

# **GRAVITY EQUATIONS IN INTERNATIONAL TRADE**

based on Chapter 5 of  
*Advanced international trade: theory and evidence*  
by R. C. Feenstra (2004, PUP)

## Intro: increasing returns to scale and international trade

IRtS as a source of gains from international trade:

- Accessing larger international markets firms could move down their AC curves and gain in efficiency (policy actions example: Canada tariffs policy, 1960s; Canada and US f.t.a., 1989);
- “A puzzle”: under free trade not all firms could expand output that much (as who would buy it?); some firms will leave the market entirely.

Background literature on IRtS & trade:

- **Ohlin**, B. 1933. *Interregional and international trade*. Cambridge: Harvard University Press.
- **Graham**, F. 1923. “*Some Aspects of Protection further Considered.*” *Quarterly Journal of Economics*, 37.
- **Ethier**, W. J. 1982. “*Decreasing Costs in International Trade and Frank Graham’s Argument for Protection.*” *Econometrica*, 50.
- **Eastman**, H. C., **Stykolt**, S. 1967. *The Tariff and Competition in Canada*. Toronto: Macmillan.

## A. Monopolistic competition model

Background literature on Monopolistic Competition, Monopolistic Competition and Trade:

- **Chamberlin**, E. 1936. *The Theory of Monopolistic Competition: a Reorientation of the Theory of Value*. Cambridge: Harvard University Press.
- **Robinson**, J. *The Economics of Imperfect Competition*. London: Macmillan.
- **Lancaster**, K. 1975. “*Socially Optimal Product Differentiation*.” *American Economic Review*, 65.
- **Lancaster**, K. 1979. *Variety, Equity, and Efficiency*. New York: Columbia University Press.
- **Lancaster**, K. 1980. “*Intra-industry Trade under Perfect Monopolistic Competition*.” *Journal of International Economics*, 10.
- **Spence**, M. A. 1976. “*Product Selection, Fixed Costs, and Monopolistic Competition*.” *Review of Economic Studies*, 43.
- **Dixit**, A. K., **Stiglitz**, J. E. 1977. “*Monopolistic Competition and Optimum Product Diversity*.” *American Economic Review*, 67.
- **Helpman**, E. 1981. “*International Trade in the Presence of Product Differentiation, Economics of Scale, and Monopolistic Competition: a Chamberlin-Heckscher-Ohlin Approach*.” *Journal of International Economics*, 11.
- **Krugman**, P. R. 1979. “*Increasing Returns, Monopolistic Competition, and International Trade*.” *Journal of International Economics*, 9.
- **Krugman**, P. R. 1980. “*Scale Economies, Product Differentiation, and the Pattern of Trade*.” *American Economic Review*, 70.
- **Krugman**, P. R. 1981. “*Intra-industry Specialization and the Gains from Trade*.” *Journal of Political Economy*, 89.
- **Helpman**, E., **Krugman**, P. R. 1985. *Market Structure and Foreign Trade*. Cambridge: MIT Press.

# A. Monopolistic competition model

## A1. The model

Consumer's problem:

$$U = \sum_{i=1}^N v(c_i), \quad v' > 0, \quad v'' < 0.$$

$$w = \sum_{i=1}^N p_i c_i.$$

F.O.C.:

$$v'(c_i) = \lambda p_i, \quad i = 1, \dots, N$$

Elasticity of demand for Variety:

$$\eta_i = - \frac{dc_i}{dp_i} \frac{p_i}{c_i} = - \left( \frac{v'}{c_i v''} \right) > 0.$$

*further assuming:*

$$d\eta_i/dc_i < 0$$

# A. Monopolistic competition model

## A1. The model

Technology and producers' behavior:

$$L_i = \alpha + \beta y_i,$$

$$AC_i = wL_i/y_i = w\alpha/y_i + w\beta.$$

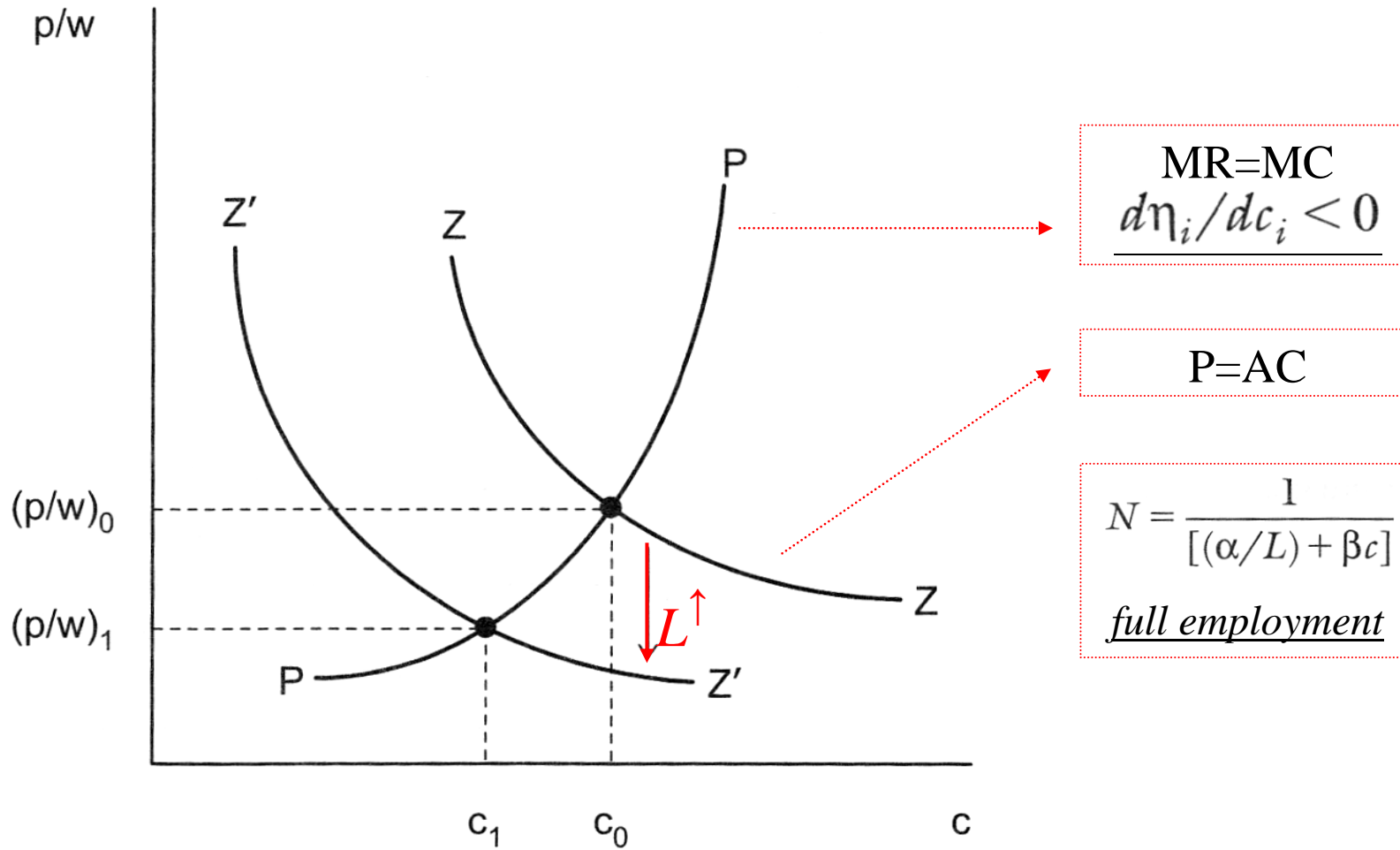
Equilibrium Conditions:

- Profit maximization:  $MR = MC:$   $p\left(1 - \frac{1}{\eta}\right) = w\beta,$  or  $\frac{p}{w} = \beta\left(\frac{\eta}{\eta - 1}\right)$
- Long-run zero profits:  $P = AC:$   $p = \left(\frac{w\alpha}{y}\right) + w\beta$  or  $\frac{p}{w} = \left(\frac{\alpha}{Lc}\right) + \beta$
- Full employment:  $L = \sum_{i=1}^N L_i:$   $= \sum_{i=1}^N (\alpha + \beta y_i) = N(\alpha + \beta y) = N(\alpha + \beta Lc)$

