and a convenient parameterization of a whole spectrum of regimes of oligopolistic competition between Cournot and Bertrand or, more generally, between tacit collusion and perfect competition.
- Also, to keep the approach simple enough to make it suitable to be inserted in a general equilibrium model as an alternative to the usual monopolistic competition approach.
- We further want to keep the approach static so as not to interfere with the dynamics of macroeconomic interactions.

Objective

- Our objective: to find a convincing characterization and a convenient parameterization of a whole spectrum of regimes of oligopolistic competition between Cournot and Bertrand or, more generally, between tacit collusion and perfect competition.
- Also, to keep the approach simple enough to make it suitable to be inserted in a general equilibrium model as an alternative to the usual monopolistic competition approach.
- We further want to keep the approach static so as not to interfere with the dynamics of macroeconomic interactions.

Clue from the empirical side

- The *New Empirical Industrial Organization* (*NEIO*) uses a generalization of Cournot's equilibrium condition:

$$\frac{P - C'_i(q_i)}{P} = \theta_i \frac{q_i / \sum_j q_j}{-D'(P)P / D(P)}$$

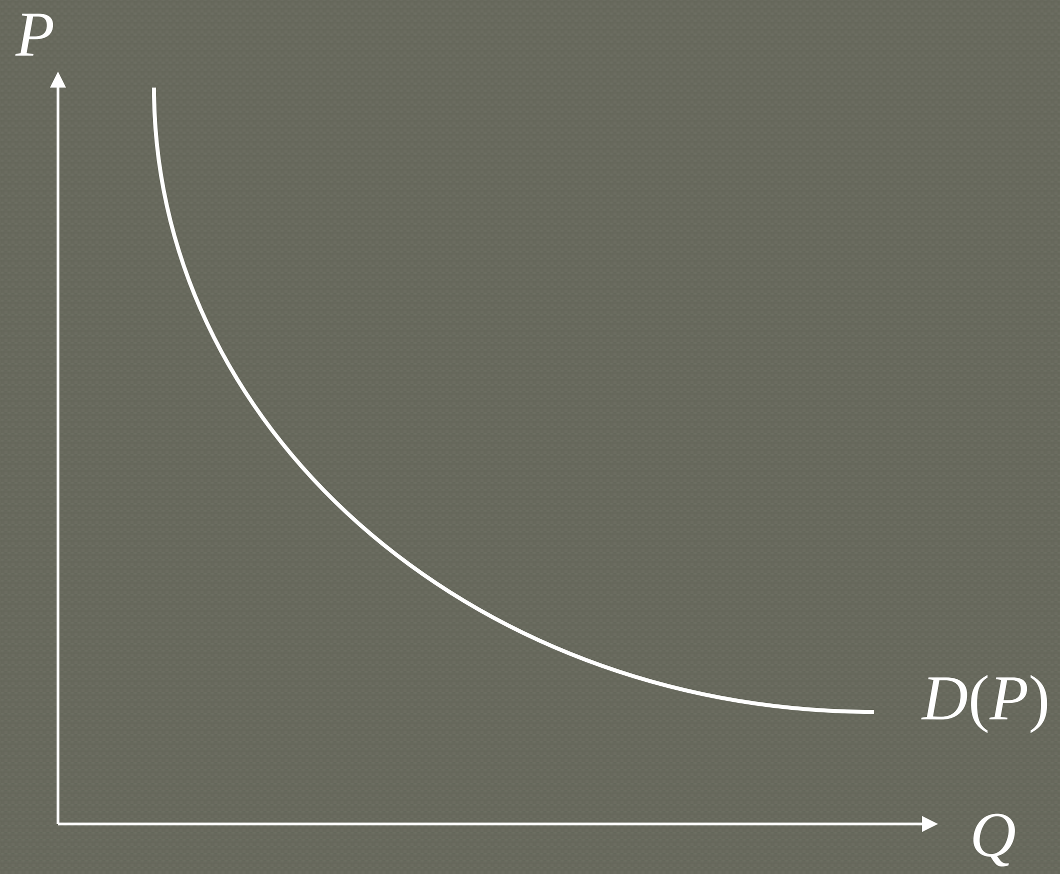
degree of monopoly =
conduct parameter × market share / demand elasticity

$\theta_i = 1$: Cournot
$\theta_i = 0$: perfect competition

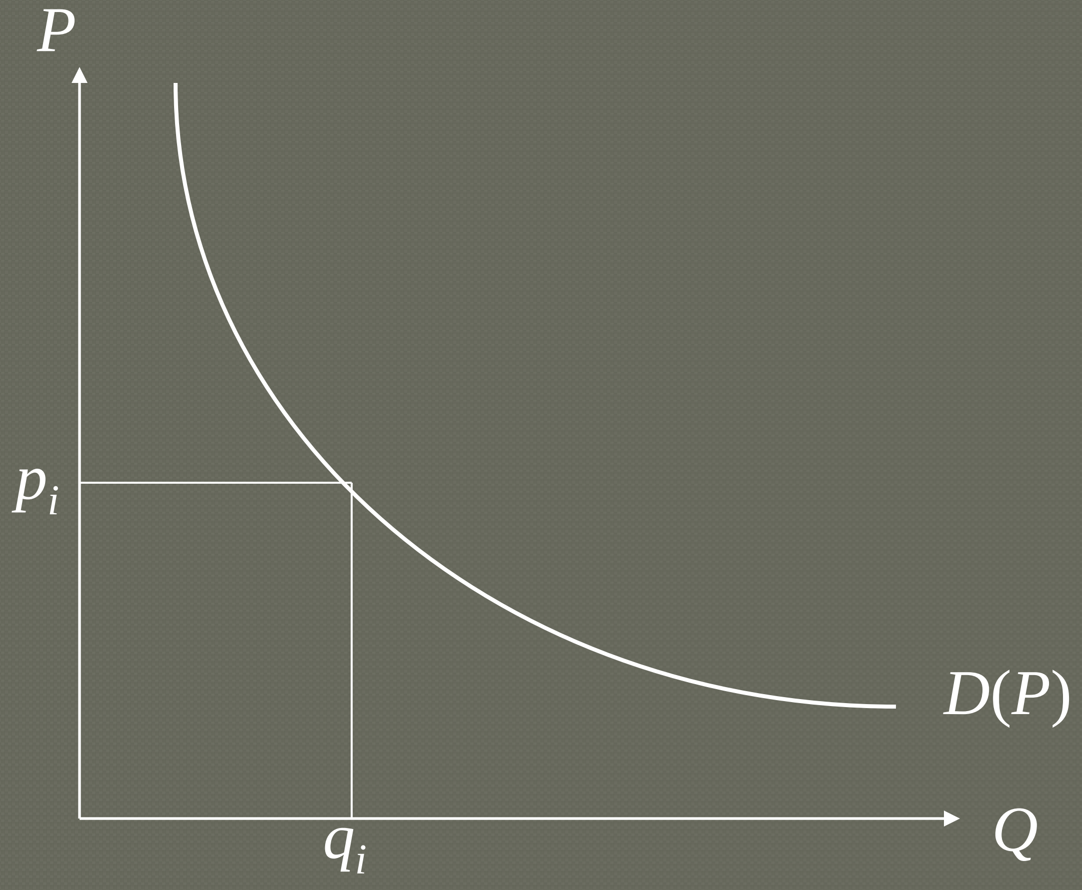
- Sole theoretical foundation: *conjectural variations*

