# Agragative Games and Entry

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# Profit maximized function

- rewrite $\widetilde{\pi}_i(Q,\ q_i) = \pi_i(Q_{-i} + q_i;\ q_i)$.
- agent's choice $q_i$ of can be written from FOC for $\widetilde{\pi}_i(Q,\ q_i)$ as

$$\frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial Q}\frac{\partial Q}{\partial q_i} + \frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial q_i} = 0,$$

where

# Oligopoly equilibrium

- for the Nash "oligopoly" equilibrium $\frac{\partial Q}{\partial q_i} = 1$, so

$$\frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial Q} + \frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial q_i} = 0,$$

- Since $\frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial Q} < 0$ we get

$$\frac{\partial \widetilde{\pi}_i^o(Q,\ q_i)}{\partial q_i} > 0$$

# Monopolistic competetion equilibrium

- for the monopolistic comptition equilibrium $\frac{\partial Q}{\partial q_i} = 0$, so
$$\frac{\partial \tilde{\pi}_i^m(Q,\ q_i)}{\partial q_i} = 0,$$
- Since $0 = \frac{\partial \tilde{\pi}_i^m(Q,\, q_i)}{\partial q_i} < \frac{\partial \tilde{\pi}_i^o(Q,\, q_i)}{\partial q_i}$, we have $q_i^m > q_i^o$ - monopolistically competition firm have higher $q_i$ than at Nash equilibruim.

# "Maximum value function"

- "Maximum value function" is $\widetilde{\pi}_i^*(Q) = \pi_i(Q, \widetilde{r}_i(Q))$,

where $\widetilde{r}_i(Q) = Q - Q_{-i}$-cumulative reaction function.

- Differentiation yields:

$$\frac{d\widetilde{\pi}(Q)}{dQ} = \frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial Q} + \frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial q_i}\widetilde{r}_i'(Q),$$

Using FOC and $\frac{\partial Q}{\partial q_i} = 1$ for the Nash equilibrium oligopoly we get

$$\frac{d\widetilde{\pi}(Q)}{dQ} = \frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial q_i}(\widetilde{r}_i'(Q) - 1) = \frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial Q}(1 - \widetilde{r}_i'(Q)).$$

# Assumptions

- Free entry of symmetric fringe.
- Welfare is a function of number of entrants:

$$W(n) = \widetilde{\phi}(Q) + n\widetilde{\pi}^*(Q),$$

- number of entrants can be varied;
- Aggregator is determined according to Nash equilibruim of the aggregate game and increases with $n$.

# Derivative of welfare

$$W'(n^e) = \left(\widetilde{\phi}'(Q) + n^e \widetilde{\pi}^{*\prime}(Q)\right) \frac{dQ}{dn},$$

where

- $n^e$ is a number of entrants at the free entry equilibrium (zero profit of fringe firms).
- $\widetilde{\phi}'(Q)$ is the non-appropriability of consumer surplus (Spence, 1976).
- $n^e \widetilde{\pi}^{*\prime}(Q)$ is the business stealing effect on other firms' profits (Spence, 1976).

# Cournot competition

- $\widetilde{\phi}'(Q)$ is the net consumer surplus, so $\widetilde{\phi}'(Q) = -Qp'(Q)$
- under symmetry $\frac{d\widetilde{\pi}(Q)}{dQ} = \frac{\partial \widetilde{\pi}_i(Q, q_i)}{\partial Q}(1 - \widetilde{r}_i'(Q)) = p'(Q)q(1 - \widetilde{r}_i'(Q))$ we get

$$\widetilde{\phi}'(Q) + n^e \widetilde{\pi}^{*\prime}(Q) = -Qp'(Q) + np'(Q)q(1 - \widetilde{r}_i'(Q)) = -p'(Q)Q\widetilde{r}_i'(Q),$$

- There sign of $W'(n^e)$ is the same to $\widetilde{r}_i'(Q)$.
- Under strategic substitutes $(\widetilde{r}_i'(Q) < 0)$ we get excessive entry (Mankiw-Whinstone, 1986)
- Under strategic complementary $(\widetilde{r}_i'(Q) > 0)$ we get opposite result.

# Bertrand model

- profit function: $\pi_i = (p_i - c_i)D_i(p_i,\ Q)$;
- aggregator: $D_i(p_i,\ Q) = -\widetilde{\phi}'(Q)g'(p_i)$;

We assume symmetric situation when marginal cost as well as demand functions are the same. Under monopolistic competition:

$$\frac{d\widetilde{\pi}(Q)}{dQ} = \frac{\partial\widetilde{\pi}_i(Q,\ q_i)}{\partial Q} = -(p_i - c)\widetilde{\phi}''(Q)g'(p_i),$$

# Bertrand model

Using FOC for producer's problem $-(p_i - c)\widetilde{\phi}'(Q)g'(p_i) \to \max_{p_i}$ we get

$$(p_i - c) = -\frac{g'(p_i)}{g''(p_i)}$$

So,

$$\frac{\partial \widetilde{\pi}_i(Q,\ q_i)}{\partial Q} = \frac{[g'(p_i)]^2}{g''(p_i)}\widetilde{\phi}''(Q),$$

We seek the sign of

$$W'(n^e) = \left(\widetilde{\phi}'(Q) + n^e \frac{[g'(p_i)]^2}{g''(p_i)}\widetilde{\phi}''(Q)\right)\frac{dQ}{dn},$$

# Monopolistic competition: logit case

$$\widetilde{\phi}(Q) = \mu lnQ;$$

$$g(p_i) = e^{-\frac{p_i}{\mu}}.$$

So, since $Q = n^e e^{-\frac{p_i}{\mu}}$

$$W'(n^e) = \frac{\mu}{Q}\left(1 - n^e \frac{e^{-\frac{p_i}{\mu}}}{Q}\right)\frac{dQ}{dn} = 0$$

This means that the optimum and the equilibrium match.

# Monopolistic competition: CES case

$$\widetilde{\phi}(Q) = \frac{1}{\lambda} lnQ;$$

$$g(p_i) = p_i^{-\lambda}.$$

So, since $Q = n^e p_i^{-\lambda}$

$$W'(n^e) = \frac{1}{\lambda Q}\left(1 - n^e \frac{\lambda p_i^{-\lambda}}{(\lambda+1)Q}\right)\frac{dQ}{dn} = \frac{1}{\lambda Q}\cdot\frac{1}{\lambda+1}\cdot\frac{dQ}{dn} > 0$$

This means that the equilibrium number of firms less then equilibrium number of firms.

*Thank you for your attention!*