Discrimination in a Dynamic Search Model

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Discrimination

Disabilities, Nationality, Gender, Language, Religion, Belief, Marital Status, Age, Sexual Orientation, Ethnic Origin. Social Origin or any other status;

- ***Benchmark***: Coate and Loury's (1993) model of discrimination in a job assignment.

- ***Contribution***: the model was expanded by introducing heterogeneity of employers while Coate and Loury assumed employers are identical.

- ***Study***: an evolution of equilibrium beliefs when unprejudiced firms enter to the market was analyzed

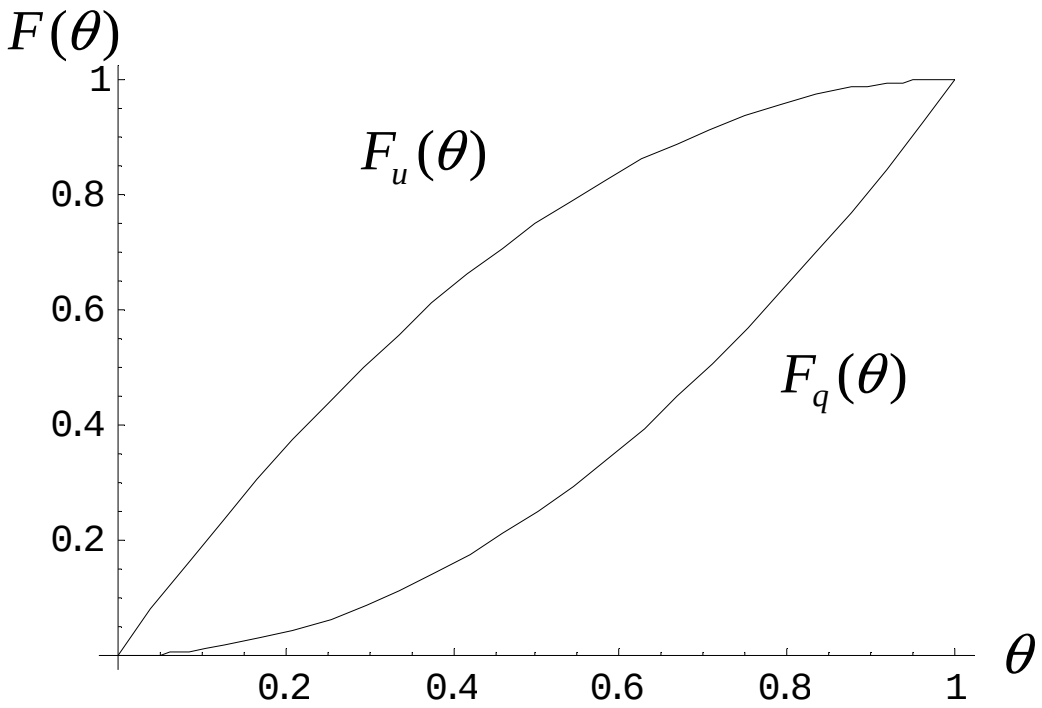
The Basic Model

- Labor market consists of two types of job, called tasks “zero” and “one”. Task “one” is the more demanding and rewarding assignment than the task “zero”.
- Workers are “qualified” or “unqualified”.
- Workers are of two types: type *M* and type *F*. Type of worker is observable.
- There are a large numbers of employers and workers at the labor market.
- Each employer is randomly matched with many workers.
- Employers are identical and have the same beliefs about the workers qualification.

Employers' Behavior

- The employer's goal is to assign each of his or her workers to one of two possible jobs in the most efficient way.
- Employers gain a net return $x_q > 0$ if they assign a qualified worker to task "one" and $(-x_u) < 0$ if they assign an unqualified worker to task "one". A ratio of net gain to loss is: $r \equiv x_q / x_u$
- Workers' and employers' returns from an assignment to task "zero" are normalized to zero.
- Employers observe each worker's group identity and a noisy signal $\theta \in [0,1]$.