2. The homogeneous oligopoly

Cournot monopoly

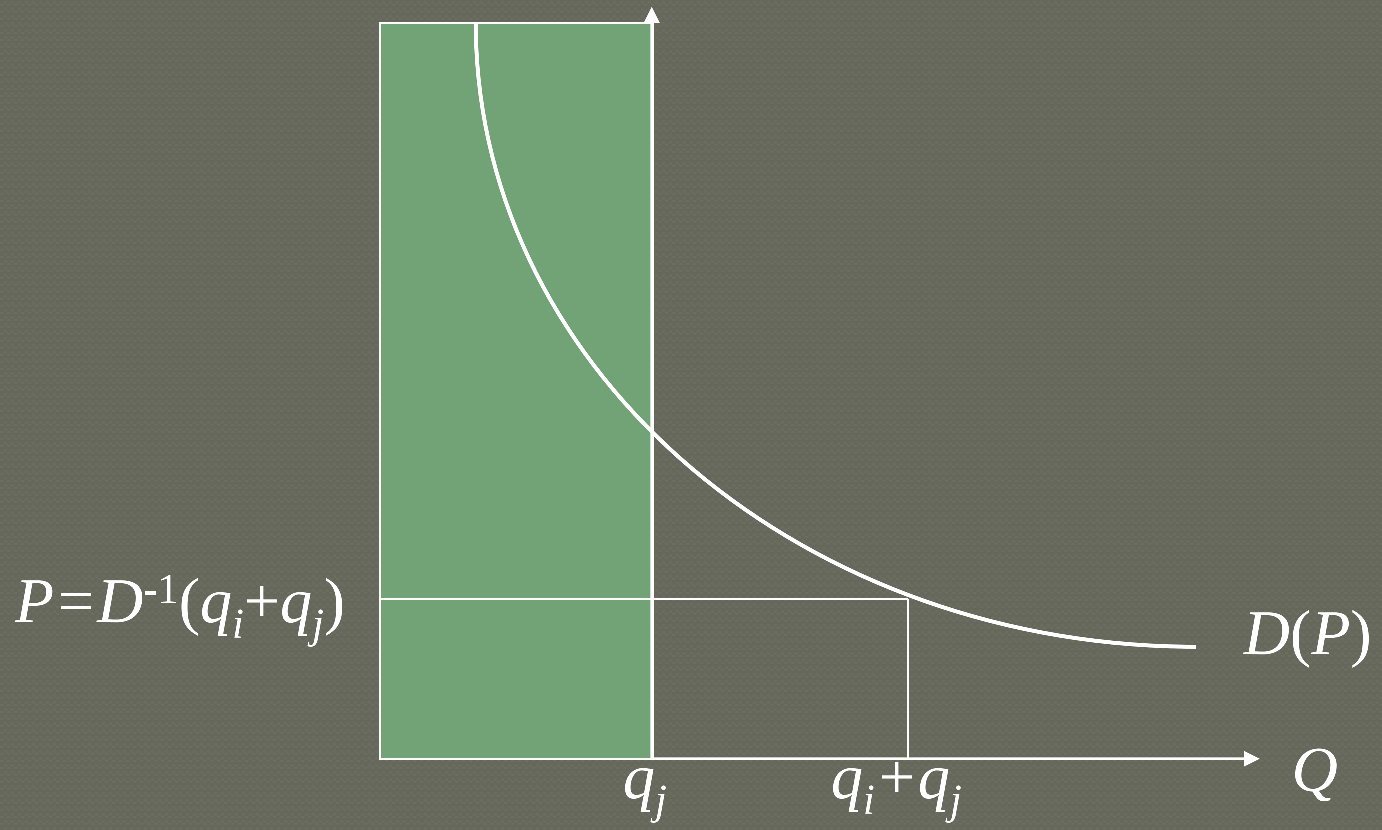


Cournot monopoly



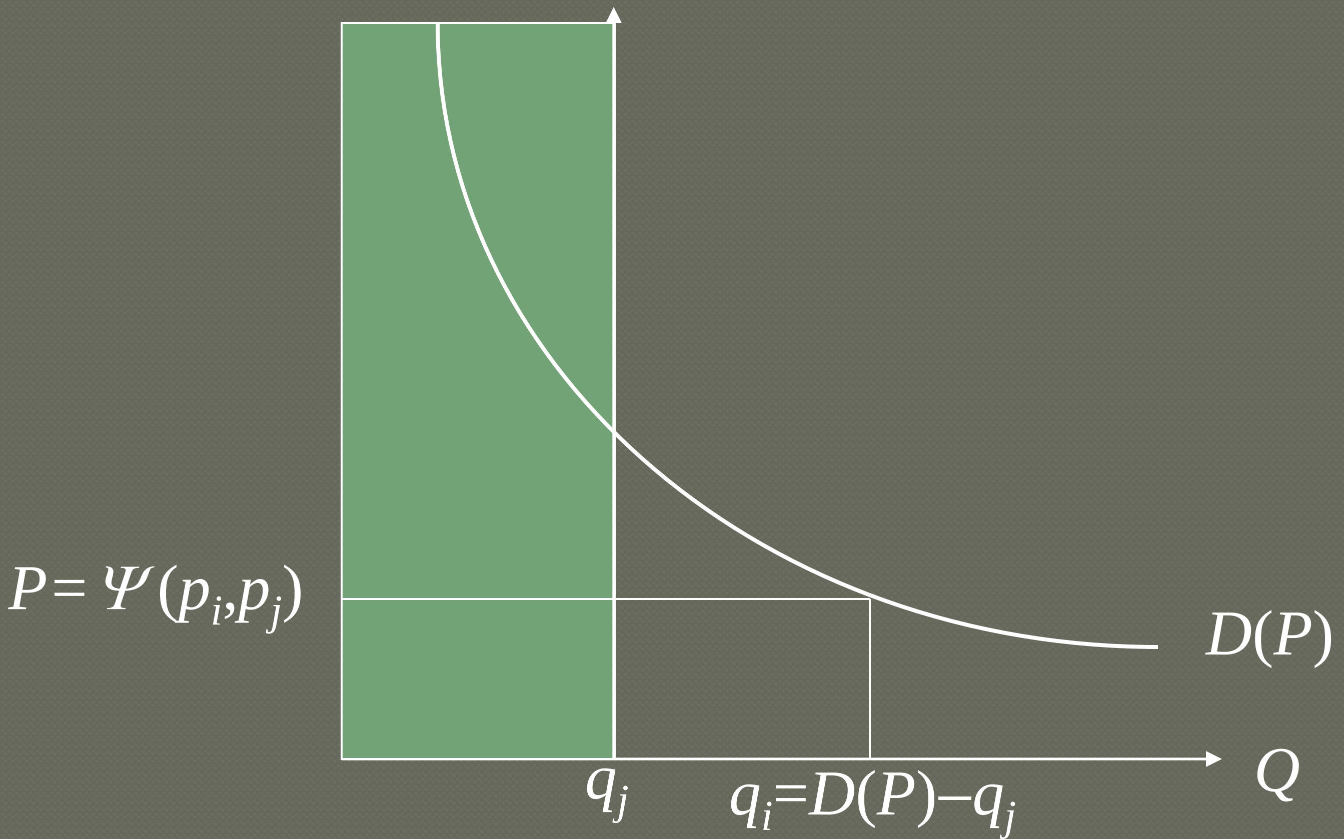
Cournot duopoly

Firms set quantities anticipating the inverse demand



Cournot duopoly

Firms set prices and quantities anticipating the residual demand

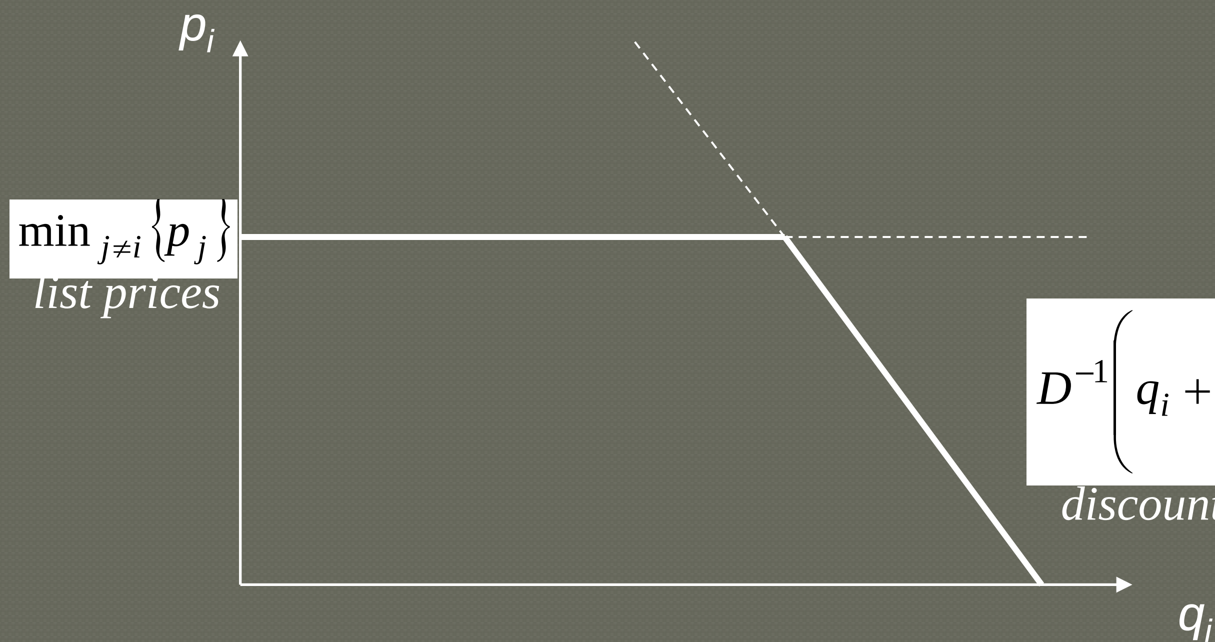


Cournot-Bertrand competition

- Each firm $i=1,\ldots,n$ chooses a pair (p_i,q_i) where p_i is a *list price* at which firm i commits to serve the whole demand [Bertrand] and q_i is the quantity to produce in advance (and hence to sell at any positive *discount price*) [Cournot].

Cournot-Bertrand competition

- Each firm i=1,…,n chooses a pair (p_i,q_i) where p_i is a *list price* at which firm i commits to serve the whole demand [Bertrand] and q_i is the quantity to produce in advance (and hence to sell at any positive *discount price*) [Cournot].



Cournot-Bertrand competition

- If there is excess demand at the lowest list price, the excess demand is allocated to firms having set this price, according to an *ex ante* sharing rule: $(s_1(p,q),\ldots,s_n(p,q))$, s.t.

$$
\begin{aligned}
s_i(p,q) &> 0 \text{ if } i \in \arg\min_j \{p_j\} \text{ and } D\left(\min_j \{p_j\}\right) > \sum_j q_j, \\
s_i(p,q) &= 0, \text{ otherwise}, \\
