Investments in Productivity under Monopolistic Competition

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Abstract

- (**Theor.question**): Impact of market size on *productivity* in monopolistic competition;
- (**Setting**): (1) variable elasticity of substitution (*VES*), (2) each firm chooses investment in decreasing marginal cost; (3) homogenous firms
- (**Results**): Impact of Growing market:
1. [Each firm's R&D investment *increases* $\uparrow$, price decreases $\downarrow$] $\Leftrightarrow$ ["Relative love for variety" (elasticity of inverse demand) increases].
2. Total R&D investment in economy go up$\uparrow$ always.
3. Socially optimal R&D investment can be bigger or smaller than market equilibrium

Outline

1. Model
2. Impact of market size on productivity
3. Social optimum

Motivation: empirics and theory

Controversy on ***Competitiveness*** and ***innovations***:

- (+) Positive **empirical** correlation between competition (***more firms***) and innovations: Baily & Gersbach (1995), Geroski (1995), Nickell (1996), Blundell, Griffith & Van Reenen (1999), Galdón-Sánchez & Schmitz (2002), Symeonidis (2002), etc.
- (+-) Non-monotone, bell-shape **empirical** correlation: Aghion et al. (2005).
- (+) Positive theoretical correlation: Vives (2008), the model of oligopolistic competition with *free entry (*$\Rightarrow$ endogenous number of firms)

We extend Vives to more realistic model:

- monopolistic competition, general equilibrium
- comparative statics of market equilibria + social optimum

Background literature

- 1. *Basic idea* of Monopolistic Competition: many firms - price-makers produce "varieties", free entry, fixed and variable costs => increasing returns: Chamberlin (1929), Dixit and Stiglitz (1977), for trade - Krugman (1979).
- 2. (Instead of CES or quadratic utility) MC model was generalized to any VES utility: Zhelodobko, Kokovin, Parenti & Thisse (2012)
- 3. Oligopolistic choice of technology in quasilinear setting: Vives (2008): firm's R&D investment in economy go up$\uparrow$ with market size always, and number of varieties can increase or decrease.

We combine *choice of technology* a'la Vives - with *monop. competition.*
It needs VES, because under CES combining is uninteresting: zero effects.

General MC assumptions

- *Increasing returns to scale* in a firm, due to investment cost f and marginal costs $c(f)$. Firms are identical.
- Each firm i produces one "variety" as a *price-maker*, but its demand $x_i(p_i, p_j...)$ is influenced by other varieties.
- L identical consumers, each $j \le L$ generates a demand function x_j, maximizing *additive utility* function $U = \int_{i \le N} u(x_i)di$. Concavity of $u(.)$ (i.e., elasticity of demand or *substitution among varieties)* - determines intensity of competition.
- *Number of firms is big enough* to ignore one firm's influence on the whole industry/economy.
- *Free entry* drives all profits to zero.
- *Labor* supply/demand is balanced.

Basic model of 1x1x1 economy. Consumers

- One diversified sector has an interval $[0, N]$ of firms=varieties i-th brand is i-th firm, $i \in [0, N]$,
- L identical consumers, each has 1 of labor and chooses an (infinite-dimensional) consumption vector $x(\cdot) : [0, N] \to \mathbb{R}_+$ i.e., a non-negative integrable function x:

$$\int_0^N u(x_i)di \to \max_{x(.)}; \quad \int_0^N p_i x_i di \leq 1.$$

- Here: utility function $u(\cdot)$, price vector $p(\cdot) : [0, N] \to \mathbb{R}_+$; price $p(i) \equiv p_i$ for i-th variety, demand $x(i) \equiv x_i$ for i-th variety. Lagrange multiplier λ, = marginal utility of income. FOC: the inverse demand $\mathbf{p}$ for i-th variety is:

$$\mathbf{p}(x_i, \lambda) = \frac{u'(x_i)}{\lambda}$$

Producers: marginal cost function of investments, FOC

- i-th firm knows its inverse-demand function $p_{\text{i}}(x_i, \lambda)$, sells $q = Lx_{\text{i}}$ and maximizes profit

$$\pi = Lx_i \cdot [p_{\text{i}}(x_i, \lambda) - c(f_i)] - f_i \to \max_{x_{\text{i}}, f_{\text{i}} \in \mathbb{R}_+} .$$

  c is marginal cost and f is fixed cost measured in labor (total cost is $cx_{\text{i}}L + f$)