- Let, the signals θ are distributed accordingly to the *Beta-distribution* at [0,1] with parameters
  - $\alpha = 1, \beta = 2$ for unqualified workers and
  - $\alpha = 2, \beta = 1$ for qualified workers.



$$F_q(\theta) = \theta^2,$$
$$f_q = 2\theta$$

$$F_u(\theta) = 2\theta - \theta^2,$$
$$f_u = 2(1-\theta)$$

Figure 1. Distribution of signals for qualified and unqualified workers.

- Let's define, the likelihood ratio at θ is:

$$\varphi(\theta) \equiv \frac{f_u}{f_q} = \frac{(1-\theta)}{\theta}$$

- $\varphi(\theta)$ is non-increasing on [0, 1],
- $F_u(\theta) \geq F_q(\theta)$ for all θ.
- Thus, higher values of the signal are more likely if the worker is qualified, and for a given prior, the posterior likelihood that a worker is qualified is larger if his signal takes a higher value.

- Besides the signals, employers have prior beliefs about workers' qualification.
- A representative member of group *j* has probability $\pi_j \in (0,1)$ of being qualified, $j = F, M$.
- Using Bayes' Rule:

$$\xi(\pi_j,\theta) \equiv \frac{\pi_j f_q(\theta)}{[\pi_j f_q(\theta)+(1-\pi_j) f_u(\theta)]} = \frac{1}{1+\frac{(1-\pi_j)}{\pi_j}\varphi(\theta)} = \frac{1}{1+\frac{(1-\pi_j)}{\pi_j}\frac{(1-\theta)}{\theta}}$$

- Employer's expected payoff from assigning a worker to task "one" is:

$$P = \xi(\pi_j,\theta)x_q - (1-\xi(\pi_j,\theta))x_u, \quad j = F, M$$

- Payoff from assigning a worker to task "zero" is zero.

- Employer assigns a worker to task “one” if and only if $P \geq 0$, i.e:

$$\xi(\pi_j,\theta)x_q - (1-\xi(\pi_j,\theta))x_u \geq 0$$

$$\frac{x_q}{x_u} \geq \frac{(1-\xi(\pi_j,\theta))}{\xi(\pi_j,\theta)} \quad \Rightarrow \quad r \geq \frac{(1-\pi_j)}{\pi_j}\frac{(1-\theta)}{\theta}, \quad j = F, M$$

- Employer chooses a threshold "standards" for each group:

$$s^*(\pi_j) \equiv \min\{\theta \in [0,1] \,|\, r \geq \frac{(1-\pi_j)}{\pi_j}\frac{(1-\theta)}{\theta}\} \quad \Rightarrow \quad s^*(\pi_j) = \frac{(1-\pi_j)}{(1-\pi_j + r\pi_j)}, \quad j = F, M$$

- Assuming $r \equiv 1, \quad s^*(\pi_j) = (1-\pi_j).$
- Employers assign a worker from a group j with π_j if and only if

$$\theta \geq s^*(\pi_j) \quad \Rightarrow \quad \theta \geq (1-\pi_j).$$

- More optimistic beliefs about a group will be reflected in easier standards.

Workers' Behavior

- All workers prefer to be assigned to task "one", whether or not they are qualified.
- Workers are qualified to do task "one" only if they have made some costly ex-ante investment.
- Cost distribution *G(c)* is the same for each group.
- Workers must decide, prior to being matched with an employer, whether making the investment is worthwhile.

