Firms' Heterogeneity and Productivity/Quality under VES

S.Kokovin, M.Parenti, J.F.Thisse, E.Zhelobodko

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Cost Heterogeneity and Productivity

Abstract

- ***Model***: Heterogeneous industry a-la Melitz but with general VES utility. Market size determines the *cutoff cost*, i.e., the worst firm.
- ***Effect***: When market grows, the cutoff cost and prices decrease, average productivity$\uparrow$ when RLV increases. Otherwise – reverse effect.
- ***Comparison***: Zero effect in CES-Melitz model. Non-robust productivity growth in Melitz-Ottaviano model with chock-off price assumption.

Cost heterogeneity literature and idea

- Empirical studies show essential and long-lasting cost heterogeneity in manufacturing as well as in natural resources industries. Intuitively, all types of heterogeneity matter, when the market expands or shrinks.
- Ricardian comparative-advantage-model suffices for natural resources. Seminal Hoppenheign (1992) and Melitz (2003) papers suggested theory for heterogeneity *in manufacturing*. In Melitz (2003) each enterpreneur in a monopolistically-competitive industry does not know her actual cost (talent), untill she experiments with the market and *learn by doing*. Applications to trade are numerous... Melitz-Ottaviano (2006) model differs in chock-off price assumption.

Model: varieties, consumer

- 1 differentiated good, 1 production factor – *labor*. Random marginal costs c have distribution $\Gamma(c)$, with density $\gamma(c)$, $c \in [c_0, \infty)$. Endogenous upper bound - cutoff cost $\bar{c}$, interval $[0, \bar{c}]$ of types operates. Endogeneous multiplier: N copies of each marginal cost c, so that operates $N \int_0^{\bar{c}} \gamma(c) dc$ varieties/firms.
- Consumption vector $X = (x_c)_{c \in [0,\bar{c}]}$ and price vector $P = (p_c)_{c \in [0,\bar{c}]}$, neoclassic utility u; $u(0) = 0$. Utility maximization:

$$\max_X \mathscr{U} = N \int_0^{\bar{c}} u(x_c) \mathrm{d}\Gamma(c), \quad N \int_0^{\bar{c}} p_c x_c \mathrm{d}\Gamma(c) \leq 1 + \Pi \Rightarrow$$

$$\mathrm{p}(x_i, \lambda) = \frac{u'(x_i)}{\lambda} \text{ with Lagrange multiplier } \lambda \tag{1}$$

Model: producers, supply

Each firm c-type faces the *same* per-consumer inverse demand function $\mathrm{p}(.,\lambda)$, where *marginal utility of income λ captures all the market statistics that matter to consumers* and firms.
Inverse-demand elasticity is $r_u \equiv -xu''(x)/u'(x)$, $\sigma = 1/r_u$.
L consumers/workers. Fixed cost f_s. Producing quantity $q_c = x_c L$ by c-firm requires cost $C(q_c) = cq_c + f_s = cLx_c + f_s$. Per-consumer maximal operational profit

$$\pi^*(c,\lambda) = \max_{x_c \geq 0}[\mathrm{p}(x_c,\lambda) - c]x_c = \frac{x^*(c\lambda)r_u(x^*(c\lambda))c}{1 - r_u(x^*(c\lambda))} \geq f_s/L \qquad (2)$$

has elasticities $\varepsilon_{\pi,c} = 1 - \sigma(c\lambda)$, $\varepsilon_{\pi,\lambda} = -\sigma(c\lambda)$.

Model: experimenting producers

Enterpreneurs emerge and die. Before starting business, each can learn her marginal cost c, if spending experiment cost f_e. Every moment firms are *born and killed stochastically*. A newborn enterpreneur's expected total profit includes cost of experiment:

$$L\Pi(\lambda) = L\int_0^{\bar{c}} (\pi^*(c,\lambda) - f_s)\mathrm{d}\gamma(c) - f_e. \tag{3}$$

When $L\Pi(\lambda) = 0$, every newcomer is indifferent to explore or not her type, at equilibrium only certain mass $\bar{N} < \infty$ of them do try.

Long-run equilibrium

(Long-run) *equilibrium* is the bundle $(\bar{\lambda}, \bar{c}, \bar{N}, \bar{X})$ including marginal utility of money, cutoff cost, replic width, and per-person consumption vector, s. t.:

- (**a**) consumption $\bar{X}$, inverse demand p and $\bar{\lambda}$ satisfy the consumer FOC;
- (**b**) under $\bar{\lambda}$, supply $\bar{X}$: is optimal for operating firms, the cutoff firm $\bar{c}$ has zero profit: $\pi_{\bar{c}} = f/L$;
- (**c**) each potential firm has zero expected profit;
- (**d**) labor balance (equivalent to budget constraint) hold.

