Endogenous Structure of Cities

City without External Trade

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Terminology Agreement

- ***Monocentric City***: there exists unique Central Business District (CBD)
- ***Polycentric City***: along with CBD there exist additional Secondary Business District(s), connected to CBD.
- Extension: ***Hierarchic Polycentricity*** – Tertiary BDs, Quaternary BDs etc.

Stylized Facts

1. Monocentric Cities and Polycentric Cities co-exist in economy space
2. "Large" cities are (in general) Polycentric, while "small" ones are Monocentric
3. City pattern may change with the lapse of time (i.e. pattern is not predefined)
4. Population growth is (in general) disproportionally larger in suburbia (i.e., in SBDs)
5. Moreover, sometimes Central city may "freezes", while suburbia population still increases (e.g., Paris)

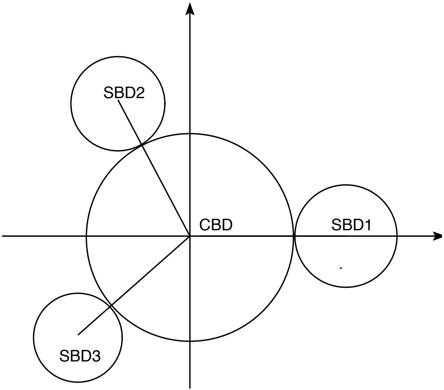
Adequate Model to Explain

- Monocentric/pre-Polycentric paradigms can't explain *WHY and HOW City pattern forms and changes*
- *ENDOGENOUS nature* as outcome of *interplay of FORCES*
- ***Agglomeration force***: Economy of Scale, i.e. decreasing average production costs force firms to gather
- ***Dispersion force***: Urban costs (e.g., commuting) force to form additional (secondary) job centers

Similar Approaches

- Main insights are based on
  - J. Cavailhès, C. Gaigné, T. Tabuchi, J.-F. Thisse (2007), *Trade and the structure of cities*, Journal of Urban Economics
- ***Differences***: drop down "long narrow city" with exactly TWO secondary centers, 2D pattern with arbitrary number of subcenters
- Insight of job sub-centers' hierarchy (secondary, tertiary, etc) borrowed from
  - T. Tabuchi (2009), *Self-organizing marketplaces*, Journal of Urban Economics

Polycentric City



CBD – Central Business District, (Axis origin placed here)
SBDi – Secondary Business Districts
$i \in \{1, 2, \ldots, m\}$,
$m = 0$ – monocentric city

Why Circles?

There is no reason to be something else

- Job centers (Central or Secondary) are dimensionless
- Plane is initially featureless
- Residents (workers) take a unite of area for residence and commuting costs depend on *Euclidean distance*
- Each worker lives in residence zone around her job center
- Thus Central Zone and Secondary Zones are circles and
- All Secondary Zones are identical

Main Feature of CBD

- SBDs lack some non-tradable goods/services presented in CBD: specific local public goods and business-to-business services such as marketing, banking, insurance
- It reflects in additional *communication* costs $\mathscr{K}(x^S) = K + k \cdot ||x^S||, \ K > 0, \ k > 0$

Parameters: Exogenous vs Endogenous

- *Size of population* L – exogenous in separated city and endogenous in economy with migration
- *Pattern (mono/polycentric)* – $m > 0$ or $m = 0$, endogenous
- *Number* of SBDs m – ambiguously, will be discussed later
- *Distance* $||x^S||$ of SBD x^S from CBD – endogenous
- *Population shares* of residence zones (Central $\theta \in (0, 1]$ and Secondary $(1 - \theta)/m$) – endogenous

Workers

Workers' welfare depends on the three goods:

- q_0 – numéraire good, homogenous and unproduced with initial endowment $\bar{q}_0$
- $q(i)$, $i \in [0, n]$ a continuum n of varieties of a horizontally differentiated good under monopolistic competition and increasing returns, using labor as the only input. Traded costlessly for price $p(i)$ within the city of origin.
- one lot of land area for residence, price (rent) $R(x)$ depend on location. Normalization: one lot of the land $= \pi \approx 3.14159\ldots$

Consumer's Problem

For CBD worker:

$$U(q_0; q(i), i \in [0, n]) \to \max$$

$$s.t. \int_0^n p(i)q(i)\mathrm{d}i + q_0 = w^C + \bar{q}_0 - R^C(x) - t||x||$$

$R^C(x)$ – land rent at location x (in fact, depends on distance $||x||$ from the CBD only).
The budget constraint of SBD worker

$$\int_0^n p(i)q(i)\mathrm{d}i + q_0 = w^S + \bar{q}_0 - R^S(x) - t||x\text{-}x^S||.$$

Firms' Problem

Producing of variety i requires a given number f of labor units (fixed costs) and costs c units of numeraire.