- Marginal cost function $c(\cdot)$ of **investment** or fixed cost f:
  - $c'(f) < 0$ (more expensive factory would have smaller marginal costs)
  - $c''(f) > 0$ (decreasing returns to scale of investments, at equilbrium)
- *Symmetric equilibrium* is (x, f, p, N, λ) satisfying all FOC and budget, free entry and labor balance:

Model: Equilibrium (x, f, p, N, λ)

- Consumers' FOC:

$$p = \mathbf{p}(x, \lambda) = u'(x)/\lambda$$

- Producers' FOC:

$$\frac{\partial \pi(x,f)}{\partial x} = 0, \qquad \frac{\partial \pi(x,f)}{\partial f} = 0$$

- Zero-profit condition (free entry):

$$\pi = (\mathbf{p}(x,\lambda) - c(f))xL - f = 0.$$

- Labor balance (equivalent to the budget constraint):

$$(f + c(f)x)N = L$$

About $\mathcal{E}_g, r_g, r_{g'}, r_{\ln g}, r'_g$, etc.

Definition of elasticity: $\mathcal{E}_g(z) = \frac{zg'(z)}{g(z)}$

Elasticity of the product is the sum of elasticities: $\mathcal{E}_{gh}(z) = \mathcal{E}_g(z) + \mathcal{E}_h(z)$

The interconnection between elasticity and Arrow-Pratt measure:

$$r_g(z) = -\frac{zg''(z)}{g'(z)} = -\mathcal{E}_{g'}(z)$$

One has: $r_{g'}(z) = -\frac{zg'''(z)}{g''(z)}, \quad r_{\ln g}(z) = -\frac{z \cdot (\ln g(z))''}{(\ln g(z))'} = \mathcal{E}_g(z) + r_g(z)$

Moreover

$$r'_g(z) \cdot z = \left(1 + r_g(z) - r_{g'}(z)\right) r_g(z)$$

$$\mathcal{E}'_g(z) \cdot z = \left(1 - \mathcal{E}_g(z) + \mathcal{E}_{g'}(z)\right) \mathcal{E}_g(z) = (1 - r_{\ln g}(z)) \mathcal{E}_g(z)$$

It is important to note:

$$\textbf{If} \quad g(z) \quad \textbf{is} \quad CES \quad \textbf{then} \quad r'_g(z) = \mathcal{E}'_g(z) = 1 - r_{\ln g}(z) = 0$$

Equilibrium equations in terms of (x, f)

We use the Arrow-Pratt measure of concavity defined for any function g :

$$r_g(z) = -\frac{zg''(z)}{g'(z)}.$$

Proposition. Equilibrium consumption/investment (x^*, f^*) is the solution to

$$\frac{r_u(x)x}{1 - r_u(x)} = \frac{f}{Lc(f)}$$

$$(1 - r_{\ln c}(f) + r_c(f))(1 - r_u(x)) = 1$$

when SOC conditions hold:

$$r_u(x) < 1, \qquad 2 - r_{u'}(x) > 0, \qquad (2 - r_{u'}(x))\, r_c(f) > 1.$$

Differentiating the system w.r.t. $L \Rightarrow$ Theorem of comparative statics:

Theorem: signs of elasticities w.r.t. market size L:

IEID (*DEID*)– Increasing (Decreasing) Elasticity of the Demand

Patterns:	*DED*	*CES*	*IED*		
Elasticities w.r.t. L of:	$r'_u < 0$	$r'_u = 0$	$r'_u > 0$		
	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
$\mathcal{E}_f$	< 0	$= 0$	> 0	$\in (0,1)$	> 0
$\mathcal{E}_{Nf}$	> 1	$= 1$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_{\frac{Nf}{L}}$	> 0	$= 0$	$\in (-1,0)$	$= 0$	> 0
$\mathcal{E}_p$	> 0	$= 0$	< 0	$= -r_u \in (-1,0)$	< 0
$\mathcal{E}_q$	< 0	$= 0$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_N$	> 1	$= 1$	$\in (0,1)$	$= r_u \in (0,1)$	< 1

- Shortly, *IED*+larger market $\Rightarrow$ bigger firms$\Rightarrow$ higher productivity
- Interpretation: bigger output - motivates higher cost-reducing investment. **But**: bigger output is guaranteed for larger market only under *IED*!