\text{and } \sum_i s_i(p,q) &= D\left(\min_j \{p_j\}\right) - \sum_j q_j.
\end{aligned}
$$

Cournot-Bertrand competition

- We thus obtain, given the cost functions C_i, a *Cournot-Bertrand game* in prices and quantities, with payoff functions (for i=1,…,n):

$$\Pi_i^{CB}\left(p_i, p_{-i}, q_i, q_{-i}\right) \equiv$$
$$\min\left\{p_i, p_{-i}, D^{-1}\left(q_i + \sum_{j\neq i} q_j\right)\right\}\left[q_i + s_i\left(p_i, p_{-i}, q_i, q_{-i}\right)\right]$$
$$-C_i\left(q_i + s_i\left(p_i, p_{-i}, q_i, q_{-i}\right)\right).$$

Oligopolistic equilibrium

- An oligopolistic equilibrium is a Nash equilibrium (p^*,q^*) of the Cournot-Bertrand game which satisfies in addition the *credibility condition*: $s_i(p^*,q^*) = 0$ for any i (no firm should be obliged to sell at equilibrium more than it would spontaneously wish to do):

$$\sum_j q_j^* = D\left(\min_j \{p_j^*\}\right)$$

Canonical characterisation of oligopolistic equilibrium

- The pair (p^*, q^*) is an oligopolistic equilibrium iff (p_i^*, q_i^*) solves, for any i,

$$\max_{(p_i,q_i)\in\Re_+^2} p_i q_i - C_i(q_i)$$

$$\text{s.t. } (1)\ \ p_i \le \min_{j\neq i}\{p_j^*\} \text{ and } (2)\ \ p_i \le D^{-1}\left(q_i + \sum_{j\neq i} q_j^*\right)$$

and satisfies in addition the credibility condition.

