An analytically solvable core-periphery model

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References

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- Krugman, P. R. (1991) Increasing returns and economic geography. *Journal of Political Economy,* 99: 483-499.

Introduction (1)

In the seminal paper by Krugman (1991), a standard new trade model with monopolistic competition a la Dixit and Stiglitz (1977) is modified to analyse industrial location. The outcome is the so called core-periphery (CP) model, which shows how economic integration may lead to a dramatic increase in the geographical concentration of industrial production via a self-reinforcing agglomeration process.

Introduction (2)

The aim is to propose some simple modifications, based on Forslid (1999) and Ottaviano (1996), that make the CP model analytically solvable in the sense that it is possible to derive closed form solutions for the endogenous variables. This allows to assess analytically the number of equilibria as well as their global stability. The additional usefulness of the simplified model is illustrated by deriving new analytical results also in the realistic cases of exogenous asymmetries between regions in terms of both size and trade barriers.

Forslid, R. (1999) Discussion paper 2102, Centre for Economic Policy Research, London.

Ottaviano, G. I. P. (1996) Discussion paper 1327, Centre for Economic Policy Research, London.

The footloose entrepreneur (FE) model

- The economy consists of two regions, 1 and 2
- There are two factors of production, skilled and unskilled labour.
- Each worker supplies one unit of his type of labour inelastically.
- Total endowments are H and L for skilled and unskilled labour respectively so that

$$H_1 + H_2 = H \; ; \; L_1 + L_2 = L$$

Skilled workers can be thought of as self-employed entrepreneurs who move freely between regions, and we will therefore refer to the model as the 'footloose entrepreneur' (FE) model.

The utility function

$$U_i = X_i^{\mu} A_i^{1-\mu}, \tag{1}$$

$$X_i = \left(\int_{s \in N} d_i(s)^{(\sigma-1)/\sigma} ds \right)^{\sigma/(\sigma-1)}, \tag{2}$$

$$\mu \in (0,\ 1) \qquad\qquad \sigma > 1 \qquad\qquad n_1 + n_2 = N.$$

A_i is consumption of agricultural products,

X_i is consumption of manufactures.

The demand by residents in location i for a variety produced in location j :

$$d_{ji}(s) = \frac{p_{ji}(s)^{-\sigma}}{P_i^{1-\sigma}} \mu Y_i,\ i,\ j = \{1,\ 2\} \tag{3}$$

Parameters of the demand functions

P_i is the local CES price index:

$$P_i = \left[\int_{s \in n_i} p_{ii}(s)^{1-\sigma} ds + \int_{s \in n_j} p_{ji}(s)^{1-\sigma} ds \right]^{1/(1-\sigma)}, \quad (4)$$

Y_i is local income consisting of skilled (w_i) and unskilled wages (w_i^L):

$$Y_i = w_i H_i + w_i^L L_i. \quad (5)$$

The budget constraint:

$$\int_{s \in n_i} p_{ii}(s) d_{ii}(s) ds + \int_{s \in n_j} p_{ji}(s) d_{ji}(s) ds + p_i^A A_i = Y_i, \quad (6)$$

Firms in sector *A*

produce a homogenous good under perfect competition and constant returns to scale and employ only unskilled labour.

Without loss of generality, units are chosen so that one unit of output requires one unit of labour. This implies that

$$p_i^A = w_i^L$$

Manufacturing firms (1)

Firms in sector **X** are monopolistically competitive and employ both skilled and unskilled workers under increasing returns to scale. Product differentiation ensures a one-to-one relation between firms and varieties.

