RATIONAL INATTENTION

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Nizhnij, 2012

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Rational inattention

- Dynamic stochastic general equilibrium models (DSGE)
- Sticky price, Calvo, 1983
- Sticky information, Mankiw, Reis, 2002
- Rational inattention, Sims, 2003
- Information constrained state-dependent pricing Woodford, 2009.
- Handbook of monetary economics, 2010

How much information in "YES"?

To be or not to be?

Does she love me?

Equally likely "Yes"/"No"

Choice	Yes	No
Prob	1/2	1/2

Pre-supposed "Yes"

Choice	Yes	No
Prob	0.99	0.01

Now answer "Yes" gives too small information. What about "No"?

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Pre-supposed "Yes"

Choice	Yes	No
Prob	0.99	0.01

Now answer "Yes" gives too small information. What about "No"?

"Yes" is very unlikely

Choice	Yes	I would think	
Prob	1/n	1/n	1/n

Intuitive answer

Information from "Yes" depends on the probability of the alternatives.

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Information about SOMETHING is the differences of chances for SOMETHING after and before the experiment

Formally,

 $\mathsf{P}(H_1) \Longrightarrow$ experiment (I ask her, does she love me) $\Longrightarrow \mathsf{P}(H_1|x)$

Bayes rule (the altermative appears)

$$\mathsf{P}\{H_1|x\} = \frac{\mathsf{P}\{x|H_1\}\,\mathsf{P}\{H_1\}}{\mathsf{P}\{x|H_1\}\,\mathsf{P}\{H_1\}+\mathsf{P}\{x|H_2\}\,\mathsf{P}\{H_2\}}$$

Under hypothesis H_i the random variable has distribution $f_i(x)$

$$\log \frac{\mathsf{P}\{H_1|x\}}{\mathsf{P}\{H_2|x\}} = \log \frac{f_1(x)}{f_2(x)} + \log \frac{\mathsf{P}\{H_1\}}{\mathsf{P}\{H_1\}}$$

Idea

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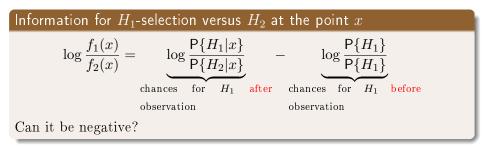
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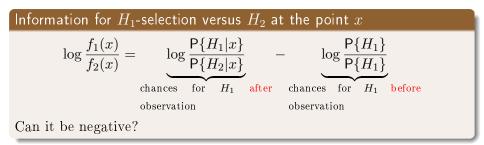
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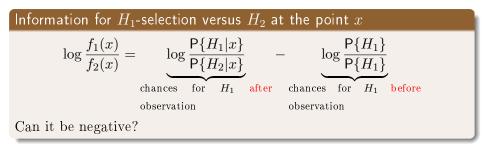
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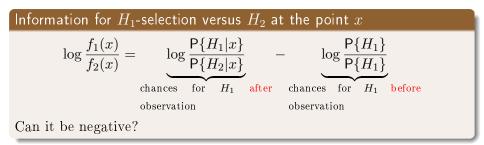
Average info for H_1 vs. H_2

$$I(1:2) = \sum_{x} \left(\log \frac{f_1(x)}{f_2(x)} \right)$$



Average info for H_1 vs. H_2

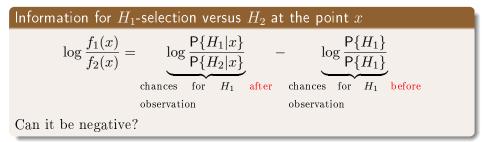
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Average info for H_1 vs. H_2

$$I(1:2) = \sum_{x} \left(\log \frac{f_1(x)}{f_2(x)} \right) f_1(x) = \int f_1(x) \log \frac{f_1(x)}{f_2(x)} dx$$

Definition



Average info for H_1 vs. H_2

$$I(1:2) = \sum_{x} \left(\log \frac{f_1(x)}{f_2(x)} \right) f_1(x) = \int f_1(x) \log \frac{f_1(x)}{f_2(x)} dx$$

Deviation

$$J(1:2) = I(1:2) + I(2:1)$$

Example

$$\mathsf{P}{H_2} = 1, \quad H_1 \subset H_2$$
$$I(1:2) = \log \left(\mathsf{P}{H_1|x}\right) - \log \left(\mathsf{P}{H_1}\right)$$
Let $\mathsf{P}{H_1|x} = 1$. Then $I(1:2) = -\log \mathsf{P}{H_1}$. The information is large, if unconditional probability of H_1 is small.