- The constraint (1) is imposed by the competition with insiders (to the industry) and concerns the market share, the constraint (2) is imposed by the competition with outsiders and concerns the market size.

Parameterization of the set of equilibria

- There is a continuum of oligopolistic equilibria, in particular those which lead to the Cournot, Bertrand (for linear cost functions), and perfect competition outcomes (but not to the collusive solution).

Parameterization of the set of equilibria

- There is a continuum of oligopolistic equilibria, in particular those which lead to the Cournot, Bertrand (for linear cost functions), and perfect competition outcomes (but not to the collusive solution).

- λ_i : Lagrange multiplier associated with constraint (1) on market share
  v_i : Lagrange multiplier associated with constraint (2) on market size
  $\theta_i = \lambda_i / (\lambda_i + v_i)$: index of *competitive toughness* (or *aggressiveness*)

- Lerner index of the degree of monopoly

Parameterization of the set of equilibria

- There is a continuum of oligopolistic equilibria, in particular those which lead to the Cournot, Bertrand (for linear cost functions), and perfect competition outcomes (but not to the collusive solution).

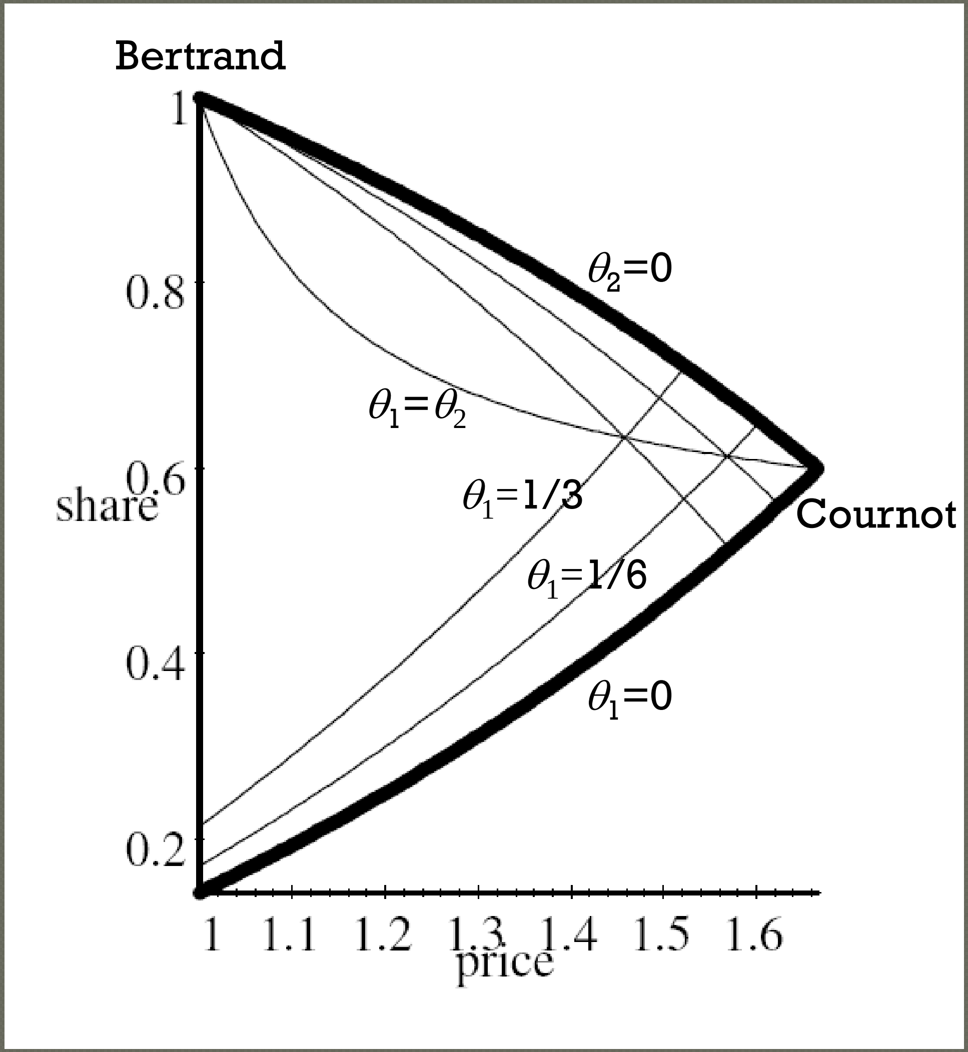
- λ_i : Lagrange multiplier associated with constraint (1) on market share
  v_i : Lagrange multiplier associated with constraint (2) on market size
  $\theta_i = \lambda_i / (\lambda_i + v_i)$: index of *competitive toughness* (or *aggressiveness*)

- Lerner index of the degree of monopoly:

$$\frac{p_i^* - C_i'(q_i^*)}{p_i^*} = (1-\theta_i)\frac{q_i^* / \sum_j q_j^*}{-\epsilon D\left(\min_j \{p_j^*\}\right)} \equiv \mu_i\left(\theta_i, \min_j \{p_j^*\}, q^*\right)$$

  $1 - \theta_i$: conduct parameter of the *NEIO*

Asymmetric duopoly: market share of the technological leader



Linear demand and costs

Equivalent models

- Our canonical characterisation may be applied to other equivalent equilibrium concepts (leading to the same sets of outcomes):

- P^{min}–equilibrium (with a meeting competition clause);
- supply function equilibrium (with *non-decreasing* functions);
- *compensating* (non-collusive) conjectural variations.

- Our parameter $1 - \theta_i$ corresponds exactly to the conduct parameter of the *NEIO*.

- In all these cases, the index of competitive intensity is endogenous: it signs an equilibrium of which we do not know how it is selected.

Exogenizing the index of competitive toughness

- The *peaceful* side of competition: θ_i as the weight put by firm *i* on collective interest.

  The payoff function of firm *i* is taken as the arithmetic mean of its profit and, with relative weight θ_i, of the sum of their rivals' profits plus the consumers' surplus:

$$\begin{aligned}\Pi_i^{cc}\left(q_i,q_{-i},\theta_i\right) &= (1-\theta_i)\left[q_i D^{-1}\left(q_i+\sum_{j\neq i}q_j\right)-C_i\left(q_i\right)\right]\\ &\quad +\theta_i\left[\int_0^{q_i+\sum_{j\neq i}q_j} D^{-1}\left(Q\right)dQ-\sum_j C_j\left(q_j\right)\right]\end{aligned}$$

Exogenizing the index of competitive toughness

- The *warlike* side of competition: θ_i as the probability that firm i choose an aggressive conduct.