In order to produce **x(s)** units of variety **s**,
a firm incurs a fixed input requirement of **α** units
of skilled labour and a marginal input requirement
of **βx** units of unskilled labour.
The total cost of production of a firm, in location **i,**
is thus given by:

$$TC_i(s) = w_i \alpha + w_i^L \beta x_i(s). \qquad (7)$$

Skilled and unskilled labour

Skilled labour market clearing implies that in equilibrium the number of firms is determined by:

$$n_i = \frac{H_i}{\alpha}, \qquad (8)$$

Unskilled workers are perfectly mobile between sectors but spatially immobile and assumed to be evenly spread across regions: $L_i = L/2.$

Good *A* is freely traded, so that its price is the same everywhere

$$p_i^A = w_i^L = 1$$

Manufacturing firms (2)

Trade in sector X is inhibited by frictional trade barriers, which are modeled as iceberg costs: for one unit of the differentiated good to reach the other region, $\tau \in [1, +\infty)$ units must be shipped.

A typical manufacturing firm located in region i maximizes profit:

$$\Pi_i(s) = p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta[d_{ii}(s) + \tau d_{ij}(s)] - \alpha w_i, \quad (9)$$

FOC for maximization

$$p_{ii}(s) = \beta\sigma/(\sigma - 1) \quad , \quad p_{ij}(s) = \tau\beta\sigma/(\sigma - 1) \quad (10)$$

After using (10) the CES price index (4) simplifies to:

$$P_i = \frac{\beta\sigma}{\sigma - 1}[n_i + \phi n_j]^{1/(1-\sigma)}, \quad (11)$$

where $\quad \phi \equiv \tau^{1-\sigma} \in (0, 1]$

is a measure of the freeness of trade

Equilibrium (1)

The total number of X firms is given by

$$n_i + n_j = H/\alpha$$

Due to free entry and exit, there are no profits in equilibrium:

$$\alpha w_i = p_{ii}(s)d_{ii}(s) + p_{ij}(s)d_{ij}(s) - \beta[d_{ii}(s) + \tau d_{ij}(s)]$$

Equilibrium (2)

After using (10):

$$w_i = \frac{\beta x_i}{\alpha(\sigma - 1)}, \qquad (12)$$

where

$$x_i = [d_{ii}(s) + \tau d_{ij}(s)]$$

is total production by a typical firm in location i.

Equilibrium (3)

Using (3), (10) - (12), x_i can be written as:

$$x_i = \frac{\sigma - 1}{\beta \sigma} \left(\frac{\mu Y_i}{n_i + \phi n_j} + \frac{\phi \mu Y_j}{\phi n_i + n_j} \right). \qquad (13)$$

and

$$w_i = \frac{\mu}{\sigma} \left[\frac{Y_i}{H_i + \phi H_j} + \frac{\phi Y_j}{\phi H_i + H_j} \right], \qquad (14)$$

where

$$Y_i = \frac{L}{2} + w_i H_i. \qquad (15)$$

Equilibrium (4)

For $i = 1, 2$, the system consisting of equations (8), (10), (12) - (15) determines the endogenous variables n_i, p_i, w_i, x_i, Y_i for a given allocation of skilled workers H_i.

Plugging (15) into (14) generate:

$$w_i = \frac{(\mu/\sigma)}{1 - (\mu/\sigma)} \frac{L}{2} \frac{2\phi H_i + [1 - (\mu/\sigma) + (1 + (\mu/\sigma))\phi^2]H_j}{\phi(H_i^2 + H_j^2) + [1 - (\mu/\sigma) + (1 + (\mu/\sigma))\phi^2]H_i H_j}. \quad (16)$$

Equilibrium (5)

$$\frac{w_1}{w_2} = \frac{2\phi h + [1 - (\mu/\sigma) + (1 + (\mu/\sigma))\phi^2](1-h)}{2\phi(1-h) + [1 - (\mu/\sigma) + (1 + (\mu/\sigma))\phi^2]h}. \quad (17)$$

where

$$h \equiv H_1/H$$

is the share of skilled workers that reside in region 1.

