Market Size, Productivity and Entrepreneurship in a Model a'la Melitz

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Outline

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1. Motivation
   - Survey
   - Review of research
2. Our Results
   - Main Assumptions
   - Comparative Statics
3. Special cases of utilities
   - Linear upper utility function
   - Lower-tier utility function is CES
   - Upper-tier utility function is logarithm

Actuality

Recently the actual direction of researching in modern IO (especially in modeling of Monopolistic Competition) and International Trade is modeling of firms heterogeneity

The most popular approaches to this problem were developed in **Melitz (2003)** and **Melitz & Ottaviano (2008)**
But the fundamental defect in these works is exogeneity of heterogeneous productivity.

Our Aim is to develop model with endogenous heterogeneity and to investigate influence of market characteristics on profits and outputs of firms

Modern Approach

The first attempt to modeling of endogenous heterogeneity is proposed in **Oyama, D., Sato, Y., Tabuchi, T. and Thisse, J.-F. (2011)**

But two significant weakness in their approach:

1. using specific utility function (*CES*)
2. agents are homogenous in entrepreneurship ability and heterogeneous in unskilled work abilities.

Unspecific Utility Function Approach

This method has been proposed in **Krugman (1979)** and **Vives (1999)**
The most common form of that was developed in **Zhelobodko E., Kokovin S., Parenti M. and Thisse J.-F. (2011)**

In the last paper this approach was applied to the Melitz's model, but the firms were been as "black box" still

Outline

Maintain Ideas

Our work is draft research of developing model with endogenous heterogeneity in Monopolistic Competition environment.

We will use approach proposed in **Lucas (1978)**:

- agents are homogenous in their preferences and unskilled work abilities, heterogeneous in entrepreneurship abilities
- agents choose their type of activity (to be an entrepreneur or to be a worker) comparing a reservation wage with a potential profit

Announce of Contributions

We have evaluated of elasticities the next characteristics of the Economy w.r.t. of market size changing and have found out their boundaries:

- share and total amount of entrepreneurs in population
- individual consumption of varieties and size of a firm
- prices and profits
- market statistics

and other interpretation

Outline

Economy

- 2-sector economy (diversified sector with increasing return in scale and traditional one with constant return in scale)
- only one factor for producing - a labor
- L agents in economy (continuously), mobile between sectors
- Agents are homogenous in their preferences
- Agents are homogenous in their work abilities, each of one has a unit of labor
- Agents are differentiated in their entrepreneurship abilities, describing by the parameter c - marginal cost of production if organizing a firm (the smaller c the higher entrepreneurship ability).
- Each agent knows her type and chooses between being an entrepreneur or a worker
- The parameter c is distributed on $[0;+\infty]$ (d.d.f. $\gamma(c)$, c.d.f $\Gamma(c)$)
- Each c-type agent has $L\gamma(c)$ copies in the population.

Consumer problem

Each agent consumes infinite-dimensional vector of varieties $\mathbf{x}(c) \equiv \{x(c) : [0; \overline{c}] \to R_+\}$ and scalar $A \geq 0$ of homogenous good
Consumer problem is

$$V\left(L\int_0^{\bar{c}} u(x_c)\gamma_c dc\right) + A \to \max_{x, A}, \text{ s.t. } L\int_0^{\bar{c}} p_c x_c \gamma_c dc + A = I$$

here:
I is agent's income, it equals the wage $w = 1$ for worker and the operating profit π_s for s-type entrepreneur
$\overline{c}$ (endogenously) is the "cutoff", type of "marginal" agent, who is indifferent between two types of activity: to be entrepreneur or to be a worker.

