Endogenous Structure of Non-Linear Cities

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Outline

Stylized Facts

1. Monocentric Cities and Polycentric Cities co-exist in economy space
2. "Large" cities are Polycentric, while "small" ones are Monocentric
3. City pattern may change with the lapse of time (i.e. pattern is not predefined)
4. Population growth is (in general) disproportionally larger in suburbia
5. Moreover, sometimes Central city may "freezes", while suburbia population still increases

Why It May Be Important?

The way the cities are organized has a major impact of the people's well-being:

- **Urban costs** (housing and commuting ones), account for a large share of consumers' expenditures:
  - US housing accounts for 20% of household budgets, car related expenses – 18% of total expenditures
  - Journeys inside the Paris metropolitan area amounted to 34.3 billion Euros (over 8% of local GDP)
  - Housing price per m^2 in Paris is 80% higher than in the rest of France
  - Moscow Metro: 2.6 billion commuting trips per year

Creation of subcenters is a natural way to alleviate the burden of urban costs!

Forces

- Agglomeration force: *Economy of Scale* force firms to gather in one place
- Dispersion force: *Urban Costs* force to form additional (secondary) job centers
- Interplay of *production costs* and *urban costs* generates the inner city structure

Compare to Other Approaches

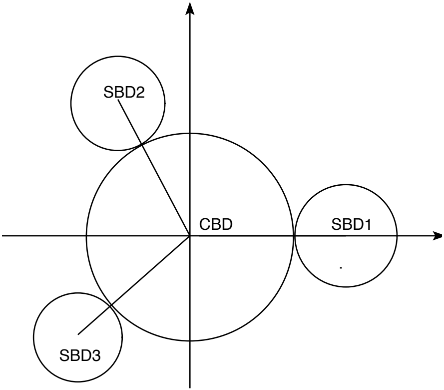
- Polycentricity is not predefined unlike
  - Sullivan (1986), *A general equilibrium model with agglomerative economies and decentralized employment*, Journal of Urban Economics
  - Wieand (1987), *An extension of the monocentric urban spatial equilibrium model to a multicenter setting: The case of the two-center city*, Journal of Urban Economics
  - Helsley and Sullivan (1991), *Urban subcenter formation*, Regional Science and Urban Economics
- We don't rely on consumer's "propensity to big malls"
  - Anas and Kim (1996), *General equilibrium models of polycentric urban land use with endogenous congestion and job agglomeration*, Journal of Urban Economics

Similar Approaches

- **Main insights** are based on
  - J. Cavailhès et al. (2007), *Trade and the structure of cities*, Journal of Urban Economics
- Yet we **don't rely** on paradigm of **"long narrow city"** with exactly TWO secondary centers, using two-dimensional pattern with arbitrary number of subcenters
- Insight of **job sub-centers' hierarchy** (secondary, tertiary, etc) borrowed from
  - T. Tabuchi (2009), *Self-organizing marketplaces*, Journal of Urban Economics
- **Empiric evidences**: MacMillen and Smith (2003), *The number of subcenters in large urban areas*, JUE
  "Number of subcenters increases with population, as as well as with commuting costs"

Outline

Polycentric City



CBD – Central Business District, (Axis origin placed here)
SBDi – Secondary Business Districts
$i \in \{1, 2, \ldots, m\}$,
$m = 0$ – monocentric city

Why Circles?

There is no reason to be something else

- Plane is initially featureless
- Residents (workers) take a unite of area for residence and commuting costs depend on *Euclidean distance*
- No cross-commuting

Why Two Dimensions?

- Urban costs depend on *distances* or *geographic size*
  - proportional to *population* or *demographic size* in linear city
  - *less than proportional* in two-dimensional model (geographic size increases as *square root* of population).
- Two-dimensional model allows to allocate *more than two* SBDs around central zone.
- Thus linear model (possibly) *overestimates* dispersion forces (caused by urban costs) in comparison to agglomeration forces (related to monopolistic competition).