CP model by Krugman (1991)

The total cost of production of a firm, in location *i,* is given by:

$$TC_i(s) = w_i[\alpha + \beta x_i(s)], \qquad \text{(A1)}$$

$$p_{ij}(s) = w_i \tau \beta \sigma/(\sigma - 1), \quad p_{ii}(s) = w_i \beta \sigma/(\sigma - 1) \qquad \text{(A2)}$$

The firm size: $\qquad x_i = \dfrac{\alpha(\sigma - 1)}{\beta}. \qquad \text{(A3)}$

CP model by Krugman (1991)

Plugging (A1) and (A3) into the skilled labour market clearing condition yields:

$$n_i = \frac{H_i}{\alpha\sigma}. \qquad \text{(A4)}$$

The manufacturing goods market clearing condition can be written as:

$$1 = \frac{w_i^{1-\sigma}\mu Y_i}{w_i^{1-\sigma}H_i + \phi w_j^{1-\sigma}H_j} + \frac{\phi w_i^{1-\sigma}\mu Y_j}{\phi w_i^{1-\sigma}H_i + w_j^{1-\sigma}H_j}. \qquad \text{(A5)}$$

$$Y_i = \frac{L}{2} + w_i H_i \qquad \text{(A6)}$$

Equations (A5) are non-linear in w_1 and w_2.

Equilibrium and stability (1)

Differentiating (17) with reference to h shows that the region with more skilled workers offers a higher (lower) skilled worker wage whenever Φ is larger (smaller) than the threshold:

$$\phi \quad > \quad \phi_w \equiv \frac{1-(\mu/\sigma)}{1+(\mu/\sigma)}. \tag{18}$$

$$\dot{h} \equiv dh/dt = \begin{cases} W(h, \phi) & \text{if } 0 < h < 1 \\ \min\{0, W(h, \phi)\} & \text{if } h = 1 \\ \max\{0, W(h,\phi)\} & \text{if } h = 0 \end{cases} \tag{19}$$

where the current indirect utility differential:

$$W(h, \phi) \equiv \eta\left(\frac{w_1}{P_1^{\mu}} - \frac{w_2}{P_2^{\mu}}\right), \quad \eta \equiv \mu^{\mu}(1-\mu)^{1-\mu}. \tag{20}$$

Equilibrium and stability (2)

$$P_1 = \frac{\beta\sigma}{\sigma - 1} \left(\frac{H}{\alpha}\right)^{1/(1-\sigma)} [h + \phi(1 - h)]^{1/(1-\sigma)} \qquad (21)$$

$$P_2 = \frac{\beta\sigma}{\sigma - 1} \left(\frac{H}{\alpha}\right)^{1/(1-\sigma)} [1 - h + \phi h]^{1/(1-\sigma)} \qquad (22)$$

A spatial equilibrium implies: $\dot{h} = 0.$

If $W(h, \Phi) > 0$, some workers will move from 2 to 1;
if it is negative, some will go in the opposite direction.

Solutions (1)