Low-tier Utility Function

Low-tier utility , $u(x)$, is the satisfaction from consuming x:

$$u(\cdot) \in C^3(\mathbb{R}_+ \mapsto \mathbb{R}_+),\ u'(\cdot) \geq 0,\ \ u''(\cdot) \leq 0$$

$$u(0) = 0,\ \ u'(0) = +\infty$$

The more convex is function u the higher is relative love for variety (that mirrors Arrow-Pratt theory of risk aversion)

$$r_u \equiv r_u(x) \equiv -\frac{u''(x)x}{u'(x)} \in (0;1)$$

Marginal utility function isn't too convex

$$r_{u'} \equiv r_{u'}(x) \equiv -\frac{u'''(x)x}{u''(x)} < 2$$

Upper-tier Utility Function

Interdependence of varieties with the numerarie is expressed by the upper-tier utility function $V(Y)$ of consumption a composite good:

$$V(\cdot) \in C^2(\mathbb{R}_+ \mapsto \mathbb{R}), \; V'(Y) \geq 0, \;\; V''(Y) \leq 0$$

The more convex is function V the higher is love for variety

$$r_V \equiv -\frac{YV''(Y)}{V'(Y)} \geq 0$$

Demand

First Order Condition for consumer problem is:

$$p_c = p(x_c) = \frac{u'(x_c)}{\lambda}, \; c \in [0; \overline{c}], \lambda \equiv 1/V'\left(L\int_0^{\bar{c}} u(x_c)\gamma_c dc\right)$$

here:
λ is market statistics, the higher its derivative the more income is spent on differentiated good whereas too concave V means quick satisfaction with varieties.
Assumptions on $u(\cdot)$ guarantee the neoclassic demand properties: $p(\cdot)$ decreases from infinity to zero.

Producer problem

A firm has only Variable Costs, proportionally to it's output

$$\pi_c = (p(x_c) - c)\, L x_c \to \max_{x_c}$$

The **First Order Condition** determines output for c-type agent:

$$M_c \equiv \frac{p_c - c}{c} = r_{u_c}$$

The **Second Order Condition** determines output for c-type agent:

$$r_{u'_c} < 2$$

Zero Profit Condition

The equilibrium is defined by formulating the “zero-profit condition”

$$\pi_{\overline{c}} = w = 1 \Leftrightarrow \frac{\overline{r}_u \overline{x}}{1 - \overline{r}_u} = \frac{1}{L\overline{c}}$$

If agent's type lets him to get profit more than wage, then that agent will chose entrepreneur activity.

Equilibrium

Definition

The equilibrium is the bundle $\left(\overline{c},\ \lambda,\ \{p_c;\ x_c\}_{c\in[0;\overline{c}]}\right)$ such that consumption x maximizes each consumer's utility under price vector $\{\mathbf{p} = p_c(x_c)\}_{c\in[0;\overline{c}]}$ and solves each producer's problem under $\overline{c},\ \lambda,\ \mathbf{p}(\cdot)$, the zero-profit condition holds and $\lambda = 1/V'\left(L\int_0^{\overline{c}} u(x_c)\gamma_c dc\right)$. For a given equilibrium, consumption of the numerarie A_c for each type follows from the budget constraint, that entails also the labor balance under our normalization $w = 1$

Outline

Marginal Agent Productivity

The elasticity of the cutoff cost to the market size is determined with love to variety.

$$\mathcal{E}_L \overline{c} = \frac{\overline{r}_u}{1-\overline{r}_u} \frac{\frac{1}{r_V} - \frac{1}{\overline{r}_u} + \frac{J_0}{J}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\overline{r}_u} \frac{\overline{u}}{\tilde{u}} \frac{\overline{\gamma_c}}{\overline{\Gamma}}}$$

here: bar signs the value expressed at the point $x_{\overline{c}}$, tilde means an average under condition $c < \overline{c}$,

$$J_0 = \int_0^{\overline{c}} \frac{u'(x_c)x_c}{r_{u_c}} \cdot \frac{1-r_{u_c}}{2-r_{u'_c}} \gamma_c dc, \quad J = \int_0^{\overline{c}} u(x_c)\gamma_c dc$$

$$\overline{\Gamma} = \int_0^{\overline{c}} \gamma_c dc, \quad \tilde{u} = \frac{J}{\overline{\Gamma}}$$

Marginal Agent Productivity, boundaries

The elasticity of the cutoff cost to the market size can't decrease/increase too fast:

	$\frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J} < 0$	$\frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J} = 0$	$\frac{1}{r_V} - \frac{1}{\bar{r}_u} + \frac{J_0}{J} > 0$
$\mathcal{E}_L \bar{c}$	$(-1;0)$	$=0$	$\left(0; \frac{\bar{r}_u}{1-\bar{r}_u}\right)$

Marginal Productivity, sufficient condition

For cutoff cost increase in response to the growing market size, any of the following two conditions are sufficient, either

$$\bar{r}_u > r_V \;\Rightarrow\; \mathcal{E}_L \bar{c} > 0$$

or concavity of u decrease and concavity of $\ln u$ is bounded from below:

$$r'_u \leqq 0,\; r_{\ln u} \geq 1 \;\Rightarrow\; \mathcal{E}_L \bar{c} \geqq 0$$

Mass of Entrepreneurs

Total mass of entrepreneurs is $E = L\overline{\Gamma} = L\int_0^{\overline{c}} \gamma_c dc$
The fraction of entrepreneurs in population is $e = \frac{E}{L} = \overline{\Gamma} = \int_0^{\overline{c}} \gamma_c dc$
The share of entrepreneurs changes like as the marginal productivity

$$\mathcal{E}_L e = \frac{\overline{\gamma_{\overline{c}}}}{\overline{\Gamma}} \mathcal{E}_L \overline{c}$$

Remark *If the marginal productivity decreases the total amount of entrepreneurs might decrease also*

$$\mathcal{E}_L E = 1 + \frac{\overline{\gamma_{\overline{c}}}}{\overline{\Gamma}} \mathcal{E}_L \overline{c} \in \left(1 - \frac{\overline{\gamma_{\overline{c}}}}{\overline{\Gamma}};\ 1 + \frac{\overline{\gamma_{\overline{c}}}}{\overline{\Gamma}} \frac{\overline{r}_u}{1 - \overline{r}_u}\right)$$

Direct and Indirect effects

ue endogenous share of entrepreneurs in population influence of expanding market on equilibrium values can be decomposed in two effects:

- **Direct effect** - influence of market size under constant share of entrepreneurs
- **Indirect effect** - effect of changing marginal entrepreneur when market expands

Market Statistics

If the market expands then influence on the competitive intensity
by the direct way is positive: $\mathcal{E}_L^d \lambda = \bar{r}_u > 0$
by the Indirect way is opposite to changing of share of entrepreneurs: $\mathcal{E}_L^i \lambda = -(1-\bar{r}_u)\mathcal{E}_L \bar{c}$

Total effect is always positive but no too strong:

$$\mathcal{E}_L \lambda = \mathcal{E}_L^d \lambda + \mathcal{E}_L^i \lambda = \frac{1 + \frac{\bar{r}_u}{1-\bar{r}_u} \frac{\bar{u}}{\tilde{u}} \frac{\overline{\gamma c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\bar{r}_u} \frac{\bar{u}}{\tilde{u}} \frac{\overline{\gamma c}}{\bar{\Gamma}}} > 0$$

$$\mathcal{E}_L \lambda \leq \max\{\bar{r}_u; r_V\}$$

Outputs, Prices, Profits and Productivity

Higher entrepreneurial ability implies, bigger output bigger profit and smaller price and vice verse:

$$Lx_{c_1} \gtreqless Lx_{c_2} \Leftrightarrow c_1 \lesseqgtr c_2$$

$$p_{c_1} \lesseqgtr p_{c_2} \Leftrightarrow c_1 \lesseqgtr c_2$$

$$\pi_{c_1} \gtreqless \pi_{c_2} \Leftrightarrow c_1 \lesseqgtr c_2$$

Individual Consumption

Total effect for individual consumption is negative but no too strong.

