

# Evolution of the System of Cities under Globalization

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## Stylized facts

### Continuing urbanization is a world trend

Share of people in urban areas	1950	2009	2050 (forecast)
The world	28,8%	50,1%	68,7%
Developed countries	52,6%	74,9%	86,2%
Developing countries	17,6%	44,6%	65,9%

### Importance as economic units

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- creating and development of human and social capital
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Progress of WTO and other international trade agreements

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⇒ declining significance of national governments

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## Challenges for theory

- How and why did some distribution of cities emerge in the economy?
- What are the forces driving this process and what is the relationship between them?
- What can we expect from this distribution in the future?
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## Some references

- Henderson (1974): increasing returns lead to specialized cities
- Tabuchi (1998), Alonso-Villar (2008): bell-shaped curve of agglomeration
- Tabuchi & Thisse (2008): hierarchical structure
- Anas(2004): optimal deurbanization

# Anas (2004)

## Setup

World is **endogenous** number of cities.

Agglomeration forces:

increasing returns in production

love for varieties + transportation costs

Dispersion force:

increasing with a city size congestion

What is the social optimum (second best)?

## Result

The only optimal trend is deurbanization (decreasing city size).

## Anas (2004)

### Possible artefact

Constant elasticity of substitution (CES) utility

### Hypothesis

The result driven by CES function

### Research aim

Extend the model on more general utility function

Examine robustness of Anas (2004) result

# Model description

## Main assumptions

- Endogenous number of cities
- Symmetric position of cities in terms of trade
  - symmetric position in space: located on a big circle trading through center
  - transportation costs do not depend on distance

## Timing

Social planner chooses number of cities to maximize agents utility  
Firms produce and price varieties of consumption good  
Consumers buy consumption good

# System of cities

## Consumer's problem

Maximize additive separable utility function subject to budget constraint:

$$\max \left\{ \iint_{l \in L, j \in J} u(x_{lj}) \, dl dj \right\} \text{ s.t. } \iint_{l \in L, j \in J} p_{lj} x_{lj} \, dl dj \leq (1 - \theta(N))w$$

They supply inelastically unit of labor for working for wage  $w$  and commuting two the work

$\theta(N)$  — congestion costs due to commuting depend on city size



## System of cities

### Consumer's FOC

Assuming interior solution (consumption of all goods) demand function:

$$p_{lj} = \frac{u'(x_{lj})}{\lambda}$$

where  $\lambda$  — Lagrange multiplier of budget constraint,  
i.e. marginal utility of income

i.e. measure of the competition level in the market

Sufficient condition:  $\lim_{x \rightarrow 0} u'(x) = \infty$

# System of cities

## Producers

- Monopolistic competition a-la Dixit-Stiglitz(1977)

Cost function :  $c(y) = (cy + F)w$

- Sell product in all the cities
- Shipping of good to any city but the one of origin requires transportation costs:  
Iceberg type — shipping to the city  $i$  1 unit of good requires sending from city  $j$   $\tau$  units
- Producers maximize total profit knowing demand functions
- There are no entrance barriers

## System of cities

### Producer's problem

Individual producer's problem:

$$\max \left\{ \pi_{li} = (p_{li}^i - cw)Nx_{li}^i + \int_{j \neq i} (p_{li}^j - wct)Nx_{li}^j dj - Fw \right\}$$

### Producer's FOC

$$\frac{\partial \pi_{li}}{\partial x_{li}^i} = N \left( \frac{u'(x_{li}^i)}{\lambda} + \frac{u''(x_{li}^i)x_{li}^i}{\lambda} - cw \right) = 0$$

$$\frac{\partial \pi_{li}}{\partial x_{li}^j} = N \left( \frac{u'(x_{li}^j)}{\lambda} + \frac{u''(x_{li}^j)x_{li}^j}{\lambda} - \tau cw \right) = 0$$

# System of cities

## Equilibrium

Focus only on the symmetric equilibria:

$$x_{li}^i = x^h \quad \text{and} \quad x_{li}^j = x^f \quad \forall l, i, j \neq i$$

## Producer's FOC

Can rewrite as pricing rules:

$$p^h = \frac{CW}{1 - r(x^h)} \quad p^f = \frac{\tau CW}{1 - r(x^f)}$$

here  $r(x) \equiv -\frac{u''(x)x}{u'(x)}$

## System of cities

### Free entry

Free entry condition brings firms to zero profit level:

$$\frac{Nr(x^h)x^h}{1 - r(x^h)} + \frac{(P - N)\tau r(x^f)x^f}{1 - r(x^f)} = \frac{F}{c}$$

### Labor market clearance

Since congestion (commuting) costs are in time:

$$m(cNx^h + c\tau(P - N)x^f + F) = N(1 - \theta(N))$$

here  $m$  — mass of firms in a city (equivalently, mass of varieties)

# System of cities

## Definition of Equilibrium

(Symmetric) trade **equilibrium** in a system with given number of cities

- consumption bundles  $(x^h, x^f)$  solving consumer's problem given prices  $(p^h, p^f, w)$
- corresponding production bundles  $(Nx^h, (P - N)x^f)$  solving producer's problem given  $\lambda$  and granting zero profit
- mass  $m$  of varieties producing in every city granting labor market clearing

