A simple model of economic geography a la Helpmann-Tabuchi *(Y. Murata, J.-F. Thisse 2005)*

Discussion

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Abstract

- This paper explores the interplay between commodities' transportation costs and workers' commuting costs within a general equilibrium framework a la Dixit–Stiglitz. Workers are mobile and choose a region where to work as well as an intraurban location where to live.
- The paper shows that a more integrated economy need not be more agglomerated. Instead, **low transportation costs** lead to **the dispersion** of economic activities.
- This is because workers are able to avoid the burden of urban costs by being dispersed, while retaining a good access to all varieties.

Motivation

- CP model does not fit well contemporary space-economies.
- First, nowadays, the main dispersion force seems to lie in the existence of urban costs (the sum of housing and commuting costs), borne by workers living in large agglomerations, and not in the agricultural sector whose share in employment and expenditure has sharply decreased in most industrialized countries.
- Second, Krugman's model fails to recognize that economic agglomerations typically generate higher costs to be paid by residents when the population size rises. Yet, such costs are unavoidable once agglomeration takes the form of a city.

Precursors

- Tabuchi (1998) is an attempt to unify urban economics a la Alonso (1964) and NEG and allows for the interplay between commuting costs and transportation costs in a spatial economy. Unfortunately, analytical results are available only for the two extreme cases of zero and infinite transportation costs.
- Independently, Helpman (1998) has introduced a housing market into an economic geography model in which all workers are mobile. However, cities have no spatial extension in his setting because Helpman abstracts from commuting costs and, in addition, his treatment of the model is purely numerical.

Findings and Contribution

- A simple model that integrates both transportation and commuting costs when labor is homogeneous and mobile.
- The agglomeration force is in the need to reduce transportation costs of goods, but the main dispersion forces is in the need for workers to commute.

1. Unlike Krugman (1991), agglomeration is a stable equilibrium when transportation costs are large but dispersion prevails when they are low.
2. Second, agglomeration is always a stable equilibrium once commuting costs are sufficiently low.
3. The model is "almost" analytically solvable. (the exact analytical expressions for both the break and sustain points, but not a complete characterization of the set of equilibria)
4. A new effect found: a larger total mass of varieties at the symmetric equilibrium than when agglomeration prevails.

Model (space)

- Two regions ($r = 1, 2$), one industrial sector producing a differentiated product by using labor as its sole input, and two goods (the differentiated product and land).
- A unit mass of identical and mobile workers, a large amount of land in each region.
- Each worker owns one unit of labor. The mass of workers in regions 1 and 2 is respectively given by $L_1 = \lambda$ and $L_2 = 1 - \lambda$.
- An iceberg transportation technology: $T > 1$.
- Land is perfectly immobile. Each region is formed by a linear city: all firms are in CBD ($x = 0$). Workers are equally distributed around the CBD of region r: $[-L_r/2, L_r/2]$.
- Effective labor supply by a worker living at a distance $|x|$ from the CBD is $s(x) = 1 - 2\theta\,|x|$, where $\theta \in [0, 1]$ is the efficiency loss due to commuting.

Model (space) 2

- Effective labor supply in r is $S_r = \int_{-L_r/2}^{L_r/2} s(x)dx = L_r(1 - \theta L_r/2)$.
- The land rent at both city edges are zero. If w_r is the wage in r, the wage net of commuting costs earned by a worker residing at either edge is:
- $s(-L_r/2)w_r = s(L_r/2)w_r = (1 - \theta L_r)w_r$.
- As workers are identical, the wage net of both commuting costs and land rent must be equal across all locations.
- Thus, $s(x)w_r - R_r(x) = s(-L_r/2)w_r = s(L_r/2)w_r$, where $R_r(x)$ is the land rent in region r at a distance $|x|$.
- Then, the equilibrium land rent in region r is $R_r^*(x) = \theta L_r - 2\,|x|\,w_r$.
- The aggregate land rent in r is $ALR_r = \int_{-L_r/2}^{L_r/2} R_r^*(x)dx = \theta L_r^2 w_r/2$.