- The expected benefit of investment is the product of two quantities:
  - the gross return from being assigned to task one (w) and
  - the increase in probability of good assignment.

$$\beta(s) \equiv w\Delta \Pr = w[(1-F_q(s))-(1-F_u(s))] = 2w\,s(1-s)$$

- Workers with investment cost c facing the standard s invests if and only if

$$c \leq \beta(s)$$

- The proportion of workers that become qualified is

$$G(\beta(s)) = G(2ws(1-s))$$

Sequence of Actions

0. nature chooses workers types
1. workers make investment decisions
2. workers matched with employers
3. test results $\theta \in [0, 1]$ observed
4. employers make assignment decisions
5. payoffs received

Equilibrium

- ***Definition 1***: A pair of beliefs for employers about the two groups (π_F, π_M) will be ***self-confirming*** if, by choosing standards optimal for those beliefs [$s^*(\pi_F)$, $s^*(\pi_M)$] employers induce workers from two groups to become qualified at precisely the rate postulated by the beliefs.

- ***Definition 2***: An *equilibrium* is a pair of employers' beliefs (π_F, π_M) satisfying $\pi_j = G(\beta(s^*(\pi_j)))$, $j = F, M$.

- ***Proposition 1***: A *discriminatory equilibrium* (say, one with $\pi_F < \pi_M$) can arise whenever the model has multiple solutions.

- ***Proposition 2***: Assume that $\varphi(\theta)$ is continuous, $\varphi'(\theta)<0$, $\varphi(\theta)>0$ on [0, 1], and that $G(c)$ is continuous, $G(0) = 0$.
  If $\exists\, s \in (0,1) : G(\beta(s)) > \varphi(\theta)/[r + \varphi(\theta)]$ then there exist at least two nonzero solutions.

