Monopolistic competition: the SDS approach

Ph.Ushchev

July 2012

Imperfect competition and spatial economics

Two crucial ingredients of economic geography models:

- increasing returns
- imperfect competition

Increasing returns is a major *centripetal force* – an incentive for the agents to agglomerate

Constant returns + homogenous space = Starret's impossibility theorem

How to model imperfect competition?

Two basic approaches:

- Monopolistic competition:
  - firms are price-makers because they produce differentiated goods under increasing returns
  - strategic interactions are either absent or weak because the number of firms is large
- Oligopolistic competition:
  - a small number of big agents (firms, local governments, land developers)
  - strategic interactions

Why monopolistic competition?

Oligopoly models

- are difficult to handle
- ignore income effect
- do not allow for endogenous number of firms
- are essentially based on partial equilibrium approach

Assumptions

The *four key assumptions* going back to Chamberlin (1933):

- Firms sell products which are of the same nature but they are not perfect substitutes – the *varieties* of a differentiated good
- Every firm produces a single variety under increasing returns and chooses its price
- The number of firms in the industry is sufficiently large for each of them to be negligible with respect to the rest of the economy
- There is free entry and exit, so profits are zero

Plan

Structure of the economy

The economy involves:

- two production sectors:
  - agriculture – a homogeneous good is produced under constant returns and is sold in a perfectly competitive market
  - industry – firms produce a differentiated good under increasing returns and compete in a monopolistic competition setting
- one production factor – labour

Consumers and their preferences

- The economy is endowed with L identical consumers
- the upper-tier utility is Cobb–Douglas:

$$U = C\, M^{\mu} A^{1-\mu}, \;\; 0 < \mu < 1,$$

where A is consumption of the agricultural good, C is a normalizing constant;

- the lower-tier utility is of CES type:

$$M = \left(\sum_{i=1}^{n} q_i^{\rho} \right)^{1/\rho}, \;\; 0 < \rho < 1,$$

where q_i is consumption of variety i of the manufacturing good, n is the total number of varieties, ρ is an inverse measure of consumers' *love for variety*

Elasticity of substitution

Instead of using ρ, it often proves more convenient to use the parameter σ, which shows the *elasticity of substitution* between varieties

The parameters σ and ρ are related as follows:

$$\rho = \frac{\sigma - 1}{\sigma}, \quad \sigma = \frac{1}{1-\rho}$$

Another representation for the lower-tier utility M:

$$M = \left(\sum_{i=1}^{n} q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1$$

Consumers: budget constraint

Introduce the following notation:

- p_a – the price for the agricultural good
- p_i – the price for the i-th variety of the differentiated manufacturing good
- y – the consumer's total revenue

Then the budget constraint is

$$\sum_{i=1}^{n} p_i q_i + p_a A \leq y$$

Sub-utility maximization

Assume that we already know the expenditure E on the manufacturing good. The distribution of these expenditure between varieties should maximize

$$M = \left(\sum_{i=1}^{n} q_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to

$$\sum_{i=1}^{n} p_i q_i \leq E$$

As the upper-tier utility is Cobb-Douglas, and preferences are identical and homothetic,

$$E = \mu L y$$

Demands for particular varieties

The aggregate demand for variety i:

$$q_i = \frac{p_i^{-\sigma}}{\sum_{j=1}^{n} p_j^{-(\sigma-1)}} E$$

The market share of variety i:

$$s_i = \frac{p_i q_i}{E} = \frac{p_i^{-(\sigma-1)}}{\sum_{j=1}^{n} p_j^{-(\sigma-1)}}$$

As $n \to \infty$, each market share in a symmetric market tends to zero, i.e. each firm is fairly small in comparison to the rest of the economy

Price index

Definition. *Expenditure function* is a function $e(\mathbf{p}, M)$ which maps price vector $\mathbf{p} = (p_1, ..., p_n)$ and utility level into the minimum expenditure yielding utility M under prices $\mathbf{p}$

It is straightforward to see that in our case

$$e(\mathbf{p}, M) = M \left(\sum_{j=1}^{n} p_i^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}$$