- $c = 0$ without loss of generality;
- No trade costs for local sales;
- For firm producing i in CBD:

$$\Pi^C(i) = I(i) - f \cdot w^C \rightarrow \max$$

  $I(i)$ – firm's revenue ($I(i) = L \cdot p(i)q(i)$ for local sales).
- For firm in SBDs:

$$\Pi^S(i) = I(i) - \mathscr{K}(x^S) - f \cdot w^S \rightarrow \max$$

City Equilibrium

- None of workers want to change her choice of job/residence (CBD or one of SBD zone and distance from job center)
- None of firms want change her choice of location (CBD or one of SBD)
- Firm's Cut-Off: profits $\Pi^C = \Pi^S = 0$.
- Mass of firms (=varieties of differentiated good) $n = \frac{L}{f}$.
- Consumer's Cut-Off:
  $w^C - C_u^C = w^C - (R^C(x) + t \cdot ||x|| - \bar{q}_0) \geq 0$

How to Obtain Equilibrium θ?

- Costs $K > 0$, $k > 0$, $f > 0$ and $L \geq 0$, $m > 0$ are given.
- If $f \cdot t \leq k$ all firms stay in CBD, i.e. $\theta \equiv 1$.
- Otherwise, let $\delta = \dfrac{f \cdot t - k}{f \cdot t + k} \in (0, 1)$ – *relative difference* of firm's commuting and communication costs
- $L^M = \left(\dfrac{K}{k} \cdot \dfrac{(1-\delta)}{2\delta} \right)^2$.

Theorem

If $L \leq L^M$, then polycentric city could not exists. i.e. $\theta \equiv 1$. Otherwise, for all $m > 0$ unique equilibrium $\theta^* \in \left(\frac{1}{1+m\delta^2}, 1 \right)$ exists.

Comparative Statics

Theorem

$$\frac{\partial \theta^*}{\partial L} < 0,\ \frac{\partial \theta^*}{\partial m} < 0 \ \textit{for}\ L > L^M,$$

$$\lim_{L\to\infty} \theta^* = \frac{1}{1+m\delta^2},\ \lim_{m\to\infty} \theta^* = \frac{1}{L}\left(\frac{K}{k}\cdot\frac{(1-\delta)}{2\delta}\right)^2 = \frac{L^M}{L}$$

Corollary

$$\lim_{m\to\infty} \theta^* \cdot L \equiv L^M$$

Redistribution of Total Rent

Initial endowment

$$\bar{q}_0 = \frac{1}{L} \int\limits_X R(x) \mathrm{d}x$$

In this case Consumer's Cut-Off:

$$w - C_u(\theta^*) = w^{C*} - t\sqrt{\theta^* L} + \frac{t}{3} \cdot \sqrt{L} \left[(\theta^*)^{3/2} + \frac{(1-\theta^*)^{3/2}}{\sqrt{m}} \right] \geq 0$$

Consumer's Utility

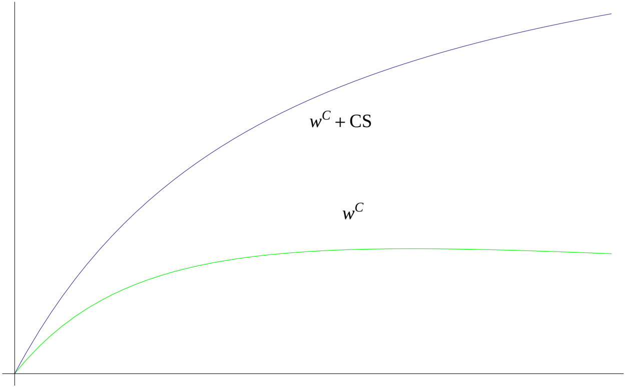
- Ottaviano's quasi-linear utility for $\alpha > 0$, $\beta > 0$, $\gamma > 0$

$$U(q_0; q(i), i \in [0,n]) = q_0 + \alpha \int_0^n q(i)\mathrm{d}i - \frac{\beta}{2} \int_0^n [q(i)]^2 \mathrm{d}i - \frac{\gamma}{2} \left[\int_0^n q(i)\mathrm{d}i \right]$$

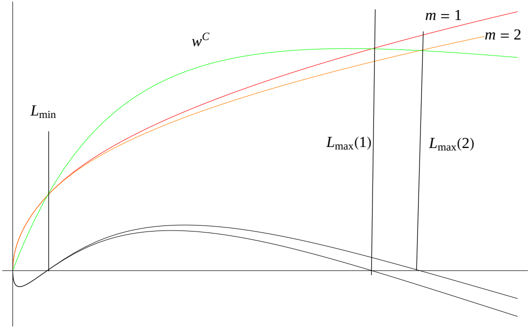
(studied for *linear cities* in J. Cavailhès, C. Gaigné, T. Tabuchi, J.-F. Thisse (2007))

- Generates *linear demand curve*

Wage and Consumer's Surplus



City Equilibrium



Cut-Off: $L \in [L_{\min}, L_{\max}] \subset (0, +\infty)$

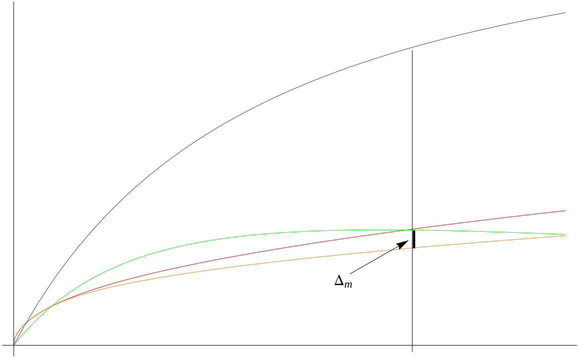
Endogenous Population L?