Model (consumption)

- CES utility: $U_r = \left[\int_{i \in I_1} c_{rr}(i)^{\frac{\sigma-1}{\sigma}} di + \int c_{rs}(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}}$

s.t. $\int_{i \in I_1} p_r(i) c_{rr}(i) di + \int p_s(i) T c_{rs}(i) di = (1 - \theta L_r) w_r + \theta L_r w_r / 2.$

- Then, $c_{rr}(i) = p_r(i)^{-\sigma} P_r^{\sigma-1} (1 - \theta L_r/2) w_r,$

$c_{rs}(i) = p_s(i)^{-\sigma} T^{-\sigma} P_r^{\sigma-1} (1 - \theta L_r/2) w_r,$

$P_r = \left[\int_{i \in I_1} p_r(i)^{1-\sigma} di + \int p_s(i)^{1-\sigma} T^{1-\sigma} di \right]^{\frac{1}{1-\sigma}},$

- Indirect utility is $V_r = \frac{(1-\theta L_r/2) w_r}{P_r}$.
- Adjustment process: $\partial L_r / \partial t = (V_r - \overline{V}) L_r, \overline{V} \equiv V_1 L_1 + V_2 L_2.$

Production

- Labor requirement to produce y_r units of good i is $l_r(i) = F + vy_r(i)$
- Profit $\pi_r(i) = p_r(i)y_r(i) - w_r\left[F + vy_r(i)\right]$,
  where $y_r(i) = c_{rr}(i)L_r + Tc_{sr}(i)L_s$
- $\Rightarrow p_r^*(i) = \frac{\sigma v}{\sigma - 1}w_r = w_r$. { normalization in v}.
- Zero-profit condition $\Rightarrow y_r^* = \sigma F = 1$ {normalization in F}.

Product diversity

Theorem

The more symmetric the spatial distribution of workers, the larger the total mass of varieties in the economy.

Intuition: dispersed economy $\Rightarrow$ lower commuting costs, thus more labor is available for the industrial sector.

More precisely, agglomeration generates two types of costs for the workers: higher urban costs as well as a narrower range of varieties.

Characterization of solution

- Consumer problem gives:

$$c_{rr}(i) = p_r(i)^{-\sigma} P_r^{\sigma-1}(1-\theta L_r/2)w_r,$$
$$c_{rs}(i) = p_s(i)^{-\sigma} T^{-\sigma} P_r^{\sigma-1}(1-\theta L_r/2)w_r,$$

- Market clearing:

$$c_{11}(i)L_1 + Tc_{21}(i)L_2 = 1,\ Tc_{12}(i)L_1 + c_{22}(i)L_2 = 1.$$

- It gives the wage equations:

$$w_1^{\sigma} = P_1^{\sigma-1}S_1 w_1 + T^{1-\sigma}P_2^{\sigma-1}S_2 w_2,\ w_2^{\sigma} = T^{1-\sigma}P_1^{\sigma-1}S_1 w_1 + P_2^{\sigma-1}S_2 w_2,$$

where

$$P_1 = \left[S_1 w_1^{1-\sigma} + S_2(w_2 T)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},\ P_2 = \left[S_1(w_1 T)^{1-\sigma} + S_2 {w_2}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

- Murata (2003) shows there exists unique solution for $\{P_1, P_2, w_1, w_2\}$.
- In general is not solvable analytically, but for special cases $\lambda = \{0, 1/2, 1\}$ may be analyzed.

Symmetric equilibrium

$V_r = \frac{(1-\theta L_r/2)w_r}{P_r}$. Then, $\frac{dV_r}{V_r} = -\frac{\theta L_r/2}{1-\theta L_r/2}\frac{dL_r}{L_r} + \frac{dw_r}{w_r} - \frac{dP_r}{P_r}$.
Then, at $\lambda = 1/2$
wage equations give $(\frac{\sigma}{Z} - 1)\frac{dw_r}{w_r} = (\sigma - 1)\frac{dP_r}{P_r} + \frac{dS_r}{S_r}$.
price indices give $(\frac{1-\sigma}{Z})\frac{dP_r}{P_r} = (1-\sigma)\frac{dw_r}{w_r} + \frac{dS_r}{S_r}$,
where $Z \equiv \frac{1-T^{1-\sigma}}{1+T^{1-\sigma}}$.
Manipulations give $\frac{L_r}{V_r}\frac{dV_r}{dL_r} = \frac{4-2\theta}{4-\theta}\left[\frac{(2\sigma-1)Z}{(\sigma-1)[\sigma(Z+1)-Z]} - \frac{\theta}{2(2-\theta)}\right]$.
If $\frac{dV_r}{dL_r} < 0$ then dispersion eq-m is stable.

Theorem

$\lambda = 1/2$ *is always equilibrium, and it is stable if and only if*

$$\Omega(Z) \equiv \frac{(2\sigma-1)Z}{(\sigma-1)\left[\sigma(Z+1)-Z\right]} < \frac{\theta}{2(2-\theta)} \equiv \Gamma(\theta).$$

Symmetric equilibrium

Theorem

If $\theta \in (0, min\{4/(\sigma+1), 1\})$, $\exists\,!$ T-break point ($T^b(\theta)$): D is a stable equilibrium if and only if $T < T^b(\theta)$. However, $\nexists$ T-break point and D is always stable regardless of T if and only if $\sigma > 3$ and $\theta \in (4/(\sigma+1), 1)$.