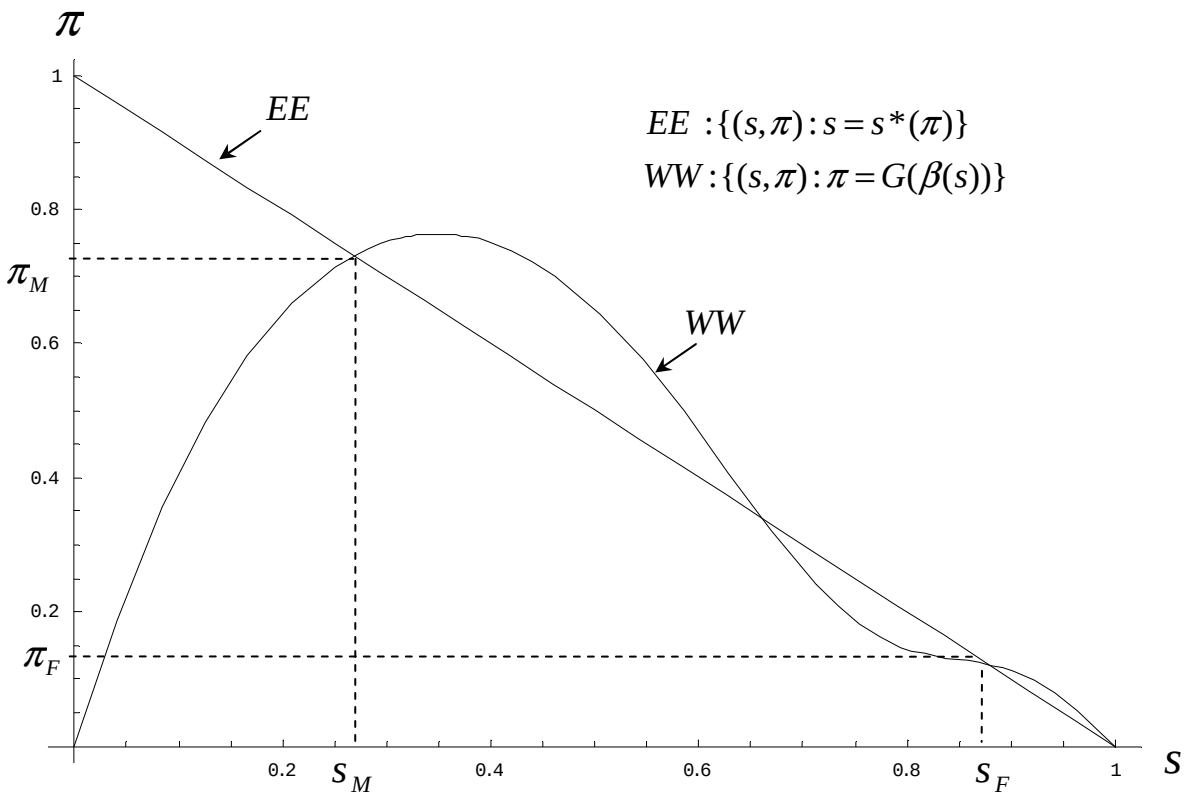


Figure 2. Equilibrium with negative stereotypes against group F workers.

There are two stable *self-confirming* equilibriums under dynamic process $\pi_{t+1} = G(\beta(s^*(\pi_t)))$, t=0,1,2,…:

$(s_F, \pi_F) = (0.87, 0.12)$
$(s_M, \pi_M) = (0.26, 0.73)$[1].

For employers' initial beliefs

- if $\pi_j \geq 0.35$ the market converges to the equilibria $(s^*_j, \pi^*_j) = (0.26, 0.73)$
- if $\pi_j < 0.35$ $(s^*_j, \pi^*_j) = (0.87, 0.12)$.

[1] For the purpose of simulation we choose the certain form of G(s) that satisfies assumptions and generates the equilibrium structure described above **$G(\beta(s)) = 5s[(1-s)^2+\sin[3\pi s^2]/65$**]. The specification of the function G(.) affects quantitative results, not qualitative ones.

The Entry of Unprejudiced Firms and Dynamic Process

- M-workers constitute *g* portion of the workers and F-workers constitute the remaining *(1-g)* portion of the workers.
- Unprejudiced firms constitute share α of whole pool of employers. Respectively, the employers with social stereotypes constitute another portion (1- α).
- Suppose, in the period $t = 0$ prejudiced employers have initial beliefs about F and M workers' qualification $\{\pi_F^0, \pi_M^0\}$, say $\pi_F^0 = 0.12$ and $\pi_M^0 = 0.73$ (which corresponds to equilibrium believes in the previous model).
- These employers choose the standards $s^*(\pi_j^0) = (1 - \pi_j^0)$, $j = F, M$.
- Unprejudiced employers do not recognized the type of workers' group.
- Their initial expectation for workers of any group being qualified for the task "one" is $\pi_N^0 = (1\text{-}g)\,\pi_F^0 + g\,\pi_M^0$
- $s_N^0 = s^*(\pi_N^0) = 1 - (1\text{-}g)\,\pi_F^0 - g\,\pi_M^0$.

The dynamic in the model goes in the following way:

- At the first period prejudiced and unprejudiced employers have the rational beliefs and choose standards that maximize their profit:
  π_F^0 π_M^0 $\pi_N^0 = (1-g)\,\pi_F^0 + g\,\pi_M^0$
  s_F^0 s_M^0 $s_N^0 = 1 - (1-g)\,\pi_F^0 - g\,\pi_M^0$

- workers have the probability α and 1- α to meet unprejudiced and prejudiced employers. Thus expectations of standards that F and M worker face are $\alpha s_F^0 + (1-\alpha) s_N^0$ and $\alpha s_M^0 + (1-\alpha) s_N^0$.

- Workers expect to meet the same signals at the next period:
  $s_F^1 = \alpha\, s_F^0 + (1-\alpha)\, s_N^0$, $s_M^1 = \alpha\, s_M^0 + (1-\alpha)\, s_N^0$, and at the end of the first period their decide invest or do not given these standards.

- The proportions those become qualified in the next period are
  $\pi_F^1 = G(\beta(s_F^1))$ and $\pi_M^1 = G(\beta(s_M^1))$.

- The next period starts and employers correctly anticipate the proportions of qualified workers and set up new standards to maximize their profit. New standards generate new proportion of qualified worker and so on.