Symbols

- *City population* ℓ – exogenous in short run and endogenous in long run
- *City Pattern (mono/polycentric)* – $m > 0$ or $m = 0$, endogenous
- *Number* of SBDs m – exogenous in short run and endogenous (??) in long run
- *Distance* $||x^S||$ between SBD and CBD – endogenous
- *Shares* of firms id BDs (Central $\theta \in (0, 1]$ and Secondary $(1 - \theta)/m$) – endogenous

Main Feature of CBD

- Putting aside WHY and HOW CBD emerges...
- It possess some specific non-tradable local public goods and business-to-business services: marketing, banking, insurance, etc
- This reflects in additional *communication* costs for SBD firms: $\mathscr{K}(x^S) = K + k \cdot ||x^S||, \ K > 0, \ k > 0$

Workers

Workers' welfare depends on the three goods:

- q_0 – homogenous numéraire good with (endogenous) initial endowment $\bar{q}_0$
- $q(i)$, $i \in [0, n]$ a continuum n of varieties of a horizontally differentiated good under monopolistic competition and increasing returns, using labor as the only input, traded costlessly for price $p(i)$ within the city of origin, trade costs for other cities are $\tau > 0$
- one lot of land area for residence, or housing price (rent) $R(x)$ depend on location. Normalization: one lot of the land $= \pi \approx 3.14159\ldots$

Production

Producing of variety i requires a given number $\varphi > 0$ of labor units (fixed costs) and $c \geq 0$ of numéraire.

- Total Production Costs

$$TPC = \varphi \cdot w + c \cdot q(i)$$

- For firm producing i is located in the CBD:

$$\Pi^C(i) = TR - TPC = I(i) - \varphi \cdot w^C \to \max$$

  where $I(i)$ – firm's net revenue
  (for local sales $I(i) = (p(i) - c) \cdot q(i)$).

- For firm in the SBDs:

$$\Pi^S(i) = I(i) - \mathscr{K}(x^S) - \varphi \cdot w^S \to \max$$

  the firm's revenue is the same as in the CBD

Consumption

For individual working in CBD

$$U(q_0; q(i), i \in [0, n]) \to \max$$

$$s.t. \int_0^n p(i)q(i)\mathrm{d}i + q_0 = w^C + \bar{q}_0 - R^C(||x||) - t||x||$$

$R^C(x)$ – rent prevailing at x

The budget constraint of SBD worker, located at x^S

$$\int_0^n p(i)q(i)\mathrm{d}i + q_0 = w^S + \bar{q}_0 - R^S(||x - x^S||) - t||x\text{-}x^S||$$

Outline

City System and Inter-City Trade

- "World Economy" consists of R regions, separated with physical distance.
- Each region can be urbanized with a single city (mono- or poly-cenric).
- Homogenous numéraire good is costlessly tradable
- Differentiated good: no trade costs for local sales, $\tau > 0$ per unit for intercity trade

Two Cities (R=2)

Assume $R=2$, World population L is given, $\lambda \in [0,1]$ share of first city, $\ell_1 = \lambda L$, $\ell_2 = (1-\lambda)L$. Masses of firms $n_1 = \frac{\lambda L}{\varphi}$, $n_2 = \frac{(1-\lambda)L}{\varphi}$. Consumer problem (1st city):

$$\max U(q_0; q(i), i \in [0, n_1+n_2])$$

$$\int_0^{n_1} p_{11}(i) q_{11}(i) \mathrm{d}i + \int_0^{n_2} p_{21}(j) q_{21}(j) \mathrm{d}j + q_0 = w_1^C + \bar{q}_{01} - R_1^C(x) - t_1 ||x||$$

Profit of firms (1st city):

$$I_1(i) = \lambda L \cdot (p_{11}(i) - c) q_{11}(i) + (1-\lambda) L \cdot [p_{12}(i) - c - \tau] q_{12}(i)$$

