Multi-product firms under monopolistic competition: the choice of scope

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Stylized facts about multi-product firms

- Multi-product firms account for the most part of industrial output;
- Intensive margins and extensive margins of industrial firms are positively correlated:
  - Bernard, A.B., S.J.Redding and P.K.Schott (2010)
  - Goldberg, P., A. Khandewal, N. Pavnik and P. Topalova (2008)
- There is positive correlation between the firm's size and the efficiency its of R&D projects:
  - Henderson, R. and I. Cockburn (1996)
  - Cockburn, I. and R. Henderson (2001)

Theoretical literature on multi-product firms

- Ottaviano G.I.P. and J.F. Thisse (1999)
- Allanson, P. and C. Montagna (2005)
- Nocke, V. and S. Yeaple (2006)
- Feenstra, R. and H. Ma (2007)
- Eckel, C. and J.P. Neary (2010)

Questions we are trying to answer

- Do large markets necessarily exacerbate product diversity?
- Could market structure depend on supply side characteristics?
- Is cannibalization effect inevitable?

Plan

Commodities and market structure

- There is a continuum of firms of measure N.
- Each firm j, $j \in [0, N]$, chooses:
  - its product line scope n_j;
  - its production plan $\mathbf{q}_j : [0, n_j] \to \mathbb{R}_+$.
- Products are assumed to be horizontally differentiated across firms as well as within the product lines of the firms.
- Each variety is produced by a single firm; consequently, firms are price makers.

Consumers

- The economy is endowed by L identical consumers, each of whom forms her individual demands x_{ij} in order to maximize her utility function:

$$\mathscr{U} = \int_0^N \int_0^{n_j} u(x_{ij})\, di\, dj,$$

subject to the budget constraint:

$$\int_0^N \int_0^{n_j} p_{ij} x_{ij}\, di\, dj \leq 1.$$

- The function u is assumed to be increasing, concave and thrice differentiable.

Inverse demand functions

- Solving the consumer's problem, we obtain the inverse demand functions:

$$p_{ij} = \frac{u'(x_{ij})}{\lambda}.$$

- λ is a Lagrange multiplier, which can be treated as some aggregate market statistics.
- NB!! As there is a continuum of firms, the individual influence of each firm on λ is negligible.

Producers

- Each firm incurs costs of three types:
  - fixed costs F;
  - variable costs $\mathscr{V}(\mathbf{q}, n)$.
- The variable cost functions $\mathscr{V}$ is convex in $\mathbf{q}$ and satisfies the *anonimity condition*:

$$\mathscr{V}(\mathbf{q}, n) = \mathscr{V}(\mathscr{S}_j \mathbf{q}, n) \, \forall \mathbf{q}, n,$$

where $\mathscr{S}_j$ is defined as follows:

$$(\mathscr{S}_j \mathbf{q})_i = \begin{cases} q_{i+j}, & \text{if} i + j \le n, \\ q_{i+j-n}, & \text{if} i + j > n. \end{cases}$$

- Firms seek to maximize their profits

$$\widetilde{\Pi}(\mathbf{q}, n) = \frac{1}{\lambda} \int_0^n u'(q_i/L)\, q_i di - F - \mathscr{V}(\mathbf{q}, n)$$

- Due to anonimity and $r_{u'} < 2$, we can reformulate the firm's problem in symmetrized terms:

$$\max \Pi(y, n) = \frac{1}{\lambda} u'\left(\frac{y}{nL}\right) y - F - V(y, n)$$

- Here $y = \int_0^n q_i di$ is firm's total output, V is the symmetrized cost function:

$$V(y, n) = \mathscr{V}(\mathbf{q}, n)|_{\mathbf{q} \equiv y/n}.$$

- We assume V to be increasing, twice continously differentiable, convex and strictly quasi-convex.

Scale-scope spillovers

- Empiricists find positive correlation between the firm's size and the efficiency its of R&D projects.
- So, we consider variable cost functions $V(y, n)$ which exhibit scale-scope spillovers.
- Call V_y the *production marginal costs* (or *y-marginal costs*) and V_n the *scope marginal costs* (or *n-marginal costs*).

