Investments in productivity and quality under monopolistic competition

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Abstract

- (**Theor.question**): Impact of market size on *productivity/quality* in monopolistic competition;
- (**Setting**): 1) variable elasticity of substitution (*VES*), 2) each firm chooses investment in decreasing marginal cost or increasing quality; 3) homogenous or heterogenous firms
- (**Results**): [Growing market $\uparrow$ pushes each firm's R&D investment *up* $\uparrow$, price down $\downarrow$] $\Leftrightarrow$
[utility shows "increasing relative love for variety" (decreasing elasticity of demand)].
Total R&D investment in economy go up$\uparrow$ always.
Similar effects hold for investments in quality .

Outline

Motivation: empirics and theory

*Controversy on **Competitiveness** and **innovations***:

- (+) Positive **empirical** correlation between competition (***more firms***) and innovations: Baily & Gersbach (1995), Geroski (1995), Nickell (1996), Blundell, Griffith & Van Reenen (1999), Galdón-Sánchez & Schmitz (2002), Symeonidis (2002), etc.
- (+-) Non-monotone, bell-shape **empirical** correlation: Aghion et al. (2005).
- (-) Negative theoretical correlation, innovations should **decline** with usual oligopolistic competition: "Common wisdom" in IO.
- (+) Positive theoretical correlation: Vives (2008), the model of oligopolistic competition with *free entry* ($\Rightarrow$ endogenous number of firms)

We extend Vives to more realistic model:

- monopolistic competition (differentiated goods), general equilibrium
- comparative statics of market equilibria + social optimum

Background literature

- 1. *Basic idea* of Monopolistic Competition: many firms - price-makers produce “varieties”, free entry, fixed and variable costs => increasing returns: Chamberlin (1929), Dixit and Stiglitz (1977), for trade - Krugman (1979).
- 2. (Instead of CES or quadratic utility) MC model was generalized to any VES utility: Zhelodobko, Kokovin, Parenti & Thisse (2010,2011)
- 3. Oligopolistic choice of technology in quasilinear setting: Vives (2008): firm's R&D investment in economy go up$\uparrow$ with market size always, and number of varieties can increase or decrease.

We combine *choice of technology* a'la Vives - with *monop. competition.*
It needs VES, like ZKPT-2011,
because under CES combining is uninteresting: zero effects.

General MC assumptions

- *Increasing returns to scale* in a firm, due to investment cost f and marginal costs $c(f)$. Firms are identical.
- Each firm i produces one "variety" as a *price-maker*, but its demand $x_i(p_i, p_j ...)$ is influenced by other varieties.
- L identical consumers, each $j \leq L$ generates a demand function x_j, maximizing *additive utility* function $U = \int_{i \leq N} u(x_i) di$. Concavity of $u(.)$ (i.e., elasticity of demand or *substitution among varieties)* - determines intensity of competition.
- *Number of firms is big enough* to ignore one firm's influence on the whole industry/economy.
- *Free entry* drives all profits to zero.
- *Labor* supply/demand is balanced.

Basic model of 1x1x1 economy. Consumers

- One diversified sector has an interval $[0, N]$ of firms=varieties i-th brand is i-th firm, $i \in [0, N]$,
- L identical consumers, each has 1 of labor and chooses an (infinite-dimensional) consumption vector $x(\cdot) : [0, N] \to \mathbb{R}_+$ i.e., a non-negative integrable function x:

$$\int_0^N u(x_i)di \to \max_{x(.)}; \quad \int_0^N p_i x_i di \leq 1.$$

- Here: utility function $u(.)$, price vector $p(.) : [0, N] \to \mathbb{R}_+$; $p(i) \equiv p_i$ is price for i-th variety, $x(i) \equiv x_i$ is demand for i-th variety. Lagrange multiplier λ, labor endowment 1. From FOC, the inverse demand for i-th variety is:

$$p_{\mathrm{i}}(x_i, \lambda) = \frac{u'(x_i)}{\lambda}.$$

Model: Producers, FOC

- i-th firm knows its inverse-demand function $p_{\text{i}}(x_i,\lambda)$, sells $Q = L_i x_{\text{i}}$ and maximizes profit

$$\pi = Lx_i \cdot [p_{\text{i}}(x_i,\lambda) - c(f_i)] - f_i \rightarrow \max_{x_{\text{i}},f_{\text{i}} \in \mathbb{R}_+} .$$

  L - consumers' mass, c - marginal cost, f -fixed cost (measured in labor, total cost is $cx_{\text{i}}L+f$), λ - marginal utility of money. FOC is:

$$\frac{\partial \pi(\bar{x},\bar{f})}{\partial f} = 0, \qquad \frac{\partial \pi(\bar{x},\bar{f})}{\partial \bar{x}} = 0$$

- *Symmetric equilibrium* is (x, f, p, N, λ) satisfying all FOC, budget constraint, free entry and labor balance (next slide).