Market growth impact

- Differentiating the free-entry condition and using $\varepsilon_{\pi,\lambda} = -\sigma(c\lambda)$:

$$\varepsilon_{\lambda,L} = \frac{L\partial\lambda}{\lambda\partial L} = \frac{-\frac{f_s\Gamma(\bar{c})}{L} - \frac{f_e}{L}}{-\int_0^{\bar{c}} \pi^*(c,\lambda)\sigma(\lambda c)\gamma(c)dc} > 0 \qquad (4)$$

- Differentating cutoff condition yields

$$\varepsilon_{\bar{c},L} = \frac{1}{\sigma(\lambda\bar{c})[\sigma(\lambda\bar{c})-1]}\left(1 - \frac{\frac{f_s\Gamma(\bar{c}_D)+f_e}{L}}{\int_0^{\bar{c}} \pi^*(c,\lambda)\frac{\sigma(\lambda c)}{\sigma(\lambda\bar{c})}\gamma(c)dc}\right)$$

that means decreasing cutoff cost c when $\sigma(\lambda c)$ decreases and reverse.

Market growth impact

Theorem. When population L increases, the impact is:

- (a) the cutoff cost $\bar{c}$ and all prices $\bar{p_c}$ decrease in pro-competitive markets ($r_u' > 0 \Rightarrow$ average productivity$\uparrow$), the reverse hold for anti-competitive markets ($r_u' > 0$), CES has 0 effect;
- (b) mass $\bar{N}$ of copies increases under ($r_u' > 0$), mass $\bar{N}\Gamma(\bar{c})$ of varieties increases when $r_u' < 0$ (conjecture: both increase in both cases);
- (c) marginal utility of income $\bar{\lambda}$ increases, individual consumption $\bar{x}_i$ of each variety decreases, in pro-competitive case ($r_u' > 0$) welfare increases;
- (d) some operating firms increase profits, some are worse off.

Market growth impact-picture

Figure: Firms' characteristics change in cost(c).

Market growth impact-explanation

Marginal utility of income $\lambda \uparrow$ with *market size* $L \uparrow$. *Under* $\uparrow$ *RLV* $r'(x) > 0$ *the cutoff cost decreases*, $\Rightarrow$ increase in aggregate productivity. In contrast with CES, when the market mimics pro-competitive behavior, a larger market makes competition tougher, which triggers the *exit of the least productive firms*. Then consumers reallocate demand among more productive firms and aggregate productivity$\uparrow$.
On the contrary, $r'(x) < 0$ is the price-increasing case, a larger market *softens* competition, which *allows less productive firms to enter*, thus decreasing aggregate productivity.

Quality model

$s \geq 0$ - quality index with a distribution $\Psi(s)$.
Consuming x_s units of variety s brings utility $u(sx_s)$.
Preferences combine horizontal and vertical differentiation:

$$\max_{x_s(.)} \mathscr{U} \equiv \int_S u(sx_s) d\Psi(s) \qquad \text{s.t.} \quad \int_S p_s x_s d\Psi(s) = 1$$

where $S \subset [s_{\min}, s_{\max}]$ is the (endogenous) range of available varieties and $0 \leq s_{\min} < s_{\max} \leq \infty$. Consumers' FOC:

$$p_s(x_s) = su'(sx_s)/\lambda.$$

$C(s)$ is marginal production cost of s-variety, $C'(s) > 0$ $C(0) = 0$. The operating profit of a firm with s-variety:

$$\pi(x_s; \lambda) = \left[\frac{su'(sx_s)}{\lambda} - C(s)\right] Lx_s = \left[\frac{u'(sx_s)}{\lambda} - \frac{C(s)}{s}\right] Lsx_s.$$

Trade

- Consumers' problem is the same

Conclusions to Heterogeneity

- We ***method*** to study heterogeneity MC models, using measure $r_u(.)$ of concavity for ***general*** utility function's, instead of specific utilities.
- Bigger pro-competitive markets should have better productivity, lower prices, higher welfare.
- Applications to trade: comparative advantage for bigger countries and welfare/productivity gains from lowering trade barriers. Exporters/non-exporters.
- Applications to economic geography: tendency to agglomeration.

Thank you.