Definition

We say that the technology exhibits *scale-scope spillovers* (or *positive scope externality*) if y-marginal costs decrease with respect to scope, or, equivalently, if n-marginal costs decrease with respect to total output y. Formally:

$$V_{yn} \leq 0.$$

Equilibrium

Definition. An *equilibrium* is a quintuple

$$\left(\{n_j^*\}_{j\in[0,N]}, \{\mathbf{q}_j^*\}_{j\in[0,N]}, \{\mathbf{p}_j^*\}_{j\in[0,N]}, N^*, \lambda^*\right)$$

such that:

- $x_{ij}^* = q_{ij}^*/L$ maximizes consumer's utility under prices $p_{ij} = p_{ij}^*$;
- λ^* is the Lagrange multiplier in the consumer's problem;
- n_j^* and $\mathbf{p}_j^*$ maximize profit of firm j conditional on $\lambda = \lambda^*$ and the inverse demand functions;
- free entry condition and labour balance hold.

Symmetric equilibrium

Definition. We call an equilibrium *symmetric* if:

- equilibrium prices are the same, both across firms and across varieties;
- equilibrium quantities are the same, both across firms and across varieties;
- equilidrium scopes are the same across firms.

Existence and uniqueness

Proposition

Assume that there exists some $\varepsilon > 0$ *such that* $\varepsilon < r_u(x) < 1 - \varepsilon \; \forall x \geq 0$. *Then:*

(a) no asymmetric equilibria exist;

(b) at least one symmetric equilibrium exists;

(c) if $r_u'(x) > 0 \; \forall x > 0$, *the equilibrium is unique.*

Symmetric equilibrium conditions

Monopoly pricing condition:

$$p = \frac{V_y}{1 - r_u}.$$

Free entry:

$$py = F + V(y, n).$$

Labour balance:

$$L = N\left(F + V(y, n)\right).$$

The "unit elasticity" condition (follows from zero profit and producer's FOC):

$$\frac{V_y\, y}{F + V(y, n)} + \frac{V_n\, n}{F + V(y, n)} = 1.$$

Elasticities of marginal costs

- The key-factor of the market outcome is the behavior of $r_u(x)$, which is the inverse demand elasticity.
- By analogy, we introduce marginal costs elasticities.
- But complications arise, for we have *two different marginal cost functions*: the y-marginal costs and the n-marginal costs.
- So, we have *four* marginal costs elasticities:
  - the y-elasticity of y-marginal costs $\dfrac{y\,V_{yy}}{V_y}$;
  - the n-elasticity of y-marginal costs $\dfrac{n\,V_{yn}}{V_y}$;
  - the y-elasticity of n-marginal costs $\dfrac{y\,V_{ny}}{V_n}$;
  - the n-elasticity of n-marginal costs $\dfrac{n\,V_{nn}}{V_n}$.

Comparative statics of q^*, p^* and n^*N^*

Proposition

The output of a specific variety q^*, the market price p^* and the total mass of varieties n^*N^* respond to an increase in market size according to three patterns, depending only on the RLV behavior:

RLV behavior	case $r_u' > 0$	case $r_u' = 0$	case $r_u' < 0$
$\mathcal{E}_{q/L}$	$0 < \mathcal{E}_{q/L} < 1$	$\mathcal{E}_{q/L} = 0$	$\mathcal{E}_{q/L} < 0$
$\mathcal{E}_{p/L}$	$-r_u < \mathcal{E}_{p/L} < 0$	$\mathcal{E}_{p/L} = 0$	$\mathcal{E}_{p/L} > 0$
$\mathcal{E}_{nN/L}$	$0 < \mathcal{E}_{nN/L} < 1$	$\mathcal{E}_{nN/L} = 1$	$\mathcal{E}_{nN/L} > 1$

Comparative statics of firm's total output y^*

Proposition

The firm's total output y^* responds to a market size increase according to the following nine patterns:

Costs behavior	RLV behavior		
	case $r_u' > 0$	case $r_u' = 0$	case $r_u' < 0$
case $\frac{V_{nn}n}{V_n} + \frac{V_{ny}y}{V_n} > 0$	$\mathcal{E}_{y/L} > 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} < 0$
case $\frac{V_{nn}n}{V_n} + \frac{V_{ny}y}{V_n} = 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} = 0$
case $\frac{V_{nn}n}{V_n} + \frac{V_{ny}y}{V_n} < 0$	$\mathcal{E}_{y/L} < 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} > 0$