Convergence of the process for different levels of α and *g* and different levels of initial believes

- Let's π^L, π^H denote equilibrium portion of qualified workers when there is no unprejudiced employers.

- **Proposition 1**. Given initial believes $\{\pi_F^0, \pi_M^0\}$ "close" to $\{\pi^L, \pi^H\}$, there exists $\alpha^B(g)$ and the level g^* such that for any $0<g<g^*$ and any $\alpha<\alpha^B(g)$ the process converges to the $\pi^*_j = \pi^L$, $j=M, F$. The function $\alpha^B(g)$ is decreasing and $0<\alpha^B(0)<1$, $\alpha^B(g^*)=0$.

- The *proposition 1* says that if the proportion of M-workers is small and the portion of unprejudiced firms is small too then in equilibrium both groups will have low rate of qualified workers. This rate is the same as in equilibrium without unprejudiced firms.

- **Proposition 2.** Given initial believes $\{\pi_F^0, \pi_M^0\}$ "close" to $\{\pi^L, \pi^H\}$, there exists $\alpha^G(g)$ such that for any g and any $\alpha^B(g) < \alpha < \alpha^G(g)$ the process converges to the $\pi^*_j = \pi^H$, $j=M, F$. The function $\alpha^G(g)$ is increasing and $\alpha^B(0) \leq \alpha^G(0)$, $\alpha^G(1) < 1$.

- The *proposition 2* says that if the proportion of M-workers is high enough and the portion of unprejudiced firms is not "very high" then in equilibrium both groups of workers will has high rate of qualified workers. This rate is the same as one in equilibrium without unprejudiced firms.

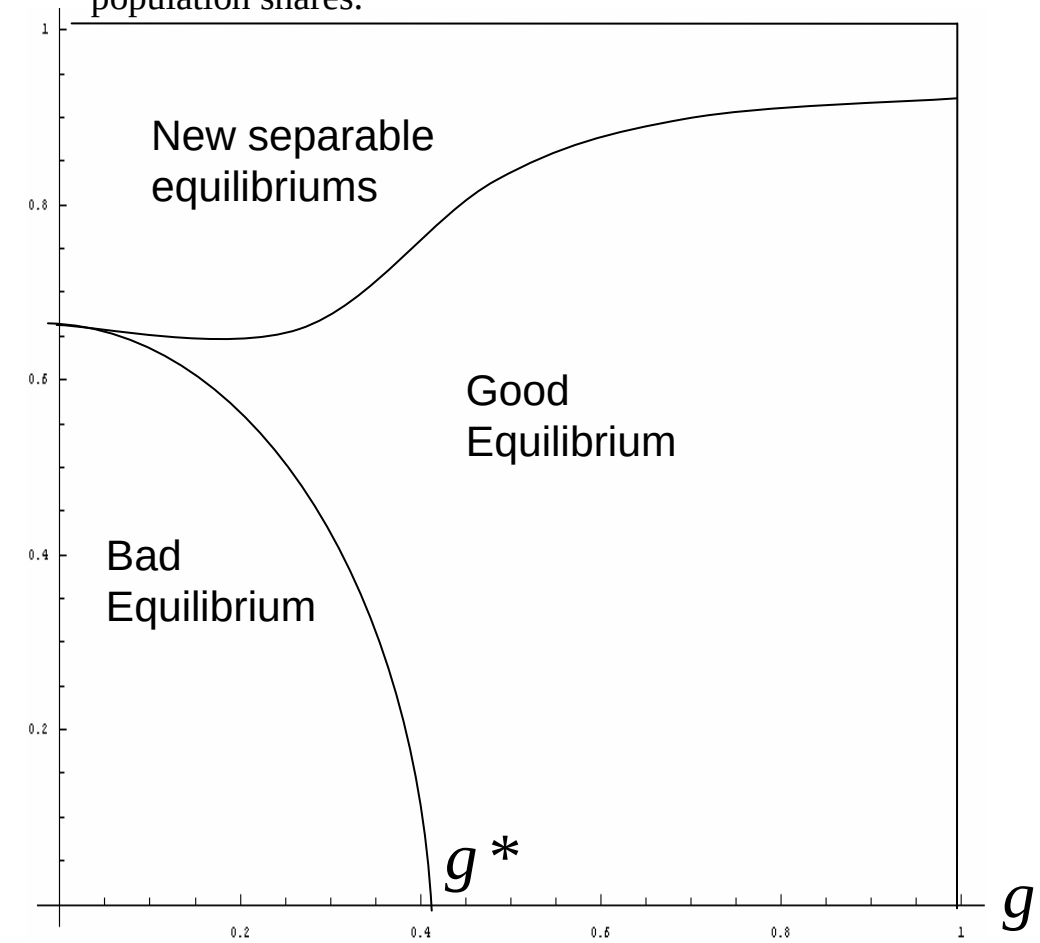
- **Proposition 3**. Given initial believes $\{\pi_F^0, \pi_M^0\}$ "closed" to $\{\pi^L, \pi^H\}$, for any g and any $\alpha > \alpha^G(g)$ the process converges to the $\{\pi^*_M(\alpha, g), \pi^*_F(\alpha, g)\}$ with $\pi^*_M(\alpha, g) > \pi^*_F(\alpha, g)$.