As $e(\mathbf{p}, M)$ is *total expenditure* on the manufactured good and M is the *quantity index*, the last term may be interpreted as the *price index* of the manufactured good:

$$P = \left(\sum_{j=1}^{n} p_i^{-(\sigma-1)} \right)^{-\frac{1}{\sigma-1}}$$

Two important properties of the price index

The price index P

- decreases with the number of varieties available
- increases with the degree of product differentiation

Namely, if $p_i \equiv p$, i,e the prices for all varieties are the same, then we get:

$$P = p\, n^{-\frac{1}{\sigma - 1}}$$

Thus, P decreases with the number of firms n and increases with the elasticity of substitution σ, which is a suitable reverse measure of product differentiation

Intuition: *more severe competitive pressure drives prices down, while higher degree of product differentiation makes competition less tough*

Demand functions revisited

Rewrite the demand functions as follows:

$$q_i = \left(\frac{p_i}{P}\right)^{-\sigma} \frac{E}{P}$$

Thus, a firm's demand accounts for

- the own price of a variety produced by a firm
- the aggregate behavior of its competitors via the price index

Lesson: competition in the SDS model is *non-localized*

Real income

Definition. The *indirect utility function* is a function of income and prices which is obtained by substitution of demand functions into utility function

Intuition: the indirect utility is a measure of *consumer's welfare*

Choose the normalizing constant C in the following way:

$$C^{-1} \equiv \mu^{\mu}(1-\mu)^{1-\mu}$$

Then the indirect utility is

$$V = \frac{y}{P^{\mu} p_a^{1-\mu}} \equiv \omega$$

We may interpret ω as the consumer's *real income*

Technology: agriculture

Agriculture

- uses only *unskilled labour*
- is a sector with *perfect competition*
- displays *constant returns to scale*

The price p_a of the agricultural good equals its marginal cost:

$$p_a = w_a m_a$$

Under appropriate choice of measurement units $m_a = 1$. Taking the agricultural good as the numeraire, we get

$$p_a = w_a = 1$$

Technology: manufacturing

In manufacturing

- there are *increasing returns to scale*
- there are *no scope economies*, so each firm produces only one variety
- each variety is produced by *only one firm*, so no relax of price competition is possible

Three alternative modeling strategies:

- labour is homogemous
- labour is heterogenous
- there are two factors of production: labour and capital

Cost functions in manufacturing

Let $f > 0$ stand for fixed costs and $m > 0$ for marginal labour requirement in manufacturing. Then:

- If labour is the only production factor and is homogemous,

$$C(q_i) = fw_a + mw_a q_i = f + mq_i$$

- If labour is the only production factor and is heterogenous,

$$C(q_i) = fw + mwq_i,$$

where w is the wage of skilled labour

- If there are two factors of production – labour and capital,

$$C(q_i) = fr + mwq_i,$$

where r is the interest rate

Profit functions and pricing

Let w be the wage rate of *skilled* labour:

$$\pi_i = p_i q_i - C(q_i) = (p_i - wm)q_i - wf$$

As firms are price-makers, the first order condition which determines the equilibrium price is the *monopoly pricing formula*:

$$p_i \left(1 - \frac{1}{\varepsilon_i}\right) = mw,$$

where ε_i is the price elasticity of demand for variety i

Prices and markups

The price elasticity of demand for variety i is given by

$$\varepsilon_i = -\frac{\partial \ln q_i}{\partial \ln p_i} = \sigma - (\sigma - 1) s_i$$

If the number of firms is sufficiently large and the market is symmetric (i.e. market shares of all firms are approximately the same), we have

$$s_i \approx \frac{1}{n} \to 0, \qquad \varepsilon_i \approx \sigma$$

Thus, the equilibrium price and markup boil down to

$$p^* = \frac{\sigma}{\sigma - 1} mw, \qquad \frac{p^* - mw}{p^*} = \frac{1}{\sigma}$$

Quantities

Due to free entry and exit,

$$\pi_i = 0$$

Combining this with the equilibrium price, we get

$$q^* = \frac{(\sigma - 1) f}{m}$$

Number of firms

The supply of skilled labour should be equal to total labour requirement of manufacturing firms:

$$L = n(f + mq)$$

Hence

$$n^* = \frac{L}{\sigma f}$$

Wages

To this end, we treated income y and expenditure E *as exogenous*.