## Social optimum (second best)

Social planner maximizes equilibrium utility of representative agent as a function of  $N$  and  $P$ :

$$V(P, N) = m(P, N)(u(x^h(P, N)) + (P/N - 1)u(x^f(P, N))) \rightarrow \max_N$$

Solving this problem she gets optimal city size and number of cities given population of the system:

$$N^* = N^*(P) \quad \text{and} \quad n^*(P) = \frac{P}{N^*(P)}$$

Achievable with help of competitive developers

## Internal structure of a city

### City

Monocentric circle

### Dwellers

- rent unit of land for renting
- divide unit of time between working and commuting to CBD

### Rent

is collected by municipal officials and divided equally between dwellers

### Stability

No costs of relocation within city  $\Rightarrow$   
Income and utility is the same in any point of a city



## Internal structure of a city

### Formally

- individual labor supply  $H(r) = 1 - sr$   
with  $s$  — commuting time costs per unit of distance
- city size  $\bar{r} = \sqrt{N/\pi}$
- aggregate labor supply  $H = \int_0^{\bar{r}} 2\pi r H(r) dr = N(1 - kN^{1/2})$
- normalization  $k \equiv 2s/3\sqrt{\pi}$
- individual income  $I = (1 - kN^{1/2})w$  or  $\theta(N) = kN^{1/2}$

## Comparative statics

### Increasing city size

Variable	$r'(x) < 0$	$r'(x) = 0$ (CES)	$r'(x) > 0$
$x^h, x^f$	↓	↓	↓
$p^h, p^f$	↑	0	↓
$m$	↑	↑	↑

Here  $r(x) \equiv -\frac{\partial p}{\partial x} \frac{x}{p} \equiv -\frac{u''(x)x}{u'(x)}$ , and  $r_{u'}(x) \equiv -\frac{u'''(x)x}{u''(x)}$

# Comparative statics

## Decreasing transportation costs

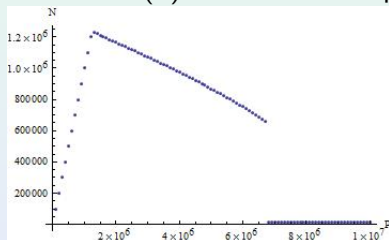
Variable	$r'(x) < 0$	$r'(x) = 0$ (CES)	$r'(x) > 0$
$x^h$	↓	↓	↓
$x^f$	↑	↑	↑
$p^h$	↑	0	↓
$p^f$	↓	↓	↓

## Social optimum

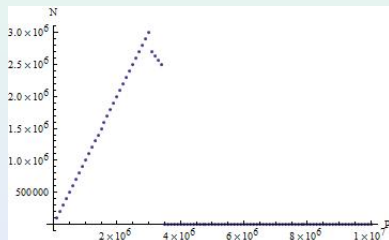
Start from example  $u(x) = x^\rho + bx$

$b < 0 \Rightarrow r'(x) > 0$  — pro-competitive effect

$b > 0 \Rightarrow r'(x) < 0$  — anti-competitive effect



$\rho = 0.9, b = -1$



$\rho = 0.9, b = 0.5$

## Limiting social optimum

### Theorem: conditions

Let  $u(x)$  satisfy following condition:

- (i)  $r_{u'}(x) \leq a_1 < 2$ , i.e.  $r_{u'}(x)$  separated from two;
- (ii)  $r(x) \leq a_2 < 1$ , i.e.  $r(x)$  separated from one;
- (iii) For any city trade is beneficial, i.e. utility in the best autarchy equilibrium is lower than in trade equilibrium for any  $N_{min} \leq N \leq N_{max}$ .

### Theorem: corollary

$\forall \bar{N} \ N_{min} \leq \bar{N} \leq N_{max}$  there exist  $\bar{P}$  such that  $\frac{\partial V}{\partial N}(\bar{N}, P) < 0$   
 $\forall P > \bar{P}$ .

In other words, with sufficiently high population of the system dispersion equilibrium is stable

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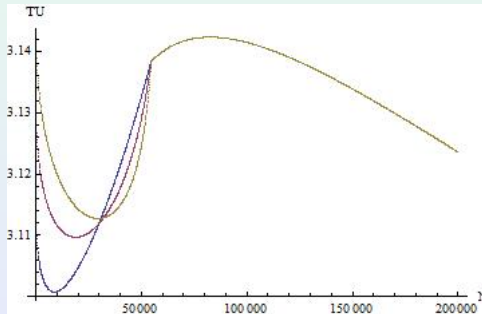
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## Corner solution (necessity of condition (iii))

Another example  $u(x) = \ln(1 + x)$

Finite derivative in zero  $\Rightarrow$  possible cut off from the  
"foreign" markets



Utility of representative dweller  
depending on the city size

# What is Missing?

Model structure suggested by Anas does not allow for developing agglomerations

Our results call for changes

Possible directions of extension:

- addition of the spacial structure with immobile second sector
- addition of the economies of scale on city (not firm) level



Thank You for Your  
Attention!