  Two-stage duopoly game (Bertrand-Edgeworth):

1)

2)

-

Exogenizing the index of competitive toughness

- The *warlike* side of competition: θ_i as the probability that firm i choose an aggressive conduct.

  Two-stage duopoly game (Bertrand-Edgeworth):

1) Each firm i chooses a pair (p_i, q_i), with p_i a list price and q_i a quantity to produce in advance. It commits to serve demand only up to quantity q_i.

2)

-

Exogenizing the index of competitive toughness

- The *warlike* side of competition: θ_i as the probability that firm i choose an aggressive conduct.

  Two-stage duopoly game (Bertrand-Edgeworth):

1) Each firm i chooses a pair (p_i,q_i), with p_i a list price and q_i a quantity to produce in advance. It commits to serve demand only up to quantity q_i.

2) With probability $1-\theta_i$ firm i adopts a compromising conduct, sticking to its list price p_i and selling $\min\{q_i, D(p_i)-q_j\}$.

-

Exogenizing the index of competitive toughness

- The *warlike* side of competition: θ_i as the probability that firm i choose an aggressive conduct.

  Two-stage duopoly game (Bertrand-Edgeworth):

1) Each firm i chooses a pair (p_i,q_i), with p_i a list price and q_i a quantity to produce in advance. It commits to serve demand only up to quantity q_i.

2) With probability $1-\theta_i$ firm i adopts a compromising conduct, sticking to its list price p_i and selling $\min\{q_i, D(p_i)-q_j\}$.
   With probability θ_i it adopts an aggressive conduct, supplying at the discount price $\min\{p_i, p_j-\varepsilon\}$ the whole quantity it can actually sell at that price (possibly beyond its residual demand, if its discount price is the lowest).

Exogenizing the index of competitive toughness

- The *warlike* side of competition: θ_i as the probability that firm *i* choose an aggressive conduct.

  Two-stage duopoly game (Bertrand-Edgeworth):

1) Each firm *i* chooses a pair (p_i, q_i), with p_i a list price and q_i a quantity to produce in advance. It commits to serve demand only up to quantity q_i.

2) With probability $1 - \theta_i$ firm *i* adopts a compromising conduct, sticking to its list price p_i and selling $\min\{q_i, D(p_i) - q_j\}$.
   With probability θ_i it adopts an aggressive conduct, supplying at the discount price $\min\{p_i, p_j - \varepsilon\}$ the whole quantity it can actually sell at that price (possibly beyond its residual demand, if its discount price is the lowest).

- Under the assumption of linear costs, we obtain at equilibrium Lerner's index:

$$\frac{P^* - c_i}{P^*} = (1 - \theta_i)\,\frac{q_i^* / (q_1^* + q_2^*)}{-\epsilon D(P^*)} + \theta_i \frac{\varepsilon}{P^*}$$

  This index tends to its value in our canonical model when ε tends to zero.

Competitive toughness as a **strategic** variable

- A preliminary stage may be added to the preceding sequential game, in which each firm *i* chooses θ_i.
  The expected profit is equal to the product:

degree of monopoly × market share × expenditure in the industry

$\theta_i \uparrow \Rightarrow$ degree of monopoly $\downarrow$
market share $\uparrow$ if $c_i < c_j$
expenditure $\uparrow$ ($\downarrow$) if D is elastic (inelastic)

Competitive toughness as a **strategic** variable

- A preliminary stage may be added to the preceding sequential game, in which each firm *i* chooses θ_i.
  The expected profit is equal to the product:

degree of monopoly × market share × expenditure in the industry

$\theta_i \uparrow \Rightarrow$ degree of monopoly $\downarrow$
market share $\uparrow$ if $c_i < c_j$
expenditure $\uparrow$ ($\downarrow$) if *D* is elastic (inelastic)

- With a linear or an isoelastic demand *D*, there always exists a sub-game perfect equilibrium corresponding to Bertrand.
  If *D* has constant elasticity $\varepsilon \in (1/2, 1)$, and if cost asymmetry is moderate, there exists another equilibrium, corresponding to Cournot.

3. The differentiated oligopoly

Consumers' behaviour

- Separable utility function $U(u(x), z)$, with $x \in \Re_+^n$ and $z \in \Re_+$

  $u(x)$: 'quantity' of a composite good

  z : quantity of a numeraire good

  Both functions U and u are assumed increasing and strongly quasi-concave except, as regards u, in the two limit cases of

  - *perfect substitutability* (homogeneous oligopoly)

  $$u(x) = \sum_i x_i$$

  - *perfect complementarity* (complementary monopoly)

  $$u(x) = \min_i (x_i)$$

Consumers' behaviour

- The maximization of U under the budget constraint $px + z \leq w$ can be performed in two stages:

  (*i*)

$$\max_{x \in \mathbb{R}^n_+} \{u(x) : px \leq b\} \equiv v(p,b)$$

with solution $X(p,b) = \times_i X_i(p,b) \in \mathbb{R}^n_+$

defining the *Marshallian demand function.*

(*ii*)

$$\max_{(b,z) \in \mathbb{R}^2_+} \{U(v(p,b), z) : b + z \leq w\}$$

with solution $B(p)$. The function $D(p) \equiv X(p, B(p))$

is the *Walrasian demand function.*

Consumers' behaviour

- Alternative decomposition of consumer's program:
  (*i*)
  $$\min_{x \in \mathbb{R}^n_+} \{px : u(x) \geq \underline{u}\} \equiv e(p, \underline{u})$$

  with solution $H(p, \underline{u}) = \times_i H_i(p, \underline{u}) \in \mathbb{R}^n_+$

  defining the *Hicksian demand function,*
  and FOC $p_i = \partial_u e(p, \underline{u}) \partial_i u(x)$

  (*ii*)
  $$\max_{(\underline{u}, z) \in \mathbb{R}^2_+} \{U(\underline{u}, z) : e(p, \underline{u}) + z \leq w\}$$
  with solution in *u* denoted $\overline{D}(p)$, the *demand for the composite good* – assumed decreasing.
  $$H(p, \overline{D}(p)) = X(p, e(p, \overline{D}(p))) = D(p)$$

The oligopolistic game

- Each firm *i* chooses a strategy $(p_i, q_i) \in \Re_+^2$ to obtain the payoff

$$\begin{aligned}\Pi_i^O(p_i, p_{-i}, q_i, q_{-i}) &= \min\{p_i, \psi_i(p_{-i}, q_i, q_{-i})\} q_i - C_i(q_i) \\ &\quad \text{if } u(q_i, q_{-i}) \leq \overline{D}(p_i, p_{-i}), \\ &= -C_i(q_i), \text{ otherwise.}\end{aligned}$$