What are incentives for "immigrants":

- Disposable Income $w - C_u$
- or Welfare (Real Wage) $w + CS - C_u$
- Anyways, answer (endogenous L) requires extension multi-regional setting with inter-city trade

Endogenous m?

Assumption. SBD's number m determine "City Developers".

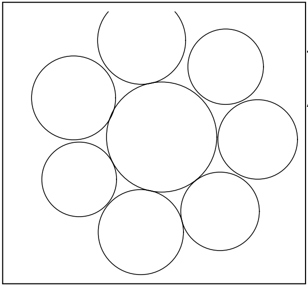


"Optimum" number m: $\Delta_m = V^C(m+1) - V^C(m) > 0$,
while $\sum_{m=0}^{\infty} \Delta_m < \overline{V}$, thus *marginal gains tent to zero* with increasing m, unlike marginal costs M to develop new SBD.

Correlation of L and m

- Empiric evidences: *L and m positively correlate* (MacMillen and Smith (2003), *The number of subcenters in large urban areas*, JUE)
- Increasing in m shifts up "optimum" population L^*;
- Increasing in L requires to increase m (if marginal gains exceed marginal developing costs)

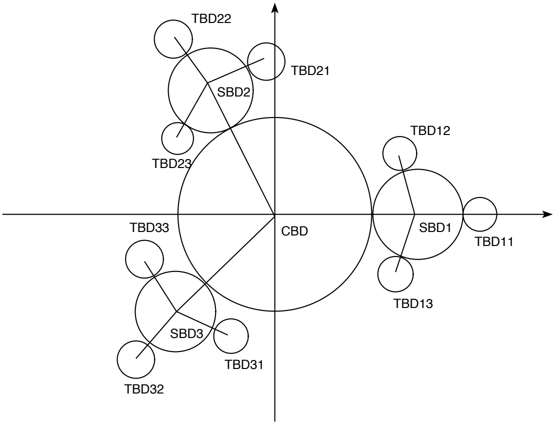
What If City Reach Its Maximum?
Does It Mean that Moscow Is NON-Rubber?



There exists theoretical maximum of SBDs.
Asymptotically (for large L):

$$M^* \approx \frac{\pi}{\arcsin\left(\frac{\delta}{1+\delta}\right)} \gtrapprox \frac{\pi \cdot (1+\delta)}{\delta}$$

Hierarchy of Business Districts



$m_0 \equiv 1$ CBD,
$m_1 \geq 1$ SBDs,
$m_2 \geq 1$ TBDs,
$\vdots$

Reduction to Two-tier Model

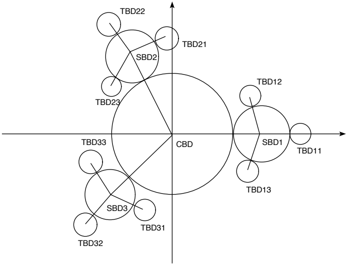
Theorem

If $L > L^M$, then for any given hierarchy $(m_1, m_2, \ldots, m_n)$ there exists unique hierarchic equilibrium.
It is technically equivalent to two-tier city equilibrium with an "effective" number of SBDs
$m_{eff} = m_1(m_2 \cdot \delta^2 + 1)(m_3 \cdot \delta^2 + 1) \ldots (m_n \cdot \delta^2 + 1)$, *where*
$\delta = \dfrac{f \cdot t - k}{f \cdot t + k} \in (0, 1)$ *– relative difference between commuting and communication costs. Moreover,* $m_{eff} \geq (1 + \delta^2)^n \to \infty$ *for* $n \to \infty$.

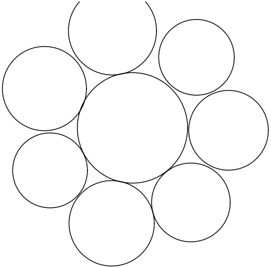
Corollary

We can do not care about theoretical maximum M^*!

"Equivalence"



$\Longleftrightarrow$



Let $m_1 = m_2 = 3,\ k = 1, f = 5, t = 1$, then $\delta = 2/3$ and
$m_{eff} = 3 \cdot \left(3 \cdot (2/3)^2 + 1\right) = 7$,
while theoretic maximum: $M^* \gtrapprox 7.85$.

That's All, Folks!

Thank You for Attention!

Any questions?