Intra-City Equilibrium

- City populations ℓ_r and number of SBDs m_r are given for each region r
- Zero-profit cut-off condition $\Pi_r^C = \Pi_r^S = 0$
- None of workers/firms want to change her choice of job/place (CBD or one of SBD zone and distance from job center)
- Local markets (labor and housing) clear $\Rightarrow$

Mass of firms $n_r = \dfrac{\ell_r}{\varphi}$ and Initial endowment (uniform redistribution) $\bar{q}_{0r} = \dfrac{1}{\ell_r} \int\limits_X R_r(x) dx$

Short Run Inter-City Equilibrium

- "Surviving" condition for City:

$$w_r^C + \bar{q}_{0r} - (R_r^C(||x||) + t||x||) \geq 0$$

- Intra-City equilibria in all (survived) Cities
- Differentiated good markets clear

Long Run Inter-City Equilibrium (R=2)

World population L assumed to be *mobile* across cities. Indirect utilities (real wages) of workers in both cities: $V_1(\lambda)$ and $V_2(\lambda)$ for any given $\lambda \in [0,1]$, $\Delta V(\lambda^*) = V_1(\lambda^*) - V_2(\lambda^*)$. Long run Inter-City equilibrium:

- Core-Periphery type: $\lambda^* = 1$, $\Delta V(1) \geq 0$ or $\lambda^* = 0$, $\Delta V(0) \leq 0$. CP-equilibria are stable when inequalities are strict.
- Interior equilibrium: $0 < \lambda^* < 1$, $\Delta V(\lambda^*) = 0$. It is stable if and only if the slope of the indirect utility differential ΔV is strictly negative in a neighborhood of the equilibrium, i.e., $\frac{\mathrm{d}\Delta V}{\mathrm{d}\lambda}(\lambda^*) < 0$.

Outline

Intra-City Equilibrium: Existence

Let $\theta \in [0,1]$ – share of firms located at CBD, then $\frac{(1-\theta)}{m}$ – share of firms located at single SBD.

Proposition

If city population $\ell \leq \ell^M = \frac{\pi K^2}{(\varphi t - k)^2}$ then there is no polycentric Intra-City Equilibrium (i.e. $m = 0$ is the only possible outcome). Otherwise, for any SBD number m there exists unique Intra-City Equilibrium

Small town cannot bear the polycentric burden!

Intra-City Equilibrium Values

Equilibrium CBD share θ^* is a (unique) solution of

$$(\varphi t - k)\sqrt{\theta\ell} = K + (\varphi t + k)\sqrt{\frac{(1-\theta)\ell}{m}}$$

Radii of CBD zone $r^C = \sqrt{\theta^*\ell}$ and SBD zones $r^S = \sqrt{\frac{(1-\theta^*)\ell}{m}}$

Distance of SBD from CBD: $||x^S|| = \sqrt{\theta^*\ell} + \sqrt{\frac{(1-\theta^*)\ell}{m}}$

Rents:

$$R^C(x) = t \cdot \left(\sqrt{\theta^*\ell} - ||x||\right), \quad \text{for} \quad ||x|| \leq \sqrt{\theta^*\ell}$$

$$R^S(x) = t \cdot \max\left\{0, \sqrt{\frac{(1-\theta^*)\ell}{m}} - ||x^S - x||\right\}, \quad \text{othewise}$$

Endogenous Endowment

Remark

Intra-City Equilibrium relies on various types of urban and production costs and does not depend on trade costs and consumer preferences

Conjecture

Total amount of rent is redistributed uniformly rent among all city population, subsidizing initial endowment of numéraire:

$$\bar{q}_0(\theta^*) = \frac{1}{\ell}\int_X R(x)dx = \frac{t}{3}\cdot\sqrt{\ell}\left[(\theta^*)^{3/2} + \frac{(1-\theta^*)^{3/2}}{\sqrt{m}}\right]$$