# A. Monopolistic competition model

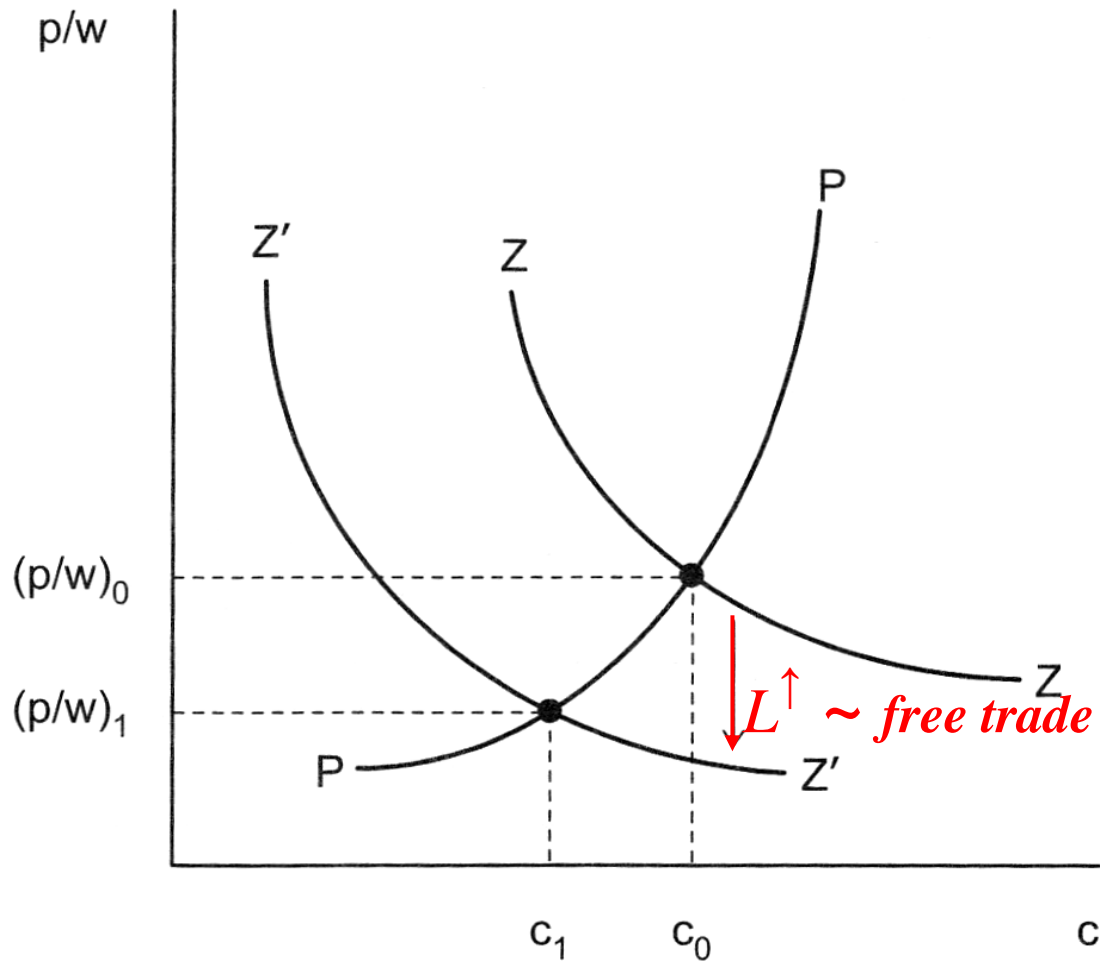
## A1. The model

Equilibrium and Properties:



# A. Monopolistic competition model

## A2. Monopolistic competition and trade



*Two identical countries move from autarky to free trade:*

- under monopolistic competition there's a rationale to trade (competition from abroad lowers AC and price);
- $L \uparrow \rightarrow ZZ \downarrow \rightarrow \left(\frac{p}{w}\right) \downarrow \sim \left(\frac{w}{p}\right) \uparrow \rightarrow c \downarrow$
- $N_{\Sigma} > N_i^{aut}$ , but  $N_i^{f.t.} < N_i^{aut}$   
 $L \uparrow \rightarrow N \uparrow$        $\left(\frac{p}{w}\right) \downarrow \rightarrow y \uparrow$
- countries are completely specialized in different varieties (*intra-industry trade*)

