International trade: the DSK approach

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The two forces

Two key forces form the distribution of economic activities across the space:

- increasing returns – a *centripetal* force
- transportation costs – a *centrifugal* force

The Dixit-Stiglitz-Krugman model (DSK) is a *spatial* version of the SDS model, which integrates both forces

Motivation

Why do we need the model?

- Classical trade theories (comparative advantage, factor endowments) fail to explain huge *bilateral trade flows* observed amply in the modern world
- The model provides basic tools for a framework convenient to explain agglomerations of *“core-periphery” type*

Structure of the economy

The economy involves

- two countries named A and B
- agricultural sector, which is identical to the one in the SDS model in both countries
- manufacturing sector, where all firms are identical with technology given by cost function:

$$C(q) = fw + mwq$$

Preferences

The lower-tier utility for country A:

$$M_A = \left\{ \int_{\mathcal{N}_A} [q_{AA}(i)]^{(\sigma-1)/\sigma} \, di + \int_{\mathcal{N}_B} [q_{BA}(i)]^{(\sigma-1)/\sigma} \, di \right\}^{\sigma/(\sigma-1)}$$

Notation:

- $\mathcal{N}_r$ – the set of varieties produced in country r, where $r \in \{A, B\}$
- $q_{sr}(i)$ – consumption level of variety $i \in \mathcal{N}_s$, produced in country s and consumed in country r
- σ – elasticity of substitution, $\sigma > 1$

NB! the expressions for country B are *fully simmetric* to those for country A, so we don't give them further on

Transportation costs

In order not to model transportation sector explicitly, it is assumed that

- agricultural good is traded at zero costs
- costs of manufacturing good transportation are of *iceberg type*

Iceberg costs mean that to deliver q units to the destination, one needs to send τq from the origin, $\tau \geq 1$

Everything works as if the fraction $(\tau - 1)/\tau$ of the freight *melts away* during the transportation (hence the term *iceberg costs*)

Mill pricing strategy

The pricing is as follows:

$$p_{rs}(i) = \tau\, p_r(i),\ r, s \in \{A, B\},\ r \neq s$$

where $i \in \mathcal{N}_r,\ p_r(i) \equiv p_{rr}(i)$ is the *mill price* of variety i in country $r \in \{A, B\}$

Demand functions

The demand functions of a consumer from country A:

$$q_{AA}(i) = \left[\frac{p_A(i)}{P_A}\right]^{-\sigma} \frac{E}{P_A} \quad \forall i \in \mathcal{N}_A$$

$$q_{BA}(i) = \left[\frac{\tau p_B(i)}{P_A}\right]^{-\sigma} \frac{E}{P_A} \quad \forall i \in \mathcal{N}_B$$

where

- E is the total expenditure on manufacturing good
- P_A is the country A price index:

$$P_A = \left\{ \int_{\mathcal{N}_A} [p_A(i)]^{-(\sigma-1)}\, di + \int_{\mathcal{N}_B} [\tau p_B(i)]^{-(\sigma-1)}\, di \right\}^{-1/(\sigma-1)}$$

Incomes of countries

Notation:

- L_a, L – the *total mass* of unskilled (respectively, skilled) workers
- θ_a, θ – the *share* of unskilled (respectively, skilled) workers residing in country A

Total income of country A:

$$Y_A = w_A \theta L + \theta_a L_a$$

Total income of country B:

$$Y_B = w_B(1-\theta)L + (1-\theta_a)L_a$$

Total demand for a variety

The total demand for a variety i produced in region A is given by

$$q_A(i) = \mu p_A(i)^{-\sigma} \left\{ P_A^{\sigma-1} Y_A + \phi P_B^{\sigma-1} Y_B \right\}$$

Here $\phi \in [0, 1]$ is the *spatial discount factor*

$$\phi \equiv \tau^{-(\sigma-1)}$$

Equilibrium

- Prices:

$$p_A^* = \frac{\sigma}{\sigma - 1} w_A m$$

- Quantities:

$$q_A^* = \frac{(\sigma - 1)f}{m}$$

- Number of firms (from the labour balance):

$$n_A^* = \frac{\theta L}{\sigma f}$$

- Price index:

$$P_A = \frac{\sigma m}{\sigma - 1} \left[n_A w_A^{-(\sigma-1)} + n_B (\tau w_A)^{-(\sigma-1)} \right]^{-1/(\sigma-1)}$$

Thank you for your attention!