Endogenous Urban Costs

$$C_U^C = R^C(x) + t||x|| - \bar{q}_0(\theta^*) \equiv t\sqrt{\theta^*\ell} - \bar{q}_0(\theta^*)$$
$$C_U^S = R^S(x) + t||x - x^S|| - \bar{q}_0(\theta^*) \equiv t\sqrt{(1-\theta^*)\ell} - \bar{q}_0(\theta^*)$$

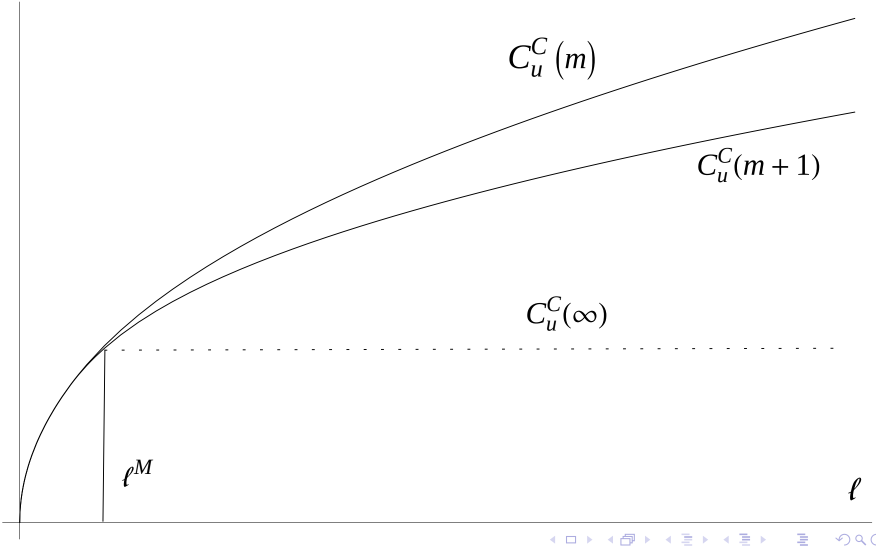
Lemma

In Intra-City equilibrium state

$$w^C - C_U^C = w^S - C_U^S \iff w^C - w^S = C_U^C - C_U^S$$

holds.

Comparative Statics of Urban Costs



Outline

Towards Inter-City Equilibrium

OTT quasi-linear utility for $\alpha > 0$, $\beta > 0$, $\gamma > 0$

$$U(q_0; q(i), i \in [0,n]) = q_0 + \alpha \int_0^n q(i)\mathrm{d}i - \beta \int_0^n [q(i)]^2 \mathrm{d}i - \gamma \left[\int_0^n q(i)\mathrm{d}i \right]^2$$

(studied for *linear cities* in J. Cavailhès et al. (2007)). **Without loss of generality**: $c = 0$!

Lemma

Trade is profitable iff

$$\tau < \tau_{trade} = \frac{2\alpha\beta\varphi}{2\beta\varphi + \gamma L}$$

(see Ottaviano et al. (2002)).

Two types of Inter-City Equilibrium: **Autarchy Equilibrium** and **Trade Equilibrium**

Autarchy Equilibrium

Set of *Intra-City Equilibria* for each city r, satisfying surviving condition

$$w_r^C - C_{Ur}^C \geq 0$$

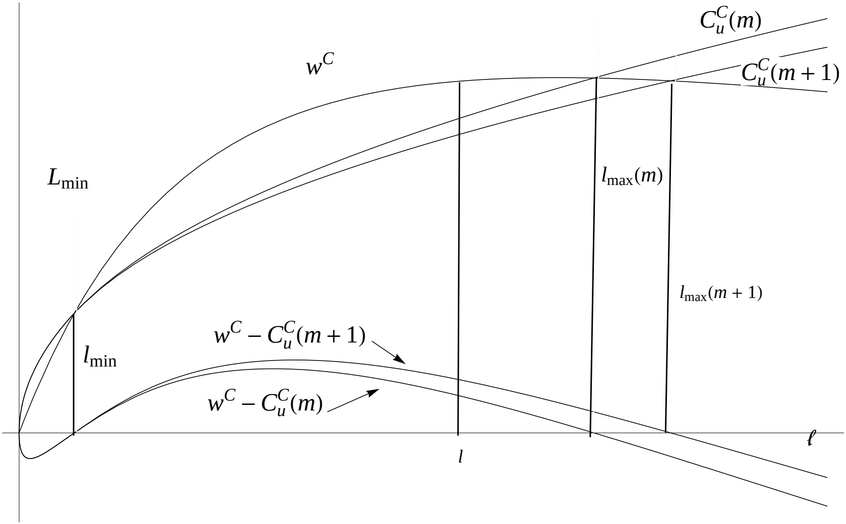
and market of differentiated good is local for each city