Theorem: Interpretation

- *CES* case is the borderline between markets with *DED* or *IED*
- $L \uparrow \Rightarrow$ *DED*: $\downarrow$ *investments*, *IED*: $\uparrow$ *investments*
- Investments are ***positively correlated*** with the size of the firm:
  - bigger output planned - motivates higher cost-reducing investment
- **But** bigger output is not guaranteed for larger market:
  - *DED* $\Rightarrow\downarrow$ **both** $q = Lx$ and f - because $N \uparrow$ **too fast**: $\Rightarrow$ **excessive** competition $\Rightarrow$ output $\downarrow \Rightarrow \downarrow$ the motive to invest in productivity
  - **But:** Nf always $\uparrow$ because growing N dominates even when $f \downarrow$
- **Prices**: as ZKPT(2012): $\uparrow$ under *DED*, $\downarrow$ under *IED*. Explanation:
  - $L \uparrow \Rightarrow$ profits $\uparrow \Rightarrow$ invite **new** firms $\Rightarrow N \uparrow \Rightarrow$ competition $\uparrow \Rightarrow x \downarrow$
  - **Paradoxically**: under *DED*: **too** convex demand function $\Rightarrow$ price $\uparrow$
- Behavior of x and N: generally as in ZKPT (2012), but a new fashion in the exotic case ($r'_u > 0$, $r_{\ln c} < 1$)
- **Interestingly**, the nature of $c(f)$ is the criterion only for $\uparrow / \downarrow$ of x. Only under sufficiently big elasticity of $c(f)$ ($r_{\ln c}(f) > 1$), $x \downarrow$

The results of ZKPT (2012) , Krugman (1979)

- Zhelodobko, Kokovin, Parenti & Thisse (2012):

	$r'_u < 0$	$r'_u = 0$	$r'_u > 0$
$\mathcal{E}_p$	$+$	0	$-$
$\mathcal{E}_q$	$+$	0	$-$
$\mathcal{E}_N$	>1	$=1$	$\in (0,1)$

- Krugman (1979) (Nobel Prize, 2008):

	$r'_u < 0$ (without strict proofs)
$\mathcal{E}_p$	$+$
$\mathcal{E}_q$	$+$
$\mathcal{E}_N$	$+$

Comparison with of ZKPT(2012), Krugman (1979)

Patterns:	DED	CES	IED		
Elasticities	$r_u' < 0$	$r_u' = 0$	$r_u' > 0$		
w.r.t. L of:	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
$\mathcal{E}_f$	< 0	$= 0$	> 0	$\in (0,1)$	> 0
$\mathcal{E}_{Nf}$	> 1	$= 1$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_{\frac{Nf}{L}}$	> 0	$= 0$	$\in (-1,0)$	$= 0$	> 0
$\mathcal{E}_p$	> 0	$= 0$	< 0	$= -r_u \in (-1,0)$	< 0
$\mathcal{E}_q$	< 0	$= 0$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_N$	> 1	$= 1$	$\in (0,1)$	$= r_u \in (0,1)$	< 1

	ZKPT(2012)				Krugman(1979)
	$r_u' < 0$	$r_u' = 0$	$r_u' > 0$		$r_u' < 0$
$\mathcal{E}_p$	$+$	0	$-$	$\mathcal{E}_p$	$+$
$\mathcal{E}_q$	$+$	0	$-$	$\mathcal{E}_q$	$+$
$\mathcal{E}_N$	> 1	$= 1$	$\in (0,1)$	$\mathcal{E}_N$	$+$