1. Full agglomeration in either region:

$$h = 0 \quad \text{or} \quad h = 1$$

is a stable spatial equilibrium if trade costs are so small that
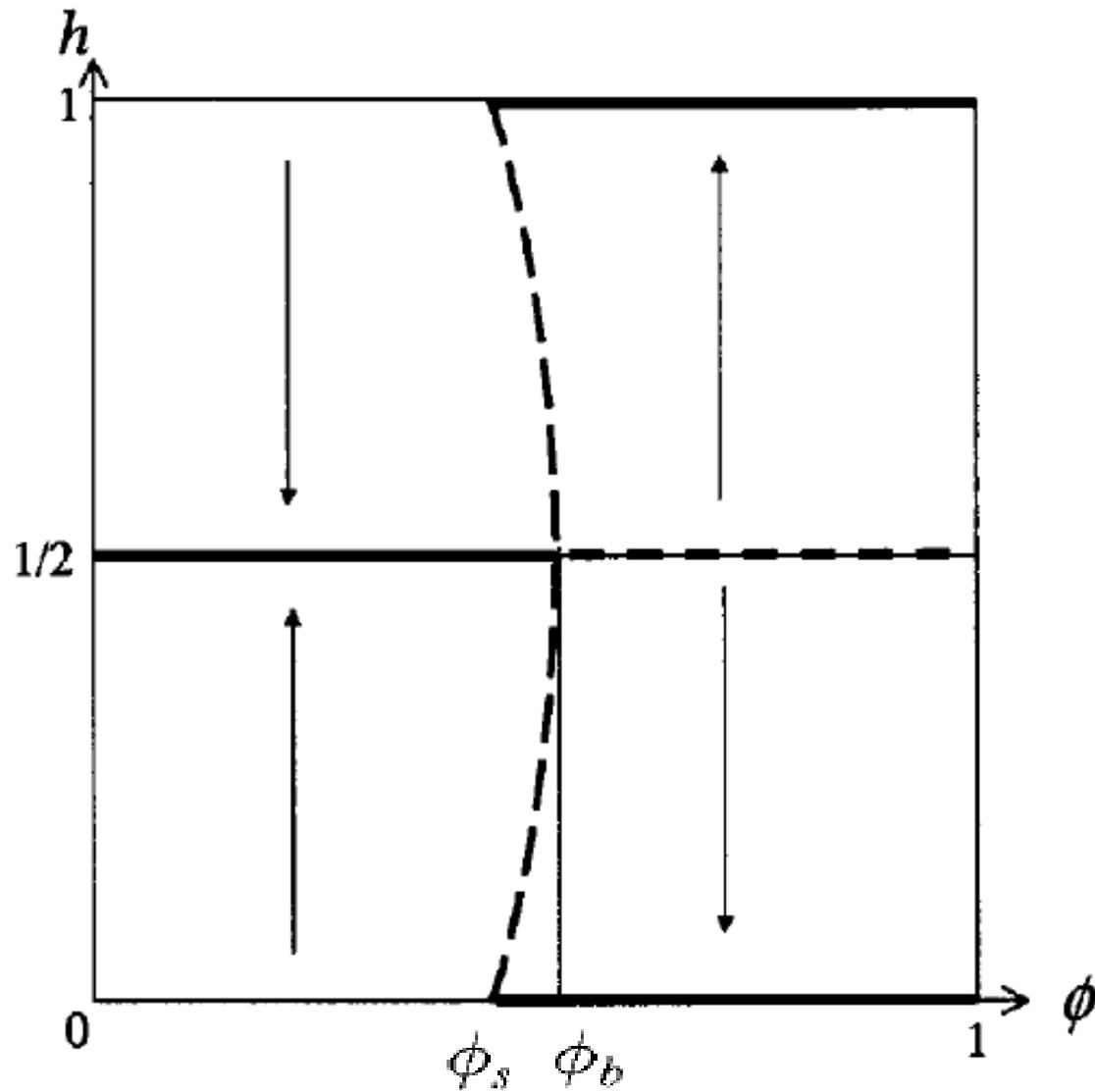
$$\phi \; > \; \phi_s$$

Equation for Φ_s :

$$1 - \frac{\mu}{\sigma} + \left(1 + \frac{\mu}{\sigma}\right)(\phi_s)^2 - 2(\phi_s)^{1+\mu/(1-\sigma)} = 0.$$

Solutions (2)

2. Symmetric solution: $h = 1/2$.
is stable if trade costs are so large that

$$\phi \quad < \quad \phi_b \equiv \phi_w \frac{(1 - 1/\sigma - \mu/\sigma)}{(1 - 1/\sigma + \mu/\sigma)}$$