Proposition

Under Autarchy there exist lower $\ell_{\min} > 0$ *and upper* $\ell_{\max} < \infty$ *threshold values for city population, i.e. consumers survive in "city jungles" only if* $\ell_{\min} \leq \ell_r \leq \ell_{\max}$.

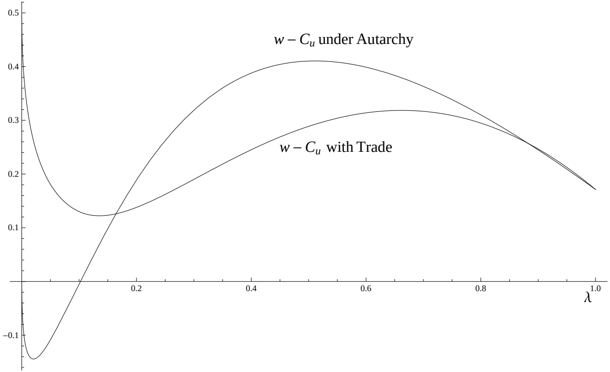
Too small cities can't survive under Autarchy! (without comparative advantages or outer support)

Existence and Comparative Statics



What Changes Trade?

Let $R = 2$, two regions Home (H) and Foreign (F), L – overall population, $\ell_H = \lambda L$, $\ell_F = (1-\lambda)L$ and trade costs τ are sufficiently small, i.e. trade is profitable. Then $\ell_{\min} = 0$!

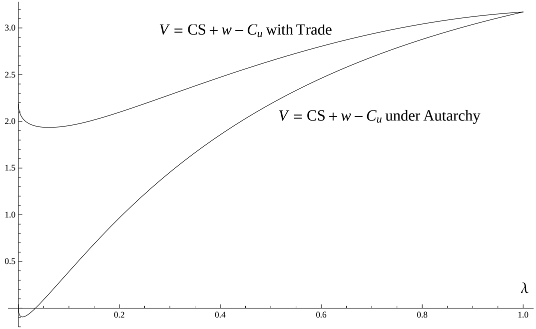


Small cities can survive as satellites of megapolis!

What About Welfare?

Indirect utility (CS – Consumer's Surplus):

$$V = CS + w - C_U$$



Outline

Endogenous Population

Welfare difference $\Delta V(\lambda) = V_H(\lambda) - V_F(\lambda)$ reflects the migration incentives:

- $\Delta V(\lambda) > 0$ – immigration (λ increases)
- $\Delta V(\lambda) > 0$ – emigration (λ decreases)
- *Ad hoc* dynamics $\dot{\lambda} = \lambda \cdot (1 - \lambda) \cdot \Delta V(\lambda)$
- And so on...

Endogenous SBD Number (Generous CD)