The 'best price' $\psi_i(p_{-i}, q_i, q_{-i})$ is the solution to

$$p_i = \partial_u e(p_i, p_{-i}, u(q_i, q_{-i}))\partial_i u(q_i, q_{-i})$$

or to

$$q_i = H_i(p_i, p_{-i}, u(q_i, q_{-i}))$$

- In the homogeneous case, $\psi_i(p_{-i}, q_i, q_{-i}) = \min_{j \neq i}(p_j)$

Oligopolistic equilibrium

- An oligopolistic equilibrium is a Nash equilibrium (p^*, q^*) of the oligopolistic game, satisfying the additional requirement:

$$u\left(q^*\right) = \overline{D}\left(p^*\right)$$

The common agency game

- We introduce an *auxiliary game* with the same set of equilibria, whose payoff function for firm *i* is:

$$\Pi_i^A\left(p_i, p_{-i}, q_i, q_{-i}\right) = p_i q_i - C_i\left(q_i\right)$$

$$\text{if } q_i \leq H_i\left(p_i, p_{-i}, u\left(q_i, q_{-i}\right)\right) \text{ and } e\left(p_i, p_{-i}, u\left(q_i, q_{-i}\right)\right) \leq B\left(p_i, p_{-i}\right)$$

$$\Pi_i^A\left(p_i, p_{-i}, q_i, q_{-i}\right) = -C_i\left(q_i\right), \text{ otherwise.}$$

- This game is interpretable as a game with *n* principals (the firms) and a common agent (the representative consumer), with

  - an *incentive compatibility* constraint: $q_i \leq H_i\left(p_i, p_{-i}, u\left(q_i, q_{-i}\right)\right)$

  - a *participation* constraint: $e\left(p_i, p_{-i}, u\left(q_i, q_{-i}\right)\right) \leq B\left(p_i, p_{-i}\right)$

Equilibrium

- A Nash equilibrium of this auxiliary game, which satisfies the additional no-rationing requirement

$$q_i^* = H_i\left(p^*, u\left(q^*\right)\right) = X_i\left(p^*, B\left(p^*\right)\right), \; i = 1, ..., n$$

is a *common agency equilibrium.*

- **Proposition:** *A vector of singleton contracts is an oligopolistic equilibrium if and only if it is a common agency equilibrium.*

Parameterization of the set of equilibria

- Canonical program:

$$\max_{(p_i,q_i)\in\mathbb{R}_+^2} \left\{p_i q_i - C_i\left(q_i\right) : f_i\left(p_i, p_{-i}^*, q_i, q_{-i}^*\right) \leq 1 \text{ and } g_i\left(p_i, p_{-i}^*, q_i, q_{-i}^*\right) \leq 1\right\}$$

- Kuhn-Tucker multipliers associated with the two constraints:

  ϕ_i and γ_i normalized to $\theta_i = \phi_i / (\phi_i + \gamma_i) \in [0,1]$.

- FOC implies a Lerner index of the *degree of monopoly*

$$\mu_i^* \equiv \frac{p_i^* - C_i'\left(q_i^*\right)}{p_i^*} = \frac{\theta_i \epsilon_{q_i} f_i\left(p^*, q^*\right) + \left(1-\theta_i\right) \epsilon_{q_i} g_i\left(p^*, q^*\right)}{\theta_i \epsilon_{p_i} f_i\left(p^*, q^*\right) + \left(1-\theta_i\right) \epsilon_{p_i} g_i\left(p^*, q^*\right)}$$

where ϵ is the elasticity operator.

The set of potential equilibria

- By applying the canonical formulation to principal *i* program, we get:

$$\max_{(p_i,q_i)\in\mathbb{R}_+^2} p_i q_i - C_i(q_i)$$

$$\text{s.t.} \quad \frac{q_i}{H_i\left(p_i, p_{-i}^*, u\left(q_i, q_{-i}^*\right)\right)} \leq 1 \text{ and } \frac{e\left(p_i, p_{-i}^*, u\left(q_i, q_{-i}^*\right)\right)}{B\left(p_i, p_{-i}^*\right)} \leq 1$$

The normalized multiplier θ_i expresses the implicit value for firm *i* of relaxing the constraint coming from its competitors inside the industry, relative to the value of relaxing the constraint coming from its outside competitors. It can be seen as a measure of the relative *competitive toughness* of firm *i* at the specific equilibrium (*p**,*q**).

Homothetic preferences

- In the particular case of homothetic (sub-)utility u ($\equiv Q$):

$$\max_{(p_i,q_i)\in\mathbb{R}_+^2}\left\{\begin{array}{c} p_i q_i - C_i(q_i) : q_i \leq Q\left(q_i, q_{-i}^*\right)\partial_i P\left(p_i, p_{-i}^*\right) \\ and\ Q\left(q_i, q_{-i}^*\right) \leq D\left(P\left(p_i, p_{-i}^*\right)\right) \end{array}\right\}$$

  First constraint on the market share.
  Second constraint on the market size.

- Dual (equivalent) formulation:

Homothetic preferences

- In the particular case of homothetic (sub-)utility u ($\equiv Q$):

$$\max_{(p_i,q_i)\in\mathbb{R}_+^2}\left\{\begin{array}{c} p_i q_i - C_i(q_i) : q_i \leq Q\left(q_i, q_{-i}^*\right)\partial_i P\left(p_i, p_{-i}^*\right) \\ and\ Q\left(q_i, q_{-i}^*\right) \leq D\left(P\left(p_i, p_{-i}^*\right)\right) \end{array}\right\}$$

First constraint on the market share.
Second constraint on the market size.

- Dual (equivalent) formulation:

$$\max_{(p_i,q_i)\in\mathbb{R}_+^2}\left\{\begin{array}{c} p_i q_i - C_i(q_i) : p_i \leq P\left(p_i, p_{-i}^*\right)\partial_i Q\left(q_i, q_{-i}^*\right) \\ and\ P\left(p_i, p_{-i}^*\right) \leq D^{-1}\left(Q\left(q_i, q_{-i}^*\right)\right) \end{array}\right\}$$

Homothetic preferences

- Equilibrium *degree of monopoly* (Lerner index):

$$\mu_i^* = \frac{\theta_i\left(1-\varepsilon_i^*\right)+\left(1-\theta_i\right)\alpha_i^*}{\theta_i\left(1-\varepsilon_i^*\right)s_i^*+\left(1-\theta_i\right)\alpha_i^*\sigma_i^*}$$

  with
  α_i* the budget share of good *i*
  ε_i* the elasticity of the Hicksian demand w.r.t. q_i
  s_i* the elasticity of substitution of good *i* for the composite good
  σ_i* the elasticity of demand to the industry via good *i*
  θ_i the relative competitive toughness of firm *i*
  μ_i^* is the harmonic mean of $1/s_i^*$ and $1/\sigma_i^*$
- **Collusive solution:**

$$\forall i, \theta_i = 0 \text{ or } \varepsilon_i^* = 1 \Rightarrow \mu_i^* = 1/\sigma_i^*$$