In order to determine the equilibrium wage of skilled labour w^*, note that

$$y = L_a + wL, \quad E = \mu y = \mu(L_a + wL)$$

Hence the equilibrium wage of skilled labour is given by

$$w^* = \frac{\mu L_a}{(1-\mu)L}$$

NB! The free entry assumption makes the question of how profits are distributed irrelevant, for *under free entry profits are zero*

Welfare

The welfare of an industrial (respectively, agricultural) worker is given by her indirect utilitiy V (respectively, V_a):

$$V = w^* \left[(n^*)^{-1/(\sigma-1)} p^* \right]^{-\mu}$$

$$V_a = \left[(n^*)^{-1/(\sigma-1)} p^* \right]^{-\mu}$$

As $w_a = 1$, we have

$$V > V_a \Leftrightarrow w^* > w_a \Leftrightarrow L_a/L > (1-\mu)/\mu$$

What we want to know?

We now discuss the results of comparative statics with respect to

- reverse degree of product differentiation measured by σ – in order to understand what happens if competition becomes more/less tough
- market size measured by $L + L_a$ – in order to understand consequences of agglomeration and trade liberalization

Comparative statics with respect to the degree of product differentiation

As σ increases , or, equivalently, the degree product differentiation *decreases*

- prices fall, and the model converges to perfect competition as $\sigma \to \infty$
- quantities increase
- the number of firms decreases, for competition becomes tougher
- wage rate of skilled labour remains the same

Comparative statics with respect to the market size

By market size increase we understand a *simultaneous proportional increase* in L and L_a (so that structural effects are eliminated)

As the market size increases

- Prices p^* , output q^* and the wage rate of skilled labour w^* *remain the same* – one of the major weaknesses of SDS approach
- an increase in the number of firms n^* is proportional to the market size increase

Both results run against the data

Why to develop alternative models?

Weaknesses of the SDS approach:

- everything works as if markups were exogenous
- firms' sizes are invariant to competitive environment (the market size, the number of firms), which runs against empirical evidence
- the model developed above is formally *not fully rigorous*, for
  - the number of firms is not necessarily an integer
  - the absence of strategic interactions is essentially based on the idea of *infinitely many firms*, but formally there is only a finite number of producers

The last contradiction can be resolved by assuming the existence of a *continuum* of firms

A continuum of firms

The lower-tier utility function takes the form

$$M = \left(\int_0^n q_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}$$

The price index takes the form

$$P = \left(\int_0^n p_i^{-(\sigma-1)} di \right)^{-\frac{1}{\sigma-1}}$$

The rest of the model is the same as before, but in a new formulation the absence of strategic interactions is *not a mere approximation*

Alternative specifications of preferences

One way to avoid exogenous markups is *to modify preferences*

Two alternative specifications of preferences which proved to be productive:

- linear-quadratic preferences (OTT, 2001):

$$U = A + \alpha \int_0^n q(i)di - \frac{1}{2}(\beta\text{-}\gamma) \int_0^n [q(i)]^2 di - \frac{1}{2}\gamma \left[\int_0^n q(i)di \right]^2, \ \beta > \gamma$$

- unspecified additively separable preferences (ZKPT, 2012):

$$U = V\left(A, \int_0^n u\left(q(i)\right) di \right)$$

where u and V are fairly general

Literature

- Chamberlin, E. (1933). The Theory of Monopolistic Competition. Cambridge, MA: Harvard University Press
- Spence, M. (1976). Product Selection, Fixed Costs, and Monopolistic Competition // The Review of Economic Studies, Vol. 43, No. 2, pp. 217-235
- Dixit, A.K. and J.E. Stiglitz (1977). Monopolistic Competition and Optimum Product Diversity // The American Economic Review Vol. 67, No. 3, pp. 297-308
- Ottaviano, G.I.P., T. Tabuchi and J.-F. Thisse (2002). Agglomeration and Trade Revisited // International Economic Review, Vol.43, pp. 409-436
- Zhelobodko E. et al. (2012). Monopolistic Competition in General Equilibrium: Beyond the CES // Econometrica, forthcoming

Thank you for your attention!