When varieties are close substitutes ($\sigma > 3$), the benefit of a better access to all varieties is small. Hence, when θ are sufficiently large ($\theta > (4/(\sigma+1))$, D is always stable, unlike what we observe in the standard CP model.
By contrast, when one of these two conditions does not hold, D may become unstable: despite the fact that the whole range of varieties in the economy shrinks when workers are agglomerated, they benefit from the access to a wider array of local varieties.
When transportation costs are high the net benefit of having all varieties locally produced is sufficiently large to outweigh the higher urban costs that workers must bear by being agglomerated. This is so if varieties are sufficiently differentiated ($\sigma < 3$), or if commuting costs are sufficiently low $\theta < 4/(\sigma+1)$, or both.

Symmetric equilibrium 2

The result may be expressed in terms of commuting cost θ.

Theorem

If $\sigma > 3$, or if both $\sigma \leq 3$ and $T < \overline{T}$, then there exists a unique θ-break point $(\theta^b(T))$ and the symmetric configuration is a stable equilibrium if and only if $\theta > \theta^b(T)$. If both $\sigma \leq 3$ and $T \geq \overline{T}$, there exists no θ-break point and the symmetric equilibrium is always unstable regardless of commuting costs.

$$\overline{T} \equiv \left[\frac{(2\sigma-1)(3-\sigma)}{5\sigma-3}\right]^{\frac{1}{1-\sigma}} \in (1, \infty),$$

$$\theta^b(T) = \frac{4(2\sigma-1)Z}{(\sigma^2+2\sigma-1)Z+\sigma(\sigma-1)},$$

$$T^b(\theta) = \left\{\frac{(2\sigma-1)[1-(\sigma-1)\Gamma(\theta)]}{(2\sigma-1)+(\sigma-1)\Gamma(\theta)}\right\}^{\frac{1}{1-\sigma}}.$$

Agglomeration

- If $\lambda = 1$, then $P_2 = TP_1$, $w_2 = T^{\frac{1-\sigma}{\sigma}} w_1$.
- For $\lambda = 1$, we have $\frac{V_2}{V_1} = \frac{T^{\frac{1-2\sigma}{\sigma}}}{1-\theta/2}$.
- If $\frac{V_2}{V_1} < 1$ then $\lambda = 1$ is stable equilibrium.

Theorem

Agglomeration is a spatial equilibrium and it is stable if and only if $T > (1-\theta/2)^{\frac{\sigma}{1-2\sigma}}$.

Authors provide more results on properties of sustain and break points of agglomeration equilibria. Graphical representation at Fig. 1 at p. 150 JUE 58 (2005).

Other equilibria

It is not possible to characterize whole set of equilibria. $\Rightarrow$ Numerical examples.
Example 1. Let $\theta = 0.200$, $\sigma = 2.5$. Then, $T^s = 1.068$, $T^b = 1.074$. Consider $\{T = 1.09, 1.072, 1.05\}$.

$T = 1.090 > T^b$. Three equilibria: agglomerations are stable, dispersion is unstable,
$T = 1.072$. Five equilibria: three equilibria, corresponding to dispersion or full agglomeration, are stable, other two (mirror) equilibria with partial agglomeration in two cities of unequal size are unstable.
$T = 1.050 < T^s$.The only D - equilibrium with two cities of equal size is stable.

Other equilibria 2

Example 2. Let $T = 1.072$, $\sigma = 2.5$. Then, $\theta^s = 0.205$, $\theta^b = 0.189$.
Consider $\theta = \{0.220, 0.200, 0.180\}$.
$\theta = 0.220 > \theta^s$. The only stable equilibrium involves two cities of equal size.
$\theta = 0.200 \in [\theta^s, \theta^b]$. Five equilibria in which dispersion and full agglomeration are the only ones that are stable.
$\theta = 0.180 < \theta^b$.Full agglomeration is the only stable equilibrium.
See the graphical representation at Fig. 2,3 at pp. 152-153 JUE 58 (2005).

Conclusions

- The paper provided a simple and unified treatment of the interactions between the transportation costs of goods and the commuting costs borne by workers.
- By being agglomerated, workers save on the transportation costs of the differentiated product, but have access to a narrower range of varieties. By contrast, by being dispersed, workers have access to a broader range of varieties, but must then bear the cost of shipping the varieties produced in the other city. The equilibrium outcome shows how the market solves this trade-off.
  Values of workers' welfare are:
  $V^A = (N^A)^{\frac{\sigma}{\sigma-1}}, V^D = (N^D)^{\frac{\sigma}{\sigma-1}} \left(\frac{1+T^{1-\sigma}}{2}\right)^{\frac{1}{\sigma-1}}$
- What really matters for the structure of the space-economy is not just the level of economic integration, but the interplay between transportation costs and urban costs.