Entropy

n hypotheses: H_1, H_2, \ldots, H_n for each hypothesis: $(H_i, \overline{H}_i), I(1:2) = -\log \mathsf{P}\{H_i\}$ Average information from an observation:

 $\mathcal{H} = -\sum \mathsf{P}\{H_i\} \log \mathsf{P}\{H_i\}$

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Equally likely Yes/No

$$\begin{array}{|c|c|c|}\hline \mathrm{Yes} & \mathrm{No} \\ \hline 1/2 & 1/2 \\ \hline \mathrm{Units:} & \mathcal{H} = \log_2 2 = 1 \text{ bit or } & \mathcal{H} = \ln 2 \text{ bits} = 1 \text{ nat.} \end{array}$$

Examples

$$\begin{array}{|c|c|c|c|c|}\hline Yes & No \\\hline \hline 0.99 & 0.01 \end{array} & \mathcal{H} = -0.99 \log 0.99 - 0.01 \log 0.01 = 0.08 \text{ bits} \\ I(1:2, Yes) = - P\{Yes\} \log(P\{Yes\}) = 0.0099. \end{array}$$

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Equally likely Yes/No

Examples

x_1	x_2	
1/2	1/2	

$\mathcal{H} = 1$ Shown before

Equally likely four values

x_1	x_2	x_3	x_4
1/4	1/4	1/4	1/4

$\mathcal{H} =$ Who knows?

- $\mathcal{H}(\text{const}) = 0$
- Increase of choices \implies Increase of entropy
- Entropy increases within uncertainty

x_1	x_2	
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$\mathcal{H} = 1$ Shown before

Equally likely four values

x_1	x_2	x_3	x_4	$\mathcal{H} = -4 \times \frac{1}{4} \log \frac{1}{4} = 2$
1/4	1/4	1/4	1/4	

• $\mathcal{H}(\text{const}) = 0$

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$$\mathcal{H} =$$

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Information about X in Y

$$X \stackrel{\text{noise, } N}{\longrightarrow} Y = X + N.$$

 H_2 is always true, $\mathsf{P}{H_2} = 1$

 H_1 : the joint distribution (X, Y) is given by the probability density f(x, y); X = x. Information when observe Y = y:

$$I(1:2, Y = y) = \log \mathsf{P}\{H_1 | Y = y\} - \log \mathsf{P}\{H_1\}$$

Compare with (discussed before)

$$I(1:2) = \log(\mathsf{P}\{H_1|x\}) - \log(\mathsf{P}\{H_1\})$$

Average information in Y about X

$$\begin{split} \langle I(1:2,Y=y)\rangle_{x,y} &= \langle \log \mathsf{P}\{X=x|Y=y\}\rangle_{x,y} - \langle \log \mathsf{P}\{X=x\}\rangle_x \\ \langle \log \mathsf{P}\{X=x,Y=y\}\rangle_{x,y} - \langle \log \mathsf{P}\{Y=y\}\rangle_y - \langle \log \mathsf{P}\{X=x\}\rangle_x \\ &= \int \int f(x,y)\log f(x,y)dxdy - \int h(y)\log h(y)dy - \int g(x)\log g(x)dx = \\ &- \mathcal{H}(X,Y) + \mathcal{H}(X) + \mathcal{H}(y) \end{split}$$

Capacity of the chanel

A noisy telegraph line, the dot or dash input reproduces itself in the output with probability of p:

$$Y = X + N, \quad \mathsf{P}\{Y = X\} = p, \quad \mathsf{P}\{X = 0\} = \alpha$$

Mutual information $I_{\alpha}(X, Y)$ depends on α Capacity is $\max_{\alpha} I_{\alpha}(X, Y)$