Comparative statics of firm's scope n^*

Proposition

The firm's scope n^* responds to a market size increase according to the following nine patterns:

Costs behavior	RLV behavior		
	case $r_u' > 0$	case $r_u' = 0$	case $r_u' < 0$
case $\frac{V_{yy}y}{V_y} + \frac{V_{yn}n}{V_y} > 0$	$\mathcal{E}_{n/L} < 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} > 0$
case $\frac{V_{yy}y}{V_y} + \frac{V_{yn}n}{V_y} = 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} = 0$
case $\frac{V_{yy}y}{V_y} + \frac{V_{yn}n}{V_y} < 0$	$\mathcal{E}_{n/L} > 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} < 0$

Comparative statics of the mass of firms N^*

Proposition

The mass of firms N^* responds to a market size increase according to the following nine patterns:

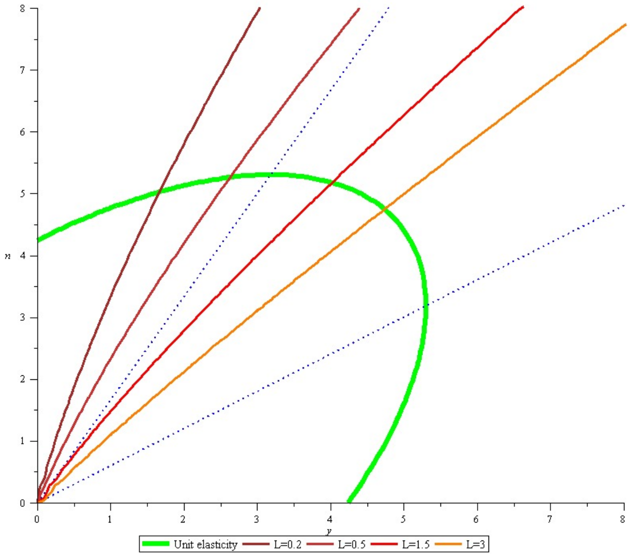
Costs behavior	RLV behavior		
	case $r'_u > 0$	case $r'_u = 0$	case $r'_u < 0$
case $\frac{V_{yy}y}{V_y} + \frac{V_{yn}n}{V_y} > \frac{V_{nn}n}{V_n} + \frac{V_{ny}y}{V_n}$	$\mathcal{E}_{N/L} > 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} < 1$
case $\frac{V_{yy}y}{V_y} + \frac{V_{yn}n}{V_y} = \frac{V_{nn}n}{V_n} + \frac{V_{ny}y}{V_n}$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$
case $\frac{V_{yy}y}{V_y} + \frac{V_{yn}n}{V_y} < \frac{V_{nn}n}{V_n} + \frac{V_{ny}y}{V_n}$	$\mathcal{E}_{N/L} < 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} > 1$

No cannibalization: an example

- Consider linear-quadratic variable costs with an interaction term:

$$V(y,n) = \frac{y^2}{2} + \frac{n^2}{2} - \gamma yn + \alpha y + \beta n.$$

- Here $\alpha, \beta \geq 0$, $0 < \gamma < 1$.
- Take $\alpha = 15.89$, $\beta = 4$, $\gamma = 0.6$, $F = 9$.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Unit elasticity
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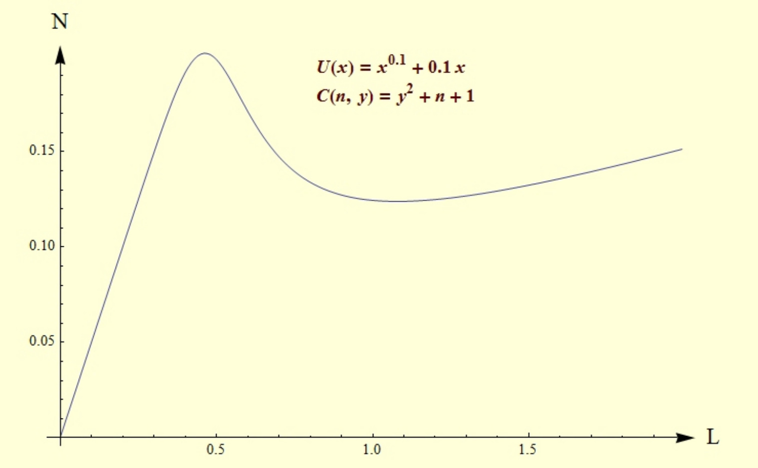
$\mathcal{E}_{N/L} < 0$: an example



Special case 1: no scale-scope spillover

Each firm incurs costs of three types:

- fixed costs F;
- R&D costs (or monitoring costs) $S(n)$, where n is the scope;
- variable production costs $V(y)$, where $y = \int_0^n q_i di$ is total output.