Model: Equilibrium (x, f, p, N, λ)

- Consumers' FOC and budget constraint:

$$p = p^*(\bar{x}) = u'(\bar{x})/\lambda$$

$$Np\bar{x} = 1$$

- Producers' FOC:

$$\frac{\partial \pi(\bar{x}, \bar{f})}{\partial f} = 0, \quad \frac{\partial \pi(\bar{x}, \bar{f})}{\partial \bar{x}} = 0$$

- Zero-profit condition (free entry):

$$\pi = (p^*(\bar{x}) - c(\bar{f}))\bar{x}L - \bar{f} = 0.$$

- Labor balance:

$$(\bar{f} + c(\bar{f})\bar{x})N = L$$

This system can be reduced to 2 equations (x, f):

Equilibrium equations for (x, f)

We use the Arrow-Pratt measure of concavity defined for any function g :

$$r_g(z) = -\frac{zg''(z)}{g'(z)}.$$

Proposition. Equilibrium consumption/investment (x^*, f^*) in a homogenous closed economy with endogenous technology is the solution to

$$\frac{r_u(x)x}{1 - r_u(x)} = \frac{f}{Lc(f)},$$

$$(1 - r_{\ln c}(f) + r_c(f))(1 - r_u(x)) = 1,$$

when SOC conditions hold:

$$r_u(x) < 1, \qquad 2 - r_{u'}(x) > 0, \qquad (2 - r_{u'}(x))\, r_c(f) > 1.$$

Differentiating the system w.r.t. $L \Rightarrow$ Theorem of comparative statics:

Theorem: signs of elasticities w.r.t. market size L:

	case IES	CES	case DES (like $\sqrt{x+a}$)		
	$r'_u < 0$	$r'_u = 0$	$r'_u > 0$		
	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
$\mathcal{E}_f = \frac{\mathcal{E}_{Lx}}{r_c}$	$-$	0	$+$	$+$	$+$
$\mathcal{E}_p = -\mathcal{E}_{Lx} \cdot r_u$	$+$	0	$-$	$-$	$-$
$\mathcal{E}_{\frac{p-c}{p}} = \mathcal{E}_x \cdot \frac{x r'_u}{r_u}$	$+$	0	$-$	0	$+$
$\mathcal{E}_x$	$-$	$-$	$-$	0	$+$
$\mathcal{E}_{Nf}$	$+$	$+$	$+$	$+$	$+$
$\mathcal{E}_N$	>1	$=1$	$\in (0,1)$	$= r_u < 1$	<1

(Only) under DES-utility a growing market *increases* firm size $Lx \uparrow$ and R&D investment $f \uparrow$, but *decreases* prices $p \downarrow$. Cost elasticity $r_{\ln c(f)} > 1$ influences per-variety consumption $x \downarrow$ and markup $\frac{p-c}{p} \downarrow$.

- Shortly, DES+larger market $\Rightarrow$ bigger firms$\Rightarrow$ higher productivity

Another interpretation: Quality

q_i = quality of $i-th$ variety, x_i = consumption, $z_i \equiv q_i x_i$ = satisfaction with variety, investment-for-quality function: $f_i = \tilde{f}(q_i)$; $\mathbf{q}(.) \equiv \tilde{f}^{-1}(.)$.

$$U(z) = \int_0^N u(q_i x_i) di, \Rightarrow \; p(x_i, q_i, \lambda) = \frac{q_i u'(q_i x_i)}{\lambda}.$$

Given $\tilde{f}(.)$, $\tilde{c}(.)$ (=marginal cost of quality), $i-th$ cost function is:

$$\tilde{c}(q_i) L x_i + \tilde{f}(q_i), \Rightarrow \textbf{auxiliary } f, c: \; f_i \equiv \tilde{f}(q_i), \; c(f_i) \equiv \frac{\tilde{c}(\mathbf{q}(f_i))}{\mathbf{q}(f_i)}.$$

Naturally, $\tilde{c}'(q_i) > 0, \tilde{f}'(q_i) > 0$ (higher quality requires spendings). Then the setting becomes identical to productivity setting:

$$\left(\frac{u'(z_i)}{\lambda} - c(f_i) \right) L z_i - f_i \rightarrow \max_{z_i \geq 0, f_i \geq 0}.$$

Same effects: DES+larger market $\Rightarrow$ bigger firms$\Rightarrow$ higher quality

Extension 1: Social optimum

In symmetric solution optimality means that x^{opt}, f^{opt} and N^{opt} are welfare- optimizing: $Nu(x) \to \max_{N,x,f}$; s.t. $N(c(f)xL + f) = L$.