- **Monopolistically competitive equilibrium :**

$$\forall i, \theta_i = 1 \text{ or } \alpha_i^* \approx 0 \Rightarrow \mu_i^* = 1/s_i^*$$

Price equilibrium

- **Price equilibrium** p^* s.t., for any i, p_i^* is the solution to

$$\max_{p_i \in \mathbb{R}_+} \left\{ p_i D_i \left(p_i, p_{-i}^*\right) - C_i \left(D_i \left(p_i, p_{-i}^*\right)\right) \right\}$$

leading to

$$\mu_i^* = -\frac{1}{\epsilon_{p_i} D_i \left(p^*\right)} = \frac{1}{\left(1 - \varepsilon_i^*\right) s_i^* + \varepsilon_i^* \sigma_i^*}$$

which can be obtained from the general formula by taking

$$\theta_i = \frac{\alpha_i^*}{\alpha_i^* + \varepsilon_i^*}$$

Quantity equilibrium

- **Quantity equilibrium** q^* s.t., for any i, q_i^* is the solution to

$$\max_{q_i \in \mathbb{R}_+} \left\{ \left(D^{-1}\right)_i \left(q_i, q_{-i}^*\right) q_i - C_i\left(q_i\right) \right\}$$

leading to

$$\mu_i^* = \frac{1-\varepsilon_i^*}{s_i^*} + \frac{\varepsilon_i^*}{\sigma_i^*}$$

which can be obtained from the general formula by taking

$$\theta_i = \frac{\alpha_i^*}{\alpha_i^* + \varepsilon_i^* s_i^* / \sigma_i^*}$$

Particular specifications: quadratic and homothetic utilities

- **Homothetic utility** ($\varepsilon_i^* = \alpha_i^*$ and, for any $i, \sigma_i^* = \sigma^*$):

$$\mu_i^* = \frac{\theta_i (1 - \alpha_i^*) + (1 - \theta_i) \alpha_i^*}{\theta_i (1 - \alpha_i^*) s_i^* + (1 - \theta_i) \alpha_i^* \sigma^*}$$

- **Quadratic symmetric utility**:

$$\mu^* = \frac{\beta - 1}{\beta + \frac{\overline{\theta}(1-\alpha)\mathbf{s}+\left(1-\overline{\theta}\right)\alpha}{\overline{\theta}(1-\alpha)+\left(1-\overline{\theta}\right)\alpha}} \equiv \mu\left(\overline{\theta}, \mathbf{s}, \alpha, \beta\right)$$

with

$\overline{\theta} \in [0,1]$	the (uniform) competitive toughness
$\mathbf{s} \in (0, \infty)$	the ratio between intra- and intersectoral substitutabilities
$\alpha = 1/n \in (0,1/2]$	the degree of concentration
$\beta \in (1, \infty)$	the market size (consumer's reservation price)

- *price equilibrium*: $\overline{\theta} = 1/2$ - *monopolistic competition*: $\overline{\theta} = 1$

- *quantity equilibrium*: $\overline{\theta} = 1/(1+\mathbf{s})$ - *collusive solution*: $\overline{\theta} = 0$

Existence

- The interval [0,1] for admissible values of θ (and the corresponding range of values of μ) refer to *potential* equilibria only.

  Indeed, firms have an incentive to deviate from strategy profiles close to the collusive solution

Existence

- The interval [0,1] for admissible values of θ (and the corresponding range of values of μ) refer to *potential* equilibria only.

  Indeed, firms have an incentive to deviate from strategy profiles close to the collusive solution

  - either by decreasing their prices, in order to compete for a higher market share, if this is responsive enough (in case of high substitutability),

Existence

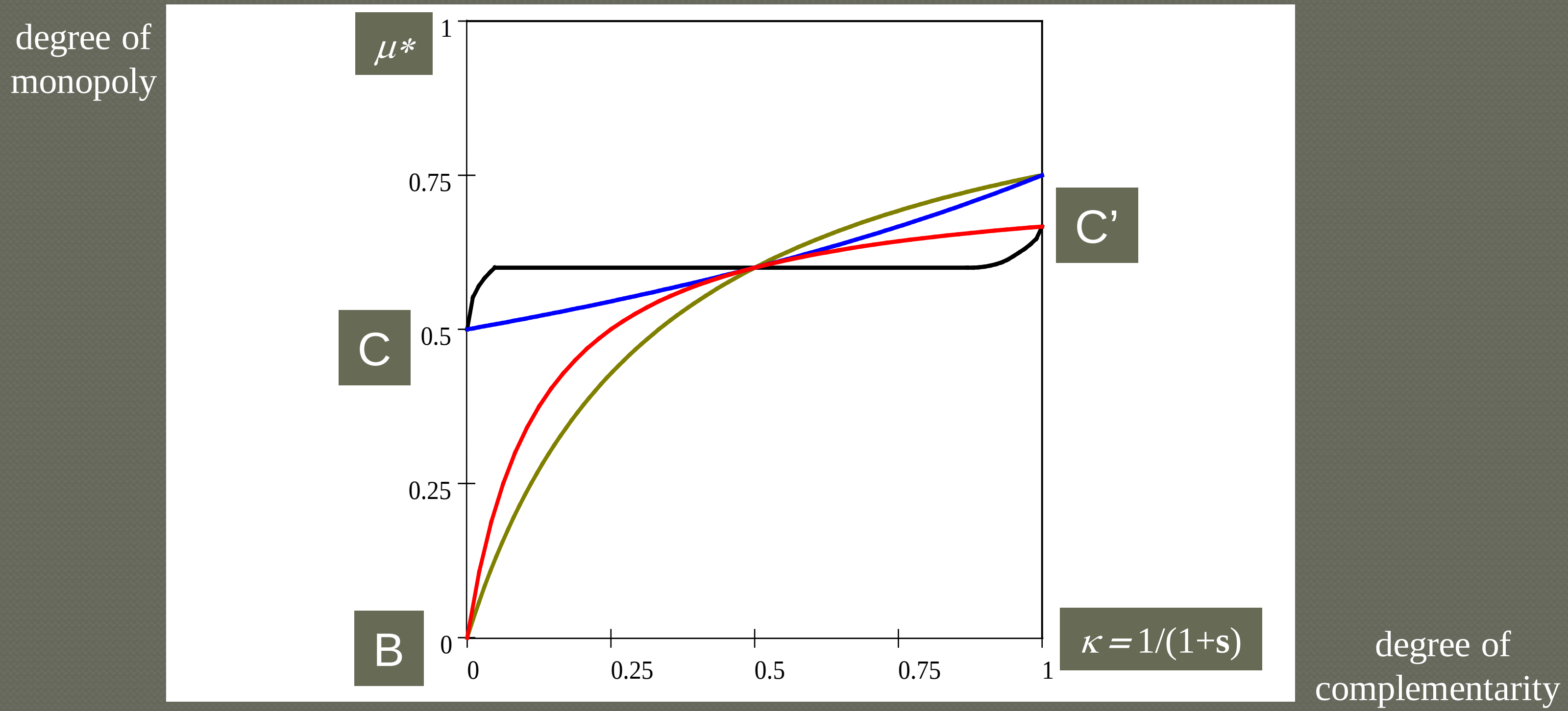
- The interval [0,1] for admissible values of θ (and the corresponding range of values of μ) refer to *potential* equilibria only.