The total costs are $F + V(y) + S(y)$.

The salient feature of this case: *no spillover effect.*

Comparative statics of firm-level variables

Proposition

The average output q^*, the market price p^* and the total mass of varieties $n^* N^*$ respond to an increase in market size according to the following three patterns, depending only on the RLV behavior:

RLV behavior	$r'_u > 0$	$r'_u = 0$	$r'_u < 0$
$\mathscr{E}_{p/L}$	$-r_u < \mathscr{E}_{p/L} < 0$	$\mathscr{E}_p = 0$	$\mathscr{E}_p > 0$
$\mathscr{E}_{y/L}$	$0 < \mathscr{E}_{y/L} < 1$	$\mathscr{E}_{y/L} = 0$	$\mathscr{E}_{y/L} < 0$
$\mathscr{E}_{q/L}$	$0 < \mathscr{E}_q < 1$	$\mathscr{E}_q = 0$	$\mathscr{E}_q < 0$
$\mathscr{E}_{n/L}$	$-1 < \mathscr{E}_{n/L} < 0$	$\mathscr{E}_n = 0$	$\mathscr{E}_n > 0$

Comparative statics of industry-level variables

Proposition

The scope n^*, the total output of a firm y^* and the total output in the industry $Y^* = N^* y^*$ respond to an increase in market size according to the following three patterns:

RLV behavior	$r_u' > 0$	$r_u' = 0$	$r_u' < 0$
$\mathcal{E}_{n/L}$	$-1 < \mathcal{E}_{n/L} < 0$	$\mathcal{E}_n = 0$	$\mathcal{E}_n > 0$
$\mathcal{E}_{Y/L}$	$\mathcal{E}_{Y/L} > 1$	$\mathcal{E}_{Y/L} = 1$	$\mathcal{E}_{Y/L} < 1$
$\mathcal{E}_{nN/L}$	$0 < \mathcal{E}_{nN} < 1$	$\mathcal{E}_{nN} = 1$	$\mathcal{E}_{nN} > 1$

Comparative statics of the mass of firms N^*

Proposition

The reactions of the number of firms N^* to the changes in the market size are as follows:

Costs behavior	RLV behavior		
	$r_u' > 0$	$r_u' = 0$	$r_u' < 0$
$\frac{V''(y)y}{V'(y)} < \frac{S''(n)n}{S'(n)}$	$0 < \mathcal{E}_{N/L} < 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} > 1$
$\frac{V''(y)y}{V'(y)} = \frac{S''(n)n}{S'(n)}$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$
$\frac{V''(y)y}{V'(y)} > \frac{S''(n)n}{S'(n)}$	$\mathcal{E}_{N/L} > 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} < 1$

Special case 2: per-variety additive costs

- Consider *per-variety additive* variable costs:

$$\mathscr{V}(\mathbf{q}, n) = \int_0^n v(q_i)di + S(n).$$

- Here v is variable costs of a separate plant, S is monitoring costs; both are increasing and convex.

Proposition

The comparative statics of total output y^*, scope n^* and the mass of firms N^* with respect to the market size L for the case of per-variety additive costs is as follows:

RLV behavior	case $r_u' > 0$	case $r_u' = 0$	case $r_u' < 0$
$\mathcal{E}_{n/L}$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} = 0$
$\mathcal{E}_{y/L}$	$0 < \mathcal{E}_{y/L} < 1$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} > 0$
$\mathcal{E}_{N/L}$	$0 < \mathcal{E}_{N/L} < 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} > 1$

The most surprizing result: scope is invariant to changes in the market size, no matter whether we have price increasing or price decreasing competition.

Plans for further work

- Heterogeneity of firms;
- The open economy case;
- Vertical linkages;
- Endogenous choice between producing a single product and multiple products.

Thank you for your attention!