Result. Three patterns of relation between social optimum $(.^{opt})$ and market equilibrium $(.^{*})$ in consumption $(x^{opt} \lesseqgtr x^{*})$, investment $(f^{opt} \lesseqgtr f^{*})$ and mass of firms $(N^{opt} \lesseqgtr N^{*})$ are subject to concavity of $\ln u$:

Case $r_{\ln u} < 1$	**Case** CES: $r_{\ln u} = 1$	**Case** $r_{\ln u} > 1$
$x^{opt} < x^{*}$	$x^{opt} = x^{*}$	$x^{opt} > x^{*}$
$f^{opt} < f^{*}$	$f^{opt} = f^{*}$	$f^{opt} > f^{*}$
$N^{opt} > N^{*}$	$N^{opt} = N^{*}$	$N^{opt} < N^{*}$

The relation between optimal total investment $(Nf)^{opt} = N^{opt} \cdot f^{opt}$ and equilibrium total investment $(Nf)^{*} = N^{*} \cdot f^{*}$ is subject to $\ln u, \ln c$:

$(1 - r_{\ln u})(1 - r_{\ln c}) < 0$	$(1 - r_{\ln u})(1 - r_{\ln c}) = 0$	$(1 - r_{\ln u})(1 - r_{\ln c}) > 0$
$(Nf)^{opt} > (Nf)^{*}$	$(Nf)^{opt} = (Nf)^{*}$	$(Nf)^{opt} < (Nf)^{*}$

Comparative statics of social optimum

Theorem. For optimality the signs of elasticities of x^{opt}, f^{opt} and N^{opt} w.r.t. market size L are

	IES	CES	DES		
	$r_{\ln u} < 1$	$r_{\ln u} = 1$	$r_{\ln u} > 1$		
	$r_{\ln c} > 1$	$r_{\ln c} \neq 1$	$r_{\ln c} > 1$	$r_{\ln c} = 1$	$r_{\ln c} < 1$
$\mathcal{E}_{x^{opt}}$	$-$	$= -1$	$-$	$= 0$	$+$
$\mathcal{E}_{Lx^{opt}}$	$-$	$= 0$	$\in (0;1)$	$= 1$	> 1
$\mathcal{E}_{f^{opt}}$	$-$	$= 0$	$+$	$\in (0;1)$	$+$
$\mathcal{E}_{N^{opt} f^{opt}}$	> 1	$= 1$	$\in (0;1)$	$= 1$	> 1
$\mathcal{E}_{N^{opt}}$	> 1	$= 1$	$\in (0;1)$	$\in (0;1)$	< 1

Extension 2: Exogenous technological progress

Let $c = c(f, \alpha)$ where $\alpha > 0$ is technological progress parameter:

$$\frac{\partial c}{\partial f} < 0, \qquad \frac{\partial^2 c}{\partial f^2} > 0(??), \qquad \frac{\partial c}{\partial \alpha} < 0, \qquad \frac{\partial^2 c}{\partial f \partial \alpha} < 0.$$

Theorem. Markets classification for technological progress:

	IES	CES	DES
	$r_u' < 0$	$r_u' = 0$	$r_u' > 0$
$\mathcal{E}_{x/\alpha}$	$+$	$+$	$+$
$\mathcal{E}_{f/\alpha}$	$+$	$+$	$+$
$\mathcal{E}_{Nf/\alpha} = \mathcal{E}_{\frac{p-c}{p}/\alpha}$	$-$	0	$+$
$\mathcal{E}_{N/\alpha}$	$-$	$-$	?
$\mathcal{E}_{p/\alpha}$	$-$	$-$	$-$

Thus, technological progress always stimulates R&D investment and pushes prices down, but impact on markup differs for DES and IES cases.

Conclusions, extensions

Main conclusion: **in a larger economy firm size and productivity (quality) is higher $\Leftrightarrow$ DES-utility**.

Further questions:

- *Hypothesis* 1: In welfare analysis, socially-optimal solutions show similar comparative statics as equilibria, and only under CES equilibria are optimal (?);
- *Hypothesis* 2: Under heterogenous firms a-la Melitz, market size yields similar effects (?);
- ***Thank you***