Simple algebra gives evidence that I_{α} attains its maximum at $\alpha = 1/2$, the dashs and dots are equally likely. In this case

$$\mathsf{P}\{X = 0, Y = 0\} = \mathsf{P}\{X = 1, Y = 1\} = p/2,$$
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$$C=I=-\mathcal{H}(X,Y)+\mathcal{H}(X)+\mathcal{H}(Y)=p\log p+(1-p)\log(1-p)-4\frac{1}{2}\log\frac{1}{2}$$

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$$C = I = -\mathcal{H}(X, Y) + \mathcal{H}(X) + \mathcal{H}(Y) = p \log p + (1-p) \log(1-p) + 2$$

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If N is normal noise that capacity is attained for normal X

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If N is normal noise that capacity is attained for normal X Can we get full information about X, if N is continious (say, normal) random variable?

Coding

Input $\stackrel{\text{coding}}{\Longrightarrow}$ Equally likely 0s and 1s $\stackrel{\text{chanel}}{\Longrightarrow}$ Output The graph is scanned into a 100×100 grid of pixels The most of symbols are zeros, their fraction is 0.98. If we send symbols one by one, we have to send 1 bit information for each symbol, totally 10000 bits. Coding: 0 represents the sequence 000, $1001 \implies 001$ Coding: 1010 represents the sequence 010, $1011 \implies 011$ Coding: 1100 represents the sequence 100, $1101 \implies 101$ Coding: 1110 represents the sequence 110, $1111 \implies 111$ Then approximately $0.98^3 = 0.94$ of three-pixel blocks are represented by a single 0, 0.06 of them are by four-element sequence. The average number of bits to transfer is

 $(0.94 \times 1 + 0.06 \times 4) \times 10\,000/3 = 3934 < 10\,000$

Entropy of the normal distribution

$$\varphi(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\mathcal{H}(x) = -\int_{-\infty}^{+\infty} \left(\log\left(\frac{1}{\sqrt{2\pi\sigma}}\right) - \frac{(x-\mu)^2}{2\sigma^2} \right) \varphi(x) dx = \log\left(\sqrt{2\pi}\right) + \log\sigma + \frac{1}{\sigma^2} \underbrace{\int_{-\infty}^{+\infty} \frac{(x-\mu)^2}{2} \varphi(x) dx}_{\text{variance}} = \frac{1}{2} \log\left(2\pi\right) + \frac{\log\sigma}{1+1/2} + + \frac{\log$$

2-dimensional vector

$$\mathcal{H}(X,Y) = \log(2\pi) + \left| \log \sigma_x + \log \sigma_y + \frac{1}{2} \log(1-\rho^2) \right| + 1$$

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2-dimensional vector

$$\mathcal{H}(X,Y) = \log(2\pi) + \frac{\log \sigma_x + \log \sigma_y + \frac{1}{2}\log(1-\rho^2)}{1 + 1} + 1$$

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Information, observing normal RV

$$I(X,Y) = -\frac{1}{2} (\log(1-\rho^2))$$

Model

$$Y = X + N$$
, $\mathbf{Cov}(X, N) = 0, (X, Y) \sim \text{normal}, \sigma_x, \sigma_y, \rho$

X is input, Y is output, N is noise; $\rho = \sigma_x / \sigma_y$, $\operatorname{Var} N = \nu^2$

$$I(X,Y) = -\frac{1}{2}\log\left(1 - \frac{\sigma_x^2}{\sigma_y^2}\right).$$

The amount of information (in bits) obtained from observation X

$$I(X,Y) = \frac{1}{2} \log \left(1 + \frac{\sigma_x^2}{\sigma_y^2 - \sigma_x^2} \right) = \frac{1}{2} \log \left(1 + \frac{\sigma_x^2}{\nu^2} \right).$$

Decrease of uncertainty

Observe $X_i + N_i$, $\mathbf{E}N_i = 0$, *m* times and choose the mean of the observations as estimation of X,

$$\mathbf{Var}\left(\frac{X_{1}+N_{1}+X_{2}+N_{2}+\ldots+X_{m}+N_{m}}{m}\right) = \frac{1}{m^{2}}\mathbf{Var}(X_{1}+N_{1}+\ldots+X_{m}+N_{m}) = \frac{\sigma_{x}^{2}+\sigma_{N}^{2}}{m}$$