  Indeed, firms have an incentive to deviate from strategy profiles close to the collusive solution

  - either by decreasing their prices, in order to compete for a higher market share, if this is responsive enough (in case of high substitutability),

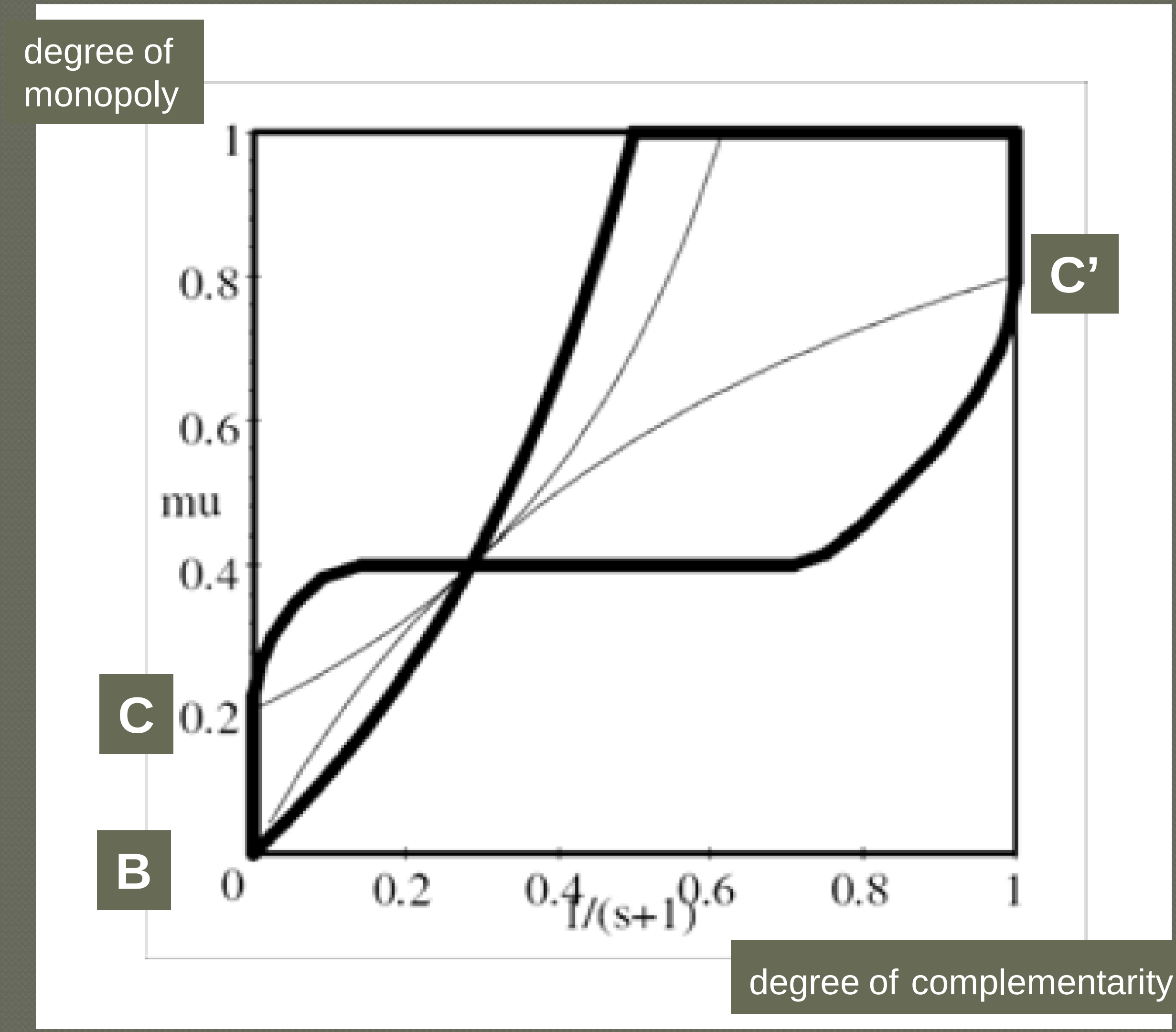
  - or by increasing their prices and their degree of monopoly, taking advantage of a sufficiently unresponsive market share (in case of high complementarity).

Symmetric duopolistic equilibria (quadratic utility)



minimum competitive toughness **quantity equilibrium**
maximum competitive toughness **price equilibrium**
C and C': **Cournot** B: **Bertrand**

Symmetric duopolistic equilibria (CES utility)



4. Conclusion

- Our approach completes the (static) theory of oligopolistic competition by adding the behavioural dimension of **competitive aggressiveness** to the structural dimensions of
  - concentration,
  - substitutability within the industry,
  - substitutability w.r.t. other industries,

  which together determine the competitors' degrees of monopoly.

- Our approach completes the (static) theory of oligopolistic competition by adding the behavioural dimension of **competitive aggressiveness** to the structural dimensions of
  - concentration,
  - substitutability within the industry,
  - substitutability w.r.t. other industries,

  which together determine the competitors' degrees of monopoly.

- Our approach is formally equivalent to the *conjectural variations* and *supply function equilibria* approaches, but it has a larger field of application, since it easily extends to the **differentiated oligopoly**.

- Our approach completes the (static) theory of oligopolistic competition by adding the behavioural dimension of **competitive aggressiveness** to the structural dimensions of
  - concentration,
  - substitutability within the industry,
  - substitutability w.r.t. other industries,

  which together determine the competitors' degrees of monopoly.

- Our approach is formally equivalent to the *conjectural variations* and *supply function equilibria* approaches, but it has a larger field of application, since it easily extends to the **differentiated oligopoly**.

- Although applicable to industrial organization studies, our approach has been designed so as to be easily integrated in simplified general equilibrium models, such are currently used in macroeconomic or international trade theories.