Social optimum compared with market equilibrium

In symmetric solution optimality means that x^{opt}, f^{opt} and N^{opt} are welfare-optimizing:

$$Nu(x) \to \max_{N,x,f} \qquad \text{s.t.} \quad N(c(f)xL + f) = L$$

IEU – increasing elasticity of utility: $\mathcal{E}'_u(x) > 0$

DEU (CEU) – decreasing (constant) elasticity of utility

IEU: $r_{\ln u} < 1 \Leftrightarrow \mathcal{E}'_u(x) > 0$	CEU: $r_{\ln u} = 1$	DEU: $r_{\ln u} > 1 \Leftrightarrow \mathcal{E}'_u(x) < 0$
purchase size $x^{opt} < x^*$	$x^{opt} = x^*$	$x^{opt} > x^*$
investment $f^{opt} < f^*$	$f^{opt} = f^*$	$f^{opt} > f^*$
mass of firms $N^{opt} > N^*$	$N^{opt} = N^*$	$N^{opt} < N^*$

Optimal total investment $(Nf)^{opt} \equiv N^{opt} \cdot f^{opt}$ and equilibrium total investment $(Nf)^* \equiv N^* \cdot f^*$ are related as

$(1 - r_{\ln u})(1 - r_{\ln c}) < 0$	$(1 - r_{\ln u})(1 - r_{\ln c}) = 0$	$(1 - r_{\ln u})(1 - r_{\ln c}) > 0$
$(Nf)^{opt} > (Nf)^*$	$(Nf)^{opt} = (Nf)^*$	$(Nf)^{opt} < (Nf)^*$

Comparative statics of social optimum

Theorem. The signs of elasticities of soc.optimal x^{opt}, f^{opt} and N^{opt} w.r.t. market size L are

	IEU	*CEU*	*DEU*		
	$r_{\ln u} < 1$	$r_{\ln u} = 1$	$r_{\ln u} > 1 \Leftrightarrow \mathcal{E}'_u(x) < 0$		
	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
$\mathcal{E}_{f^{opt}}$	< 0	$= 0$	> 0	$\in (0,1)$	> 0
$\mathcal{E}_{N^{opt} f^{opt}}$	> 1	$= 1$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_{\frac{N^{opt} f^{opt}}{L}}$	> 0	$= 0$	$\in (-1,0)$	$= 0$	> 0
$\mathcal{E}_{q^{opt}}$	< 0	$= 0$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_{N^{opt}}$	> 1	$= 1$	$\in (0,1)$	$\in (0,1)$	< 1

Behavior of optimal investment follows the 3 patterns governed by *DEU*, *CEU* and *IEU* cases of preferences: in DEU case each firm's investments goes up.

Compared effects of equilibrium and optimum

Elasticity	$r'_u < 0$	$r'_u = 0$	IED: $r'_u > 0$		
	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
$\mathcal{E}_{f^*}$	< 0	$= 0$	> 0	$\in (0,1)$	> 0
$\mathcal{E}_{N^* f^*}$	> 1	$= 1$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_{\frac{N^* f^*}{L}}$	> 0	$= 0$	$\in (-1;0)$	$= 0$	> 0
$\mathcal{E}_{q^*}$	< 0	$= 0$	$\in (0;1)$	$= 1$	> 1
$\mathcal{E}_{N^*}$	> 1	$= 1$	$\in (0,1)$	$\in (0,1)$	< 1

Elasticity	$r_{\ln u} < 1$	$r_{\ln u} = 1$	$DEU: r_{\ln u} > 1 \Leftrightarrow \mathcal{E}'_u(x) < 0$		
	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
$\mathcal{E}_{f^{opt}}$	< 0	$= 0$	> 0	$\in (0,1)$	> 0
$\mathcal{E}_{N^{opt} f^{opt}}$	> 1	$= 1$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_{\frac{N^{opt} f^{opt}}{L}}$	> 0	$= 0$	$\in (-1,0)$	$= 0$	> 0
$\mathcal{E}_{q^{opt}}$	< 0	$= 0$	$\in (0,1)$	$= 1$	> 1
$\mathcal{E}_{N^{opt}}$	> 1	$= 1$	$\in (0,1)$	$\in (0,1)$	< 1

Conclusions, main directions of study

- In a larger economy firm size and productivity is higher $\Leftrightarrow$ *IED*-utility;
- In welfare analysis, socially-optimal solutions show similar comparative statics as equilibria, and only under *CES* equilibria are optimal;
- Under heterogenous firms a-la Melitz, market size yields similar effects (?);
- Open economy case
- ***Thank you***