At cost κ for each bit of information

$$\kappa m \frac{1}{2} \log \left(1 + \frac{\sigma_x^2}{\sigma_N^2} \right)$$

A decision maker is uncertain about some economic variable x. Observing y = x + N as many times as she wishes she obtaines information I about x at a cost and chooses \tilde{x} based on this information:

 $U(\tilde{x},x) \to \max \label{eq:static} I \leqslant I^* \quad \text{information constraint}$

Optimization problem

Autoregression process

$$y_{t} = ay_{t-1} + \varepsilon_{t}, \quad (y_{t}, y_{t-1}) \sim N(\sigma_{t}^{2}, \sigma_{t-1}^{2}, \rho), \mathbf{Var}(\varepsilon_{t}) = \nu^{2}$$
$$\mathbf{Cov}(y_{t}, y_{t-1}) = a\sigma_{t-1}^{2}, \quad \rho = \frac{a\sigma_{t-1}}{\sigma_{t}}$$
$$I(y_{t}, y_{t-1}) = -\frac{1}{2}\log(1 - \rho^{2}) = \frac{1}{2}\log\frac{a^{2}\sigma_{t-1}^{2} + \nu^{2}}{\nu^{2}}$$

Minimization of losses, given an observation of y_t

$$\sum \beta^t \left(\mathbf{E}(y_t - x_t)^2 + \frac{\kappa}{2} \log \frac{a^2 \sigma_{t-1}^2 + \nu^2}{\nu^2} \right) \xrightarrow{x_t} \min$$
$$\sum \beta^t \left(\sigma_t^2 + \frac{\kappa}{2} \log \frac{a^2 \sigma_{t-1}^2 + \nu^2}{\nu^2} \right) \xrightarrow{x_t} \min$$

$$\sum \beta^t \left(\sigma_t^2 \qquad \qquad + \kappa m_t \log \frac{a^2 \sigma_{t-1}^2 + \nu^2}{\nu^2} \right) \xrightarrow{m_t} \min$$

Given yellow constants

$$m_t = \sqrt{C/\kappa}$$

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$$\sum \beta^t \left(\frac{a^2 \sigma_{t-1}^2 + \nu^2}{m_t} + \kappa m_t \log \frac{a^2 \sigma_{t-1}^2 + \nu^2}{\nu^2} \right) \xrightarrow{m_t} \min$$

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School algebra

$$\frac{A}{z} + Bz \stackrel{z}{\longrightarrow} \min$$

Solution:

$$\frac{A}{z^*} = Bz^*, \quad z^* = \sqrt{A/B}$$

Given yellow constants

$$m_t = \sqrt{C/\kappa}$$

$$\sum \beta^t \left(\frac{a^2 \sigma_{t-1}^2 + \nu^2}{m_t} + \kappa m_t \log \frac{a^2 \sigma_{t-1}^2 + \nu^2}{\nu^2} \right) \xrightarrow{m_t} \min$$

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$$m_t = \sqrt{C/\kappa}$$

κ → 0, then m_t → ∞, information is cheap and therefore in use
κ → ∞, then m_t → 0; if integer m_t < 1, the information is not processed.

Let σ_{t-1} and ν be such that $m_t^* < 1$ and $\sigma_t^2 = a^2 \sigma_{t-1}^2 + \nu^2 > \sigma_{t-1}^2$. Then the information is not processed, $x_t = ax_{t-1}$ and the variance increases. The first term $(x_t - y_t)^2$ in the objective is increasing in t, optimal (unconditional) solution m_t^* is increasing in σ_t . The choice x_t deviates from y_t more and more with t so that (since m_t^* increases) the optimal stratege at some moment t' is to collect information.