## B. The Gravity Equation

### B1. The equation

The bilateral trade between two countries is directly proportional to the product of the countries' GDPs (thus two countries will trade more with each other if they are rather similar in relative sizes)

- multicountry framework, free trade (i.e. equal prices, normalized to 1), identical and homothetic demand across the world;
- each variety produced in a country is partially consumed domestically sent to all other countries;
- Country  $i$ 's GDP:  $Y^i = \sum_{k=1}^N y_k^i$  ; World's GDP:  $Y^w = \sum_{i=1}^C y^i$ .
- Country  $j$ 's share of World's expenditure:  $s^j = Y^j / Y^w$ .
- Country  $i$ 's export to country  $j$ :  $X_k^{ij} = s^j y_k^i$

$$X^{ij} = \sum_k X_k^{ij} = s^j \sum_k y_k^i = s^j Y^i = \frac{Y^j Y^i}{Y^w} = s^j s^i Y^w = X^{ji}$$

$$X^{ij} + X^{ji} = \left( \frac{2}{Y^w} \right) Y^i Y^j \quad \text{OR} \quad X^{ij} + X^{ji} = 2s^i s^j Y^w$$



## B. The Gravity Equation

### B2. Empirics of gravity: trade within and outside OECD

*Helpman's theorem (1987) and related studies*

Consider a region A consisting of two countries  $\{i, j\}$ :

- the regions' GDP:  $Y^A = Y^i + Y^j$
- country  $i$ 's share in the region:  $s^{iA} = Y^i / Y^A$
- the region's share in the world:  $s^A = Y^A / Y^w$

$$\begin{aligned}(X^{ij} + X^{ji}) / Y^A &= 2s^{iA} s^{jA} s^A \\ 2s^{iA} s^{jA} &= 1 - (s^{iA})^2 - (s^{jA})^2\end{aligned}$$

a generalization

#### Helpman (1987) theorem:

If countries are completely specialized in their outputs, tastes are identical and homothetic, and there's free trade worldwide, then the volume of trade among countries in region A relative to their GDP is

$$\frac{\text{Volume of trade in A}}{\text{GDP}^A} = s^A \left( 1 - \sum_{i \in A} (s^{iA})^2 \right)$$

Size dispersion index

Related studies:

- **Helpman**, E. 1987. "Imperfect Competition and International Trade: Evidence from Fourteen Industrial Countries." *Journal of the Japanese and International economies*, 1.
- **Hummels**, D., **Levinsohn**, J. 1995. "Monopolistic Competition and International Trade: Reinterpreting the Evidence." *Quarterly Journal of Economics*, 110.
- **Debaere**, P. 2002. "Testing 'New' Trade Theory with Testing for Gravity: Re-interpreting the Evidence." University of Texas at Austin, manuscript.

## B. The Gravity Equation

### B3. Empirics of gravity: trade within and outside OECD

*Debaere's study (2002)*

$$\ln\left(\frac{X^{ij} + X^{ji}}{Y^i + Y^j}\right) = \ln(s^i + s^j) + \ln\left[1 - \left(\frac{Y^i}{Y^i + Y^j}\right)^2 - \left(\frac{Y^j}{Y^i + Y^j}\right)^2\right]$$

$$\ln\left(\frac{X_t^{ij} + X_t^{ji}}{Y_t^i + Y_t^j}\right) = \alpha_{ij} + \gamma \ln(s_t^i + s_t^j) + \beta \ln(\text{Dispersion}_t^{ij})$$

## B. The Gravity Equation

### B3. Empirics of gravity: trade within and outside OECD

*Debaere's study (2002)*

$$\ln\left(\frac{X_t^{ij} + X_t^{ji}}{Y_t^i + Y_t^j}\right) = \alpha_{ij} + \gamma \ln(s_t^i + s_t^j) + \beta \ln(\text{Dispersion}_{t}^{ij})$$