Increasing $m \to m+1$

- Gains of increasing

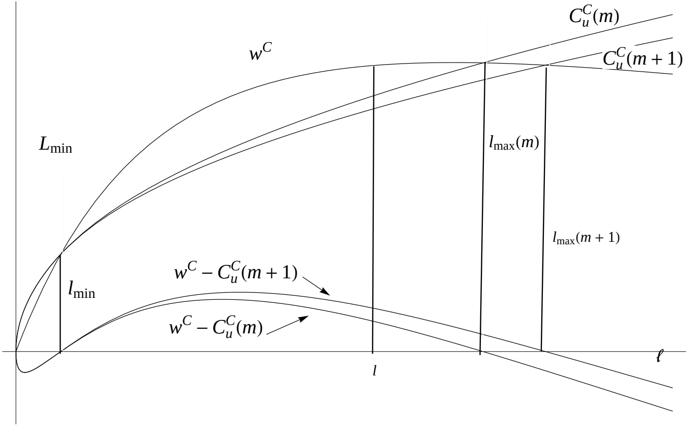
$$\Delta_m(\ell) = (V(\ell, m+1) - V(\ell, m)) \cdot \ell = (C_U(\ell, m) - C_U(\ell, m+1)) \cdot \ell > 0$$

- Costs of city developing: $D > 0$
- $\Delta_m \to 0$, $\dfrac{\partial \Delta_m}{\partial \ell} > 0$, $\dfrac{\partial \Delta_m}{\partial t} > 0$, where t – commuting costs
- $m^* = \max\{m \mid \Delta_m \geq D\}$ – “optimum” number of SBDs

Corollary

Optimum number of subcenters m^* increases with respect to city population ℓ and commuting costs t.

Endogenous SBD Number (Stingy CD)



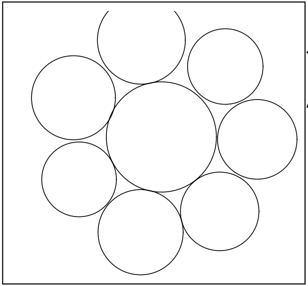
$m \to m+1$ increases City capacity

$m^{**} = \min\left\{m \mid C_U^C(\ell, m) \geq w^C(\ell)\right\}$ increases in population ℓ, yet *decreases* in commuting costs t

Outline

What If City Reach Its Maximum?

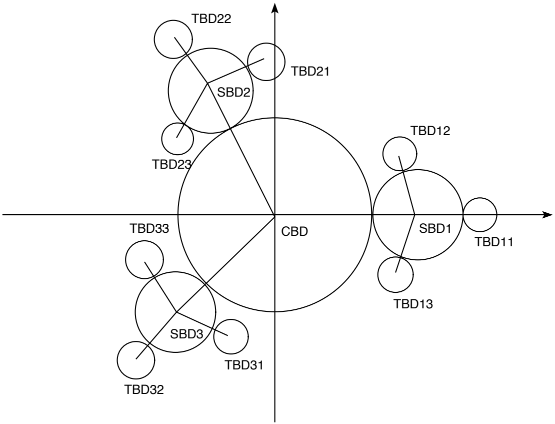
Does It Mean that Moscow Is NON-Rubber?



There exists theoretical maximum of SBDs.
Asymptotically (for large ℓ):

$$M^* \approx \frac{\pi}{\arctan\left(\frac{\varphi \cdot t - k}{2\varphi \cdot t}\right)} \gtrsim \frac{2\pi \cdot \varphi \cdot t}{\varphi \cdot t - k}$$

Hierarchy of Business Districts



$m_0 \equiv 1$ CBD,
$m_1 \geq 1$ SBDs,
$m_2 \geq 1$ TBDs,
$\vdots$

Reduction to Two-Tier Model

Theorem

For any given hierarchy $(m_1, m_2, \dots, m_n)$ *there exists unique hierarchic equilibrium.*
It is **technically equivalent** *to two-tier city equilibrium with an "effective" number of SBDs*
$m_{eff} = m_1(m_2 \cdot \delta^2 + 1)(m_3 \cdot \delta^2 + 1) \dots (m_n \cdot \delta^2 + 1)$, *where*
$\delta = \dfrac{\varphi \cdot t - k}{\varphi \cdot t + k} \in (0, 1)$. *Moreover,* $m_{eff} \geq (1 + \delta^2)^n \to \infty$ *for* $n \to \infty$.

Corollary

It is possible to overcome theoretical maximum M^**!*

That's All, Folks!

Thank You for Attention!