$\mathcal{E}_L x_c$	$\mathcal{E}_L \overline{c} < 0$	$\mathcal{E}_L \overline{c} = 0$	$\mathcal{E}_L \overline{c} > 0$
$r_u' > 0$	$< -1^+$	$(-1^+; 0)$	$(-1^+; 0)$
$r_u' = 0$	< -1	$= -1$	$(-1; 0)$
$r_u' < 0$	$< -1^-$	$< -1^-$	$(-1^-; 0)$

Output

*Output **hardly** holds pro- and anti-competitive patterns OR **hardly** corresponds with cut-off cost*

$\mathcal{E}_L Lx$	$\mathcal{E}_L \overline{c} < 0$	$\mathcal{E}_L \overline{c} = 0$	$\mathcal{E}_L \overline{c} > 0$
$r_u' > 0$	$\lesseqgtr 0?$	$(0;1)$	$(0;1)$
$r_u' = 0$	< 0	$= 0$	$(0;1)$
$r_u' < 0$	< 0	< 0	$\lesseqgtr 0?$

Consumption, direct effect

If the market expands then influence on the individual consumption ***by the direct way*** *is negative always*:

$$\mathcal{E}_L^d x_c = -\frac{\bar{r}_u}{r_{u_c}} \cdot \frac{1 - r_{u_c}}{2 - r_{u'_c}} \leqq 0, \qquad \left|\mathcal{E}_L^d x\right| \gtreqqless 1 \Leftrightarrow r'_u \lesseqqgtr 0$$

The sign of output elasticity under direct effect influence is different in pro- and anti-competitive cases:

	$r'_u > 0$	$r'_u = 0$	$r'_u < 0$
$\mathcal{E}_L^d Lx$	> 0	$= 0$	< 0

Consumption, indirect effect

If the market expands then influence on the individual consumption ***by the indirect way*** *is correlate with changing of share of entrepreneurs*:

$$\mathcal{E}_L^i x_c = \frac{1-\overline{r}_u}{r_{u_c}} \cdot \frac{1-r_{u_c}}{2-r_{u'_c}} \mathcal{E}_L \overline{c}$$

Relative Output of low-productive firms

The ratio $\frac{Lx_c}{L\overline{x}}$ *of outputs for firms with productivity close to cut-off productivity - increases if and only if the entrepreneurial sector expands:*

$$\lim_{c \to \overline{c}} \mathcal{E}_L \frac{Lx_c}{L\overline{x}} = \frac{1}{\overline{r}} \cdot \frac{1-\overline{r}}{2-\overline{r}_{u'}} \cdot \mathcal{E}_L \overline{c} \gtreqless 0 \Leftrightarrow \mathcal{E}_L \overline{c} \gtreqless 0$$

	$\mathcal{E}_L \overline{c} < 0$	$\mathcal{E}_L \overline{c} = 0$	$\mathcal{E}_L \overline{c} > 0$
$\lim\limits_{c \to \overline{c}} \mathcal{E}_L \frac{Lx_c}{L\overline{x}}$	$\left(-\frac{1}{\overline{r}} \frac{1-\overline{r}}{2-\overline{r}_{u'}}; 0\right)$	$= 0$	$\left(0; \frac{1}{2-\overline{r}_{u'_c}}\right)$

Prices

*The **direct effect** on prices is determined by measure of concavity of utility function:*

$$\mathcal{E}_L^d p_c = -\frac{\bar{r}_u r'_{u_c}}{2 - r_{u'_c}} \gtreqless 0 \;\Leftrightarrow\; r'_{u_c} \lesseqgtr 0$$

*The **indirect effect** depends on directions of changing both RLV and share of entrepreneurs in the population:*

$$\mathcal{E}_L^i p_c = \frac{1 - \bar{r}_u}{2 - r_{u'_c}} r'_{u_c} \mathcal{E}_L \bar{c}$$

The total influence of expanding market holds pro- and anti-competitive effects, CES is a borderline:

	$r'_u > 0$	$r'_u = 0$	$r'_u < 0$
$\mathcal{E}_L p_c$	< 0	$= 0$	> 0

Prices, boundaries

Boundaries are determined with increasing/decreasing share of entrepreneurs and pro- and anti-competitive cases.