Dixit-Stiglitz consumption |

$$U = \left(\int_0^1 q^{1-\nu}(i)di\right)^{1/(1-\nu)} \longrightarrow \max$$
$$\int_0^1 p(i)q(i) = I$$

Solution

$$\mathcal{L} = \left(\int_0^1 q^{1-\nu}(i)di\right)^{1/(1-\nu)} - \lambda \int_0^1 p(i)q(i) = I$$

FOC:

$$q(i) = \tilde{\lambda}^{-1/\nu} p^{-1/\nu}(i), \quad \tilde{\lambda} = \lambda U^{-\nu/(1-\nu)}$$

Price index

Substitution of FOC into budget constraint:

$$\tilde{\lambda}^{-1/\nu} P^{-(1-\nu)/\nu} = I, \quad P = \left(\int_0^1 p^{-(1-\nu)/\nu}(i) di\right)^{-\nu/(1-\nu)}$$

The Lagrange multiplier

$$\tilde{\lambda} = I^{-\nu} P^{-(1-\nu)}$$

Optimal consumption

$$q(i) = IP^{(1-\nu)/\nu}p^{-1/\nu}(i)$$



Choose x_t for unknown q_t to get profit

$$\pi_t = \begin{cases} p_t x_t - c \mathbf{q}_t, & \text{if } x_t > q_t; \\ p_t x_t - c \mathbf{x}_t, & \text{if } x_t < q_t \end{cases}$$

Expected profit for optimal $q_t = p_{t-1}^{-1/ u}$ at period t

$$\mathbf{E}\pi_{t} = (p_{t}x_{t} - cq_{t}) \mathsf{P}\{x_{t} > q_{t}\} + (p_{t}x_{t} - cx_{t}) \mathsf{P}\{x_{t} < q_{t}\} = (p_{t}x_{t} - cp_{t-1}^{1/\nu}) \left(1 - \Phi\left(\frac{\log x_{t} - p_{t-1}^{-1/\nu}}{\sigma_{t}\sqrt{m_{t}}}\right)\right) + (p_{t} - c)x_{t}\Phi\left(\frac{\log x_{t} - p_{t-1}^{-1/\nu}}{\sigma_{t}\sqrt{m_{t}}}\right) - \inf_{t} O(t)$$

 Φ is the normal distribution function

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Maximization of profit

$$\sum_{t=1}^{\infty} \left(\left(p_t x_t - c p_{t-1}^{1/\nu} \right) \left(1 - \Phi \left(\frac{\log x_t - p_{t-1}^{-1/\nu}}{\sigma_t \sqrt{m_t}} \right) \right) + (p_t - c) x_t \Phi \left(\frac{\log x_t - p_{t-1}^{-1/\nu}}{\sigma_t \sqrt{m_t}} \right) \right) - \kappa m_t \frac{1}{2} \log \frac{\sigma_t^2}{\nu^2} \xrightarrow{x_t, p_t, m_t} \max$$

subject to

$$\begin{cases} \sigma_t^2 = (\sigma_{t-1}^2 + \nu^2)/m_t, & \text{if } m \ge 1; \\ \sigma_t^2 = \sigma_{t-1}^2 + \nu^2, & \text{if } m = 0 \end{cases}$$

Solution

- $m_t = 0$: price is not adjusted on $[t_1, t_2]$
- 2 $m_t > 1$: price is adjusted on $[t_2, t_3]$, and so on

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Model

- Cost of price adjustment
- Information cost
- Objective: given the distribution of uncertain price, set (probabilistic) strategy of price adjustment

Results

- Stationary distribution of prices (under optimal price review)
- The cost of the information in terms of the firms revenue per time unit.
- The average rate of price review

THANKYOU!

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$$x_t = ax_{t-1}\theta + (1-\theta)y_t + \xi_t$$

random variable ξ_t does not depend on x_t, x_{t-1}, y_t .

$$\mathbf{Var}(ax_{t-1}\theta + (1-\theta)y_t + \xi_t) \stackrel{\theta}{\longrightarrow} \min$$
$$\sigma_t^2 \theta^2 + \sigma_{t-1}^2 (1-\theta)^2 + 2\rho\theta(1-\theta)\sigma_t \sigma_{t-1} \stackrel{\theta}{\longrightarrow} \min$$
$$\theta^* = \frac{\sigma_t^2 - \rho\sigma_{t-1}\sigma_t}{\sigma_{t-1}^2 + \sigma_t^2 - 2\rho\sigma_{t-1}\sigma_t}$$

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