	<i>OECD Countries</i>				<i>Non-OECD Countries</i>			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Measure								
of GDP	PWT	IFS	PWT	IFS	PWT	IFS	PWT	IFS
Estimator	OLS	OLS	IV <sup>a</sup>	IV <sup>a</sup>	OLS	OLS	IV <sup>a</sup>	IV <sup>a</sup>
	(a) With constant GDP shares							
ln (Dispersion)	1.01 (0.10)	0.55 (0.04)	1.97 (0.21)	2.10 (0.34)	-2.05 (0.85)	-0.14 (0.20)	-2.30 (1.69)	1.54 (0.71)
R <sup>2</sup>	0.59	0.43	0.58	0.39	0.02	0.12	0.02	0.13
N	1820	1820	1820	1820	1320	1320	1320	1320
	(b) With time-varying GDP shares							
ln (Dispersion)	1.57 (0.11)	0.89 (0.06)	3.28 (0.25)	3.52 (0.24)	-0.96 (0.99)	0.40 (0.24)	-1.43 (1.77)	2.10 (0.73)
ln (s <sub>t</sub> <sup>i</sup> + s <sub>t</sub> <sup>j</sup> )	1.30 (0.13)	0.47 (0.06)	2.54 (0.28)	2.76 (0.26)	1.98 (0.95)	0.99 (0.10)	7.51 (2.83)	4.39 (1.18)
R <sup>2</sup>	0.61	0.45	0.60	0.43	0.02	0.14	0.02	0.14
N	1820	1820	1820	1820	1320	1320	1320	1320

## B. The Gravity Equation

### B4. Empirics of gravity: trade within and between Canada and the US

Related studies:

- **McCallum**, J. 1995. “*National Border Matters.*” *American Economic Review*, 85.
- **Anderson**, J. A., van **Wincoop**, E. 2003. “*Gravity with Gravitas: a Solution to the Border Puzzle.*” *American Economic Review*, 93.
- **Feenstra**, R. C. 2002. “*Border Effects and the Gravity Equation: Consistent Methods for Estimation.*” *Scottish Journal of Political Economy*, 49.

McCallum’s (1995) regression:

$$\ln X^{ji} = \alpha + \beta_1 \ln Y^i + \beta_2 \ln Y^j + \gamma \delta^{ij} + \rho \ln d^{ij} + \varepsilon_{ij}$$

## B. The Gravity Equation

### B4. Empirics of gravity: trade within and between Canada and the US

	<i>McCallum (1995) and Other Samples</i>		<i>Anderson and van Wincoop 2001</i>		<i>With Fixed Effects<sup>a</sup></i>
	(1)	(2)	(3)	(4)	(5)
Year of data	1988	1993	1993	1993	1993
Regions included	CA-CA CA-US	CA-CA CA-US	US-US CA-CA CA-US	US-US CA-CA CA-US	US-US CA-CA CA-US
Independent variables					
$\ln Y^i$	1.21 (0.03)	1.22 (0.04)	1.13 (0.02)	1	1
$\ln Y^j$	1.06 (0.03)	0.98 (0.03)	0.97 (0.02)	1	1
$\ln d^{ij}$	-1.42 (0.06)	-1.35 (0.07)	-1.11 (0.03)	-0.79 (0.03)	-1.25 (0.04)
Indicator Canada	3.09 (0.13)	2.80 (0.14)	2.75 (0.11)		
Indicator U.S.			0.40 (0.05)		
Indicator border				-1.65 (0.08)	-1.55 (0.06)
Border effect Canada <sup>b</sup>	22.0 (2.9)	16.4 (2.0)	15.7 (1.9)	10.5 (1.2)	
Border effect U.S. <sup>b</sup>			1.5 (0.1)	2.6 (0.1)	
Border effect-average <sup>c</sup>			4.8 (0.3)	5.2 (0.4)	4.7 (0.3)
$R^2$	0.81	0.76	0.85	n.a.	0.66
Observations	683	679	1511	1511	1511

# C. Border effects in the Gravity Model

## C1. The equation

Background literature on Gravity Equations under border effects (i.e. unequal prices, price effects):