$\mathcal{E}_L p_c$	$\mathcal{E}_L \overline{c} < 0$	$\mathcal{E}_L \overline{c} = 0$	$\mathcal{E}_L \overline{c} > 0$
$\overline{r}'_u < 0$	$> -r'_{u_c} \frac{\overline{r}_u}{2 - r_{u'_c}}$	$= -r'_{u_c} \frac{\overline{r}_u}{2 - r_{u'_c}}$	$\left(0; -r'_{u_c} \frac{\overline{r}_u}{2 - r_{u'_c}}\right)$
$\overline{r}'_u = 0$	$= 0$	$= 0$	$= 0$
$\overline{r}'_u > 0$	$< -r'_{u_c} \frac{\overline{r}_u}{2 - r_{u'_c}}$	$= -r'_{u_c} \frac{\overline{r}_u}{2 - r_{u'_c}}$	$\left(-r'_{u_c} \frac{\overline{r}_u}{2 - r_{u'_c}}; 0\right)$

Relative Price of low-productive firms

The ratio $\frac{p_c}{\overline{p}}$ of prices for firms with productivity close to cut-off productivity decreases if and only if the entrepreneurial sector expands:

$$\lim_{c \to \overline{c}} \mathcal{E}_L \frac{p_c}{\overline{p}} = -\frac{1-\overline{r}}{2-\overline{r}_{u'}} \mathcal{E}_L \overline{c} \lesseqqgtr 0 \Leftrightarrow \mathcal{E}_L \overline{c} \gtreqqless 0$$

Profits

If the market expands then influence on the profits ***per capita by the direct way*** *is negative:*

$$\mathcal{E}_L^d \frac{\pi_c}{L} = -\frac{\overline{r}_u}{r_{u_c}}(1 - r_{u_c}), \quad \mathcal{E}_L^d \frac{\overline{\pi}}{L} = -(1 - \overline{r}_u)$$

the influence on the profits **by the Indirect way** is correlate/anti-correlate to changing of share of entrepreneurs:

$$\mathcal{E}_L^i \pi_c = (1 - \overline{r}_u) \cdot \frac{1 - r_{u_c}}{r_{u_c}} \cdot \mathcal{E}_L \overline{c}, \quad \mathcal{E}_L^i \overline{\pi} = -(1 - \overline{r}_u) \cdot \mathcal{E}_L \overline{c}$$

Profits

Total influence on the profits per capita is negative but no too strong:

	$\mathcal{E}_L \bar{c} < 0$	$\mathcal{E}_L \bar{c} = 0$	$\mathcal{E}_L \bar{c} > 0$
$\mathcal{E}_L \frac{\pi_c}{L}$	$\left(\frac{r_{u_c}-1}{r_{u_c}}; \bar{r}_u \frac{r_{u_c}-1}{r_{u_c}}\right)$	$= \bar{r}_u \frac{r_{u_c}-1}{r_{u_c}};$	$\left(\bar{r}_u \frac{r_{u_c}-1}{r_{u_c}}; 0\right)$

Profit Inequality

Expanding market reduces profit inequality between producers if and only if concavity measure r_u is decreasing:

$$\mathcal{E}_L \frac{\pi_{c_2}}{\pi_{c_1}} \gtreqless 0 \;\Leftrightarrow\; r_{u_{c_1}} \gtreqless r_{u_{c_2}} \;\Leftrightarrow\; r'_{u_c} \gtreqless 0$$

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Specification

Under this utility specification the consumer's program is:

$$L\int_0^{\bar{c}} u_c\gamma_c dc + A \quad \to \quad \max_x,$$

$$\text{s.t. } L\int_0^{\bar{c}} p_c x_c \gamma_c dc + A \quad = \quad I$$

The markets are not related in this specification. The results are trivial in this case