- Thus if the fraction of unprejudiced firms is "very high" we obtain a new class of equilibriums which characterized by following properties:

  - equilibrium values $\{\pi^*_M(\alpha, g), \pi^*_F(\alpha, g)\}$ are different for different values of α and g;

  - if $\pi^*_M(\alpha, g) > \pi^H$ then $\pi^*_F(\alpha, g) < \pi^L$, and If $\pi^*_M(\alpha, g) < \pi^H$ then $\pi^*_F(\alpha, g) > \pi^L$.

Figure 3. Effect of prejudiced employers share and population shares.



- we call *"bad"* equilibrium the one with $\pi^*_j = \pi_L$, $j=M, F$

- and *"good"* equilibrium the one with $\pi^*_j = \pi_H$, $j=M, F$.

Proposition 4.

- If initial believes $\{\pi_F^0, \pi_M^0\}$ "close" to $\{\pi_L, \pi_L\}$ for any g and any α there is equilibrium is $\{\pi^*_M, \pi^*_F\} = \{\pi_L, \pi_L\}$.

- If initial believes $\{\pi_F^0, \pi_M^0\}$ "close" to $\{\pi_H, \pi_H\}$ for any g and any α there is equilibrium is $\{\pi^*_M, \pi^*_F\} = \{\pi_H, \pi_H\}$.

The *proposition 4* says that if initial believes are about the same for both groups then believes converge to corresponding equilibrium regardless the share of unprejudiced firms and proportions of group.

Conclusions

- different **self-confirming believes** in respect to different groups of workers may be realized in equilibrium.
- entering of unprejudiced firms may lead to new type of separable equilibrium, “good” equilibrium, and also may result in low rate of qualified workers in both groups.
- discrimination results in:
  - low rates of qualified workers
  - overeducation of discriminated workers
- considering α as endogenous, prejudiced firms’ should leave the market…
- BUT: in reality some inefficient institutes survive. They are really persistent and for the moment there are NO good economic instruments to overcame it...
- BUT: many researches, lawers, policymakers try to develop and improve such instruments…

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LOCALS
IMMIGRANTS
WAY OUT

- Гипотезы:
- Дискриминация мигрантов:
  - влияет негативно на потоки миграции как на международном, так и региональном уровне?
  - имеет негативный эффект на рост популяции и развитие городов?
  - сосредотачивает мигрантов в low-skill и low-paid отраслях, тем самым
    - снижая качество жизни переселенцев
    - вытесняя локальных рабочих из данных секторов
  - порождает under- и overeducation для местных и приезжих соотв.

Thank You For Your Attention