- **Anderson**, J. A. 1979. “*A Theoretical Foundation for the Gravity Equation.*” *American Economic Review*, 69.
- **Bergstrand**, J. H. 1985. “*The Generalized Gravity Equation, Monopolistic Competition, and the Factor-Proportions Theory in International Trade.*” *Review of Economics and Statistics*, 71.
- **Bergstrand**, J. H. 1989. “*The Gravity Equation in International Trade: Some Microeconomic Foundations and Empirical Evidence.*” *Review of Economics and Statistics*, 67.
- **Baier**, S., **Bergstrand**, J. H. 2001. “*The Growth of World Trade: Tariffs, Transport Costs, and Income Similarity.*” *Journal of International Economics*, 53.
- **Anderson**, J. A., van **Wincoop**, E. 2003. “*Gravity with Gravitas: a Solution to the Border Puzzle.*” *American Economic Review*, 93.
- **Redding**, S., **Venables**, J. V. 2000. “*Economics Geography and International Inequality.*” Center for Economic Policy Research, dp 2568.
- **Rose**, A. K., van **Wincoop**, E. 2001. “*National Money as a Barrier to International Trade: The Real Case for Currency Union.*” *American Economic Review*, 91.
- **Harrigan**, J. 1996. “*Openness to Trade in Manufactures in the OECD.*” *Journal of International Economics*, 40.
- **Hummels**, D. 1999. “*Towards a Geography of Trade Costs.*” Purdue University, manuscript.
- **Feenstra**, R. C. 2002. “*Border Effects and the Gravity Equation: Consistent Methods for Estimation.*” *Scottish Journal of Political Economy*, 49.
- **Wolf**, H. C. 1997. “*Patterns of Intra- and Inter-state Trade.*” NBER wp 5939.
- **Wolf**, H. C. 2000. “*Intranational Bias in Trade.*” *Review of Economics and Statistics*, 82.
- **Helliwell**, J. F., **Verdier**, G. 2001. “*Measuring Internal Trade Distances: A New Method Applied to Estimate Provincial Border Effects in Canada.*” *Canadian Journal of Economics*, 34.

## C. Border effects in the Gravity Model

### C1. The equation

- Border effects (trade or transport costs, tariffs etc.) lead to unequal prices across the world. Accounting for those in the gravity equation like model needs utility function specification.
- Under CES utility function the representative consumer's in country  $i$  problem is

$$\text{to maximize } U^j = \sum_{i=1}^C N^i (c^{ij})^{(\sigma-1)/\sigma} \text{ under budget constraint } Y^j = \sum_{i=1} N^i p^{ij} c^{ij}.$$

- The resulting demand for each product is

$$c^{ij} = (p^{ij}/P^j)^{-\sigma} (Y^j/P^j).$$

$$P^j = \left( \sum_{i=1}^C N^i (p^{ij})^{(1-\sigma)} \right)^{1/(1-\sigma)}$$

- The total value of exports from country  $i$  to country  $j$  is  $X^{ij} \equiv N^i p^{ij} c^{ij}$ .
- The resulting Gravity Equation takes form

$$X^{ij} = N^i Y^j \left( \frac{p^{ij}}{P^j} \right)^{1-\sigma}$$

## C. Border effects in the Gravity Model

### C2. Some aspects of estimation

*Using price index data*

$$X^{ij} = N^i Y^j \left( \frac{p^{ij}}{P^j} \right)^{1-\sigma} \xrightarrow{Y^j = N^i p^i \bar{y}} X^{ij} = \frac{Y^i Y^j}{p^i \bar{y}} \left( \frac{p^{ij}}{P^j} \right)^{1-\sigma} \xrightarrow{p^{ij} = T^{ij} p^i} X^{ij} = \frac{Y^i Y^j}{p^{i\sigma} \bar{y}} \left( \frac{T^{ij}}{P^j} \right)^{1-\sigma}$$

Bergstrand (1985, 1989) and Baier and Bergstrand (2001) regression equation:

$$\Delta \ln X^{ij} = 2\Delta \ln(Y^i + Y^j) + \Delta \ln(s^i s^j) + (1 - \sigma)\Delta \ln T^{ij} - \sigma\Delta \ln p^i + (\sigma - 1)\Delta \ln P^j.$$