Consumption and Output

z	$\mathcal{E}_L z$	bounds	behavior	comments
$\bar{c}$	$\frac{\bar{r}}{1-\bar{r}}$	$(0;+\infty)$	$\nearrow$in $\bar{r}$	under $\bar{r} > 0.5$ cutoff grows too fast
λ	0		*const*	market statistic is unchanged
x_c	0		*const*	consumption remains unchanged
$y_c = Lx_c$	1		*const*	output is proportional to the market size
p_c	0			price remains unchanged
$\frac{\pi_c}{L}$	0		*const*	profit is unchanged

Outline

Specification

Under this utility specification the consumer's program is:

$$V\left(L\int_0^{\bar{c}} x_c^{\rho}\gamma_c dc\right) + A \quad \rightarrow \quad \max_{x_c},\ c\in[0;\bar{c}]$$

$$s.t.\ L\int_0^{\bar{c}} p_c x_c \gamma_c dc + A \quad = \quad I$$

In this specification with condition condition $r_V < 1$ CES isn't borderline due two-sector economy

Results (1), productivity, consumption and output

z	$\mathcal{E}_L z$	bounds	behavior	comments
$\bar{c}$	$\frac{\left(\frac{1}{r_V}-1\right)^{\frac{1-\rho}{\rho}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$\left(0;\frac{1-\rho}{\rho}\right)$	$\searrow$in r_V	under $\rho \longrightarrow 0$, upper boundary expands; under $\rho > 0.5$ growth is slow
$\bar{x}$	$-\frac{\frac{1}{\rho}\left(1+\frac{1}{r_V}-\frac{\rho}{1-\rho}+\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}\right)}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$\left(-\frac{1}{\rho};-1\right)$	$\nearrow$in r_V	consumption declines speedily; lower boundary expands respect to the ρ
$\bar{y}=L\bar{x}$	$-\frac{\left(\frac{1}{r_V}-1\right)^{\frac{1-\rho}{\rho}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$\left(-\frac{1-\rho}{\rho};0\right)$	$\nearrow$in r_V	output is reciprocal to the marginal cost of producing;
x_c	$-\frac{\frac{1}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$(-1;0)$	$\searrow$in r_V	consumption is reduced slowly; it is changed slower than market size expands
				output is increased slowly,

Results (2), market statistic, prices and profits

z	$\mathcal{E}_L z$	bounds	behavior	comments
λ	$\frac{1+\frac{1-\rho}{\rho}\frac{\overline{u}}{\tilde{u}}\frac{\overline{\gamma_c}}{\overline{\Gamma}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\overline{u}}{\tilde{u}}\frac{\overline{\gamma_c}}{\overline{\Gamma}}}$	$(0;1-\rho)$	$\searrow$in r_V $\nearrow$in ρ	level of market statistic increases slowly, lower boundary expands with ρ
$\overline{p}$	$\frac{\frac{1-\rho}{\rho}\frac{1}{r_V}-1}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\overline{u}}{\tilde{u}}\frac{\overline{\gamma_c}}{\overline{\Gamma}}}$	$\left(0;\frac{1}{\rho}-1\right)$	$\searrow$in r_V	upper boundary shrinks with ρ
p_c	0		$const$	price is unchanged
$\frac{\pi_{\overline{c}}}{L}$	$-\frac{1+\frac{1}{r_V}-\frac{\rho}{1-\rho}+\frac{\overline{u}}{\tilde{u}}\frac{\overline{\gamma_c}}{\overline{\Gamma}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\overline{u}}{\tilde{u}}\frac{\overline{\gamma_c}}{\overline{\Gamma}}}$	$(-1;-\rho)$	$const$	the value is exactly -1
$\frac{\pi_c}{L}$	$-\frac{\frac{\rho}{1-\rho}+\frac{\overline{u}}{\tilde{u}}\frac{\overline{\gamma_c}}{\overline{\Gamma}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\overline{u}}{\tilde{u}}\frac{\overline{\gamma_c}}{\overline{\Gamma}}}$	$(-\rho;0)$	$\searrow$in r_V	profit slowly increases

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Specification

Under this utility specification the consumer's program is:

$$\ln\left(L\int_0^{\bar{c}} u_c \gamma_c dc\right) + A \quad \rightarrow \quad \max_{x_c}, \; c \in [0;\bar{c}]$$

$$\text{s.t. } L\int_0^{\bar{c}} p_c x_c \gamma_c dc + A \quad = \quad I$$

This specification is is pretty border case, because measure of concavity for logarithm is maximum: $r_V = 1$. In this case *CES* is borderline.