Bifurcation diagram

Exogenous regional differences

1. Regions differ in terms of market size (region 2 is 'larger' than region 1) :
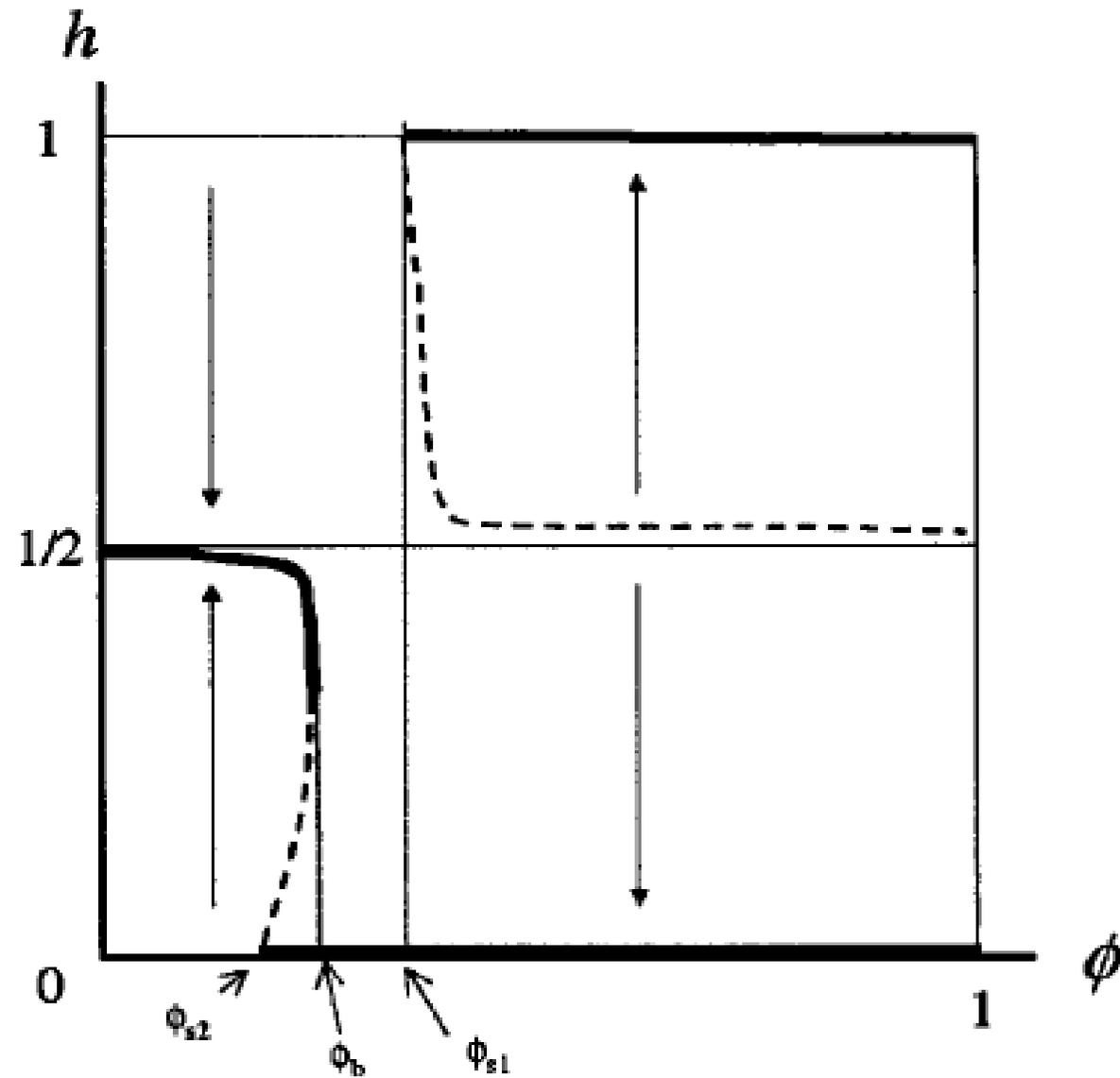
$$L_2 = \varepsilon L_1 \qquad \varepsilon > 1$$

2. Trade costs between regions are asymmetric : $\phi^* = \varepsilon \phi$, $\qquad 0 < \varepsilon < 1.$

ϕ - the freeness of trade from region 2 to region 1;

ϕ^* - the freeness of trade from region 1 to region 2.

Bifurcation diagram

Conclusions

- The paper presents an analytically solvable version of the central 'new economic geography' model, the so called core-periphery model by Krugman (1991)
- Analytical solvability allows to assess the exact number of equilibria and their global stability properties, which is not possible in the original core-periphery model.

Thank you for your attention!