Baier and Bergstrand (2001) estimates for 16 OECD countries:

$$\begin{aligned}
 \Delta \ln X^{ij} = & 0.05 + 2.37 \Delta \ln(Y^i + Y^j) + 0.60 \Delta \ln(s^i s^j) \\
 & (0.56) \quad (0.38) \qquad \qquad \qquad (0.34) \\
 & - 4.49 \Delta \ln(1 + \tau^{ij}) - 3.19 \Delta \ln(1 + a^{ij}) \\
 & (1.00) \qquad \qquad \qquad (0.37) \\
 & - 0.68 \Delta \ln Y^j - 0.25 \Delta \ln(p^i/P^j) - 0.08 \ln X_0^{ij}, \\
 & (0.24) \qquad \qquad (0.09) \qquad \qquad \qquad (0.03)
 \end{aligned}$$

$$R^2 = 0.40, N = 240.$$



## C. Border effects in the Gravity Model

### C2. Some aspects of estimation

*Using estimated border effects*

Objections to using price index data to measure  $p^i, P^j$ :

- the indices do not reflect many costs involved in making transactions across borders;
- the indices are measured relatively to a basis period, which may differ for different countries, making it impossible to compare price levels among the countries;

An alternative approach is to model foreign prices as differing from domestic ones due to distance and other factors of border effects:

$$p^{ij} = T^{ij} p^i \quad \text{and} \quad \ln T^{ij} = \tau^{ij} + \rho \ln d^{ij} + \varepsilon_{ij}$$

*Theorem (Anderson, van Wincoop, 2003)*: suppose that the transportation costs are symmetric,  $T_{ij} = T_{ji}$ . Then an implicit solution to the market-clearing condition is,  $\tilde{p}^i = (s^i / N^i)^{1/(1-\sigma)} / \tilde{P}^i$ , in which case the price indexes are solved as

$$(\tilde{P}^j)^{1-\sigma} = \sum_{i=1}^C s^i (T^{ij} / \tilde{P}^i)^{1-\sigma}.$$

Resulting Gravity Equation:

$$X^{ij} = s^i Y^j \left( \frac{T^{ij}}{\tilde{P}^i \tilde{P}^j} \right)^{1-\sigma} = \left( \frac{Y^i Y^j}{Y^w} \right) \left( \frac{T^{ij}}{\tilde{P}^i \tilde{P}^j} \right)^{1-\sigma}$$

## C. Border effects in the Gravity Model

### C2. Some aspects of estimation

*Using estimated border effects*

$$X^{ij} = s^i Y^j \left( \frac{T^{ij}}{\tilde{P}^i \tilde{P}^j} \right)^{1-\sigma} = \left( \frac{Y^i Y^j}{Y^w} \right) \left( \frac{T^{ij}}{\tilde{P}^i \tilde{P}^j} \right)^{1-\sigma}$$

$$\ln(X^{ij}/Y^i Y^j) = \rho(1-\sigma) \ln d^{ij} + (1-\sigma)\tau^{ij} + \ln(\tilde{P}^i)^{\sigma-1} \\ + \ln(\tilde{P}^j)^{\sigma-1} + (1-\sigma)\varepsilon_{ij}.$$

$$\ln(X^{ij}/Y^i Y^j) = \alpha \ln d^{ij} + \gamma(1-\delta^{ij}) + \ln(\tilde{P}^i)^{\sigma-1} \\ + \ln(\tilde{P}^j)^{\sigma-1} + (1-\sigma)\varepsilon_{ij}.$$

## C. Border effects in the Gravity Model

### C2. Some aspects of estimation

*Using fixed effects*

Resolving for unobserved price indexes and constrained optimization for regression estimation needs custom programming. So as to be able performing OLS for the Gravity Equation estimation fixed effects can be used to capture unobserved price indexes influence.

$$\ln(X^{ij} / Y^i Y^j) = \alpha \ln d^{ij} + \gamma(1 - \delta^{ij}) + \beta_1^i \delta_1^i + \beta_2^j \delta_2^j + (1 - \sigma)\varepsilon_{ij}$$

$$\beta_1^i = \ln(\tilde{P}^i)^{\sigma-1}$$

$$\delta_1^i = \begin{cases} 1, & \text{if region } i \text{ is the exporter} \\ 0, & \text{otherwise} \end{cases}$$

$$\beta_2^j = \ln(\tilde{P}^j)^{\sigma-1}$$

$$\delta_2^j = \begin{cases} 1, & \text{if region } j \text{ is the importer} \\ 0, & \text{otherwise} \end{cases}$$