Results (1), productivity

The result about expand/shrinking entrepreneur sector when population grows is displayed in the table:

$\mathcal{E}_L \bar{c}$	$r'_{u_c} < 0$	$r'_{u_c} = 0$	$r'_{u_c} > 0$
$r_{\ln u_c} < 1$	$\left(-1; \frac{\bar{r}_u}{1-\bar{r}_u}\right)$	< 0	< 0
$r_{\ln u_c} = 1$	> 0	$= 0$	
$r_{\ln u_c} > 1$		> 0	$\left(-1; \frac{\bar{r}_u}{1-\bar{r}_u}\right)$

The market statistics is bounded from below: $\mathcal{E}_L \lambda \leq r_u$

Results (2), consumption, output and profit

<table>
<tr><th>$\mathcal{E}_L \frac{\pi_c}{L}$
$\mathcal{E}_L x_c$
$\mathcal{E}_L y_c$</th><th>$r'_{u_c} < 0$</th><th>$r'_{u_c} = 0$</th><th>$r'_{u_c} > 0$</th></tr>
<tr><td>$r_{\ln u_c} < 1$</td><td>?</td><td>$> r'_{u_c} - 2$
> -1
> 0</td><td rowspan="2">$> r'_{u_c} - 2$
> -1
> 0</td></tr>
<tr><td>$r_{\ln u_c} = 1$</td><td rowspan="2">$< r'_{u_c} - 2$
< -1
< 0</td><td>$= r'_{u_c} - 2$
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$= 0$</td></tr>
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</table>

Consumer problem

Each agent consumes infinite-dimensional vector of varieties
$\mathbf{x}(c) \equiv \{x(c) : [0; \overline{c}] \to R_+\}$
Consumer problem is

$$L \int_0^{\bar{c}} u(x_{ic}) \gamma_c dc \to \max_x, \text{ s.t. } L \int_0^{\bar{c}} p_c x_{ic} \gamma_c dc = I_i$$

here:
I_i is i-type agent's income, it equals the wage $w = 1$ for worker and the operating profit π_i for i-type entrepreneur
$\overline{c}$ (endogenously) is the "cutoff", type of "marginal" agent, who is indifferent between two types of activity: to be entrepreneur or to be a worker.

Low-tier Utility Function

Low-tier utility , $u(x)$, is the satisfaction from consuming x:

$$u(\cdot) \in C^3(\mathbb{R}_+ \mapsto \mathbb{R}_+),\ u'(\cdot) \geq 0,\ \ u''(\cdot) \leq 0$$

$$u(0) = 0,\ \ u'(0) = +\infty$$

The more convex is function u the higher is relative love for variety (that mirrors Arrow-Pratt theory of risk aversion)

$$r_u \equiv r_u(x) \equiv -\frac{u''(x)\,x}{u'(x)} \in (0;1)$$

Marginal utility function isn't too convex

$$r_{u'} \equiv r_{u'}(x) \equiv -\frac{u'''(x)\,x}{u''(x)} < 2$$

Demand

First Order Condition for consumer problem is:

$$p_c = \frac{u'(x_{ic})}{\lambda_i}, \; c \in [0; \overline{c}]$$

here:
λ_i is shadow price for Income
Assumptions on $u(\cdot)$ guarantee the neoclassic demand properties:
$p(\cdot)$ decreases from infinity to zero.

