AGGREGATIVE GAMES WITH ENTRY

Part V: Heterogeneous Fringe

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Heterogeneous entry costs

- All fringe firms have the same profit functions, except that they differ by idiosyncratic K
- $K(n)$- the entry cost of the nth lowest cost frienge entrant
- Marginal fringe entrant earns zero profit at $ZPFE$
- The equilibrium solution is a fixed point given by equation with LHS less than 1 (this is quaranteed for strategic substitutes and declared for strategic complements):

$$\sum_{i \in A} \tilde{r}_i(Q) = Q$$

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Proposition

Let entry costs be strictly increasing across fringe firms. Suppose one of the insiders ($j \in A$) experiences a change that increases its marginal profit ($d\tilde{r}_j(Q) > 0$). Let A and A' stands for the set of firms in the two *ZPFE* before and after the change.

Then:

1. $Q_A < Q_{A'}$
2. the change causes some fringe firms to exit
3. each fringe firm choses a higher action iff actions are strategic complements
4. affected insider Firm j chooses a higher action

Fixed Point Condition is:

$$n\tilde{r}_f(Q) + \sum_{i \in A_I} \tilde{r}_i(Q) = Q$$

Zero Profit Condition for marginal fringe firm is:

$$\tilde{\pi}_f(Q) = K(n)$$

1.Totaly differentiating this equatons together leads to the inequality, which means that affected Firm j change cumulative response more aggresive:

$$\frac{d\tilde{r}_j(Q)}{dQ} > 0$$

It must be case $Q_A < Q_{A'}$.
2. Zero Profit Condition states Q and n vary inversely and that proves the second statement

Change of equilibrium in Heterogeneneous case, ctd.

3. Type of action of fringe firm depends on sign $\tilde{r}_f'(Q)$, which is positive in the case $Q_A < Q_{A'}$ iff actions are strategic complements

4. If action is strategic complements see 3. In the other case the result is getting by contradiction to the fact increasing of aggregator and decreasing of number of fringe firms and their lower action.

Logit Model with differentiated quality-costs

Now consider case of a fringe with different quality costs and the same entry cost.
The profit is:

$$\pi_i = (p_i - c_i) \frac{\exp(s_i - p_i)/\mu}{\sum\limits_{j=0,\dots,n} \exp(s_j - p_j)/\mu}$$

here:
s_j represent vertical "quality", $j = 0$ - is outside option (has zero price and "quality" s_0)
denominator - is aggregator,can be written as the sum of individual choices: $Q = \sum q_i$

Logit Model, ctd.

$$\pi_i = (s_i - \mu \ln q_i - c_i)\frac{q_i}{Q}$$

Label firms by decreasing quality cost, so that:

$$s_1 - c_1 \geq \ldots \geq s_n - c_n$$

Zero Profit Condition for marginal firm is:

$$\tilde{\pi}(Q; n) = K$$

Fixed Point Condition is:

$$\sum_{i \in A} \tilde{r}_i(Q; n) = Q$$

Total differentiating leads to the Firm j change cumulative response more aggresive::

$$\frac{d\tilde{r}_j(Q; j)}{dQ} > 0$$