Producer problem

A firm has only Variable Costs, proportionally to it's output

$$\pi_c = (p_c - c)\, L z_c \to \max_{p_c}$$

here:

$z_c = \int_0^\infty x_{ic} \gamma_i di$ - output per capita of c-type entrepreneur

The **First Order Condition** for c-type agent is:

$$M_c \equiv \frac{p_c - c}{c} = \tilde{r}_u (\mathbf{x}_{\bullet c})$$

here:

$\widetilde{r}_c = \mathscr{E}_{z_c}(p_c)$- is a weighted measure of concavity for low-tier utility function with weights $w_{ic} = \frac{\frac{x_{ic}}{ru(x_{ic})}}{\int_0^\infty \frac{x_{ic}}{ru(x_{ic})} \gamma_i di} = \frac{\frac{u'(x_{ic})}{u''(x_{ic})}}{\int_0^\infty \frac{u'(x_{ic})}{u''(x_{ic})} \gamma_i di}$- is a relative inverse measure of absolute risk averse

Second Order Condition

The **Second Order Condition** for c-type agent is:

$$\tilde{r}\left(\mathbf{x}_{\bullet\mathbf{c}}\right)\left(\widetilde{\frac{r_{u'}\left(\mathbf{x}_{\bullet\mathbf{c}}\right)}{r_u\left(\mathbf{x}_{\bullet\mathbf{c}}\right)}}\right) < 2$$

Here - wave means average with weights

$$w_{ic} = \frac{\frac{x_{ic}}{r_u\left(x_{ic}\right)}}{\int_0^\infty \frac{x_{ic}}{r_u\left(x_{ic}\right)}\gamma_i di} = \frac{\frac{u'\left(x_{ic}\right)}{u''\left(x_{ic}\right)}}{\int_0^\infty \frac{u'\left(x_{ic}\right)}{u''\left(x_{ic}\right)}\gamma_i di}$$

Zero Profit Condition

The equilibrium is defined by formulating the "zero-profit condition"

$$\Pi_{\overline{c}} = \left(p_{\overline{c}} - \overline{c}\right) L \int_0^{\infty} x_{i\overline{c}} \gamma_i di = w = 1$$

$$\frac{\Pi_{\overline{c}}}{L\overline{c}} = \frac{1}{\overline{c}L}$$

$$\frac{\widetilde{r_u}\left(\mathbf{x}_{\bullet\overline{c}}\right)}{1 - \widetilde{r_u}\left(\mathbf{x}_{\bullet\overline{c}}\right)} z_{\overline{c}} = \frac{1}{\overline{c}L}$$

This expression is a full analog for quasi-linear model

CES specification

Inverse Demand Function:

$$p_c = \frac{\rho x_{ic}^{\rho-1}}{\lambda_i}$$

First order condition for producer problem:

$$M_c = \frac{p_c - c}{p_c} = 1 - \rho$$

Labor Balance:

$$L\left(1 - \Gamma\left(\overline{c}\right)\right) = L \int_0^{\bar{c}} z_c \gamma_c dc$$

Elasticities

The bigger market - the bigger share of entrepreurs, moreover: the bigger market - the bigger growth rate of share entrepreneurs

$$\mathscr{E}_L\overline{c} = \frac{1}{\frac{\rho}{1-\rho} + \frac{\rho}{1-\rho}\frac{1}{1-\overline{\Gamma}}\frac{1}{L} + \frac{\overline{\gamma c}}{1-\overline{\Gamma}}} \in (0;1)$$

The total output per capita decreases when the market expands, but total output increases less proportional market size:

$$\mathscr{E}_L z_c = -\frac{\rho + \overline{\gamma c}L(1-\rho)}{\rho\left(1-\overline{\Gamma}\right)L + \rho + \overline{\gamma c}L(1-\rho)} \in (-1;0)$$

The price is const:

$$\mathscr{E}_L p_c = 0$$

Profit per capita deacreses, but total profit increases when market

Outcome

- Structure of employment/entrepreneurship can be influenced by the economy size
- Population growth changes the equilibrium
- *CES* isn't borderline in heterogeneous case

Further directions

- One sector model in general case (or another specific cases)
- Trade in one-sector model with *CES* specificaton and
- Investigations of Income Inequality in Trade for One-Sector Model with *CES* specification