Multi-product firms under monopolistic competition: the choice of scope

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Stylized facts about multi-product firms

- Multi-product firms account for the most part of industrial output;
- Intensive margins and extensive margins of industrial firms are positively correlated:
  - Bernard, A.B., S.J.Redding and P.K.Schott (2010). Multi-Product Firms and Product Switching // American Economic Review 100:70-97.
  - Goldberg, P., A. Khandewal, N. Pavnik and P. Topalova (2008). Multi-product Firms and Product Turnover in the Developing World: Evidence from India. NBER Working Paper No. 14127.
- There is positive correlation between the firm's size and the efficiency its of R&D projects:
  - Henderson, R. and I. Cockburn (1996). Scale, Scope, and Spillovers: The Determinants of Research Productivity in Drug Discovery // The RAND Journal of Economics Vol. 27, No. 1 (Spring, 1996), pp. 32-59
  - Cockburn, I. and R. Henderson (2001). Scale and scope in drug development: unpacking the advantages of size in pharmaceutical research // Journal of Health Economics, Vol. 20, No 6, pp. 1033 - 1057.

Theoretical literature on multi-product firms

- Ottaviano G.I.P. and J.F. Thisse (1999). Monopolistic Competition, Multiproduct Firms and Optimum Product Diversity. CORE discussion paper 9919.
- Nocke, V. and S. Yeaple (2006). Globalization and Endogenous firm scope. NBER working paper 12322.
- Feenstra, R. and H. Ma (2007). Optimal choice of product scope for multiproduct firms under monopolistic competition. NBER working paper 13703.
- Eckel, C. and J.P. Neary (2010). Multi-Product Firms and Flexible Manufacturing in the Global Economy // The Review of Economic Studies, Vol. 77, pp. 188217.

Questions we are trying to answer

- How do variations of the market size (induced by migrations) affect the scope, the firm's size, pricing decisions and the mass of firms?
- What is the role of scale-scope spillovers in the choice of scope?
- Does cannibalization effect necessarily arise in a setting with homogenous firms?

Our starting point

- Allanson, P. and C. Montagna (2005). Multi-product Firms and Market Structure: an Explorative Application to the Product Life Cycle // International Journal of Industrial Organization, Vol. 23, No. 7 – 8, pp. 587 – 597.
- Zhelobodko, E., S.Kokovin, M. Parenti and J.-F. Thisse. Monopolistic competition in general equilibrium: beyond the CES (2012) // Econometrica, forthcoming.

Plan

Commodities and market structure

- There is a continuum of firms of measure N.
- Each firm j, $j \in [0, N]$, chooses:
  - its product line scope n_j;
  - its production plan (q_{ij}).
- Products are assumed to be horizontally differentiated across firms as well as within the product lines of the firms.
- Each firm is a monopolist on the market of each product it chooses to produce.

Consumers

- The economy is inhabited by L identical consumers, each of whom forms her individual demands x_{ij} in order to maximize her utility function:

$$\mathscr{U} = \int_0^N \int_0^{n_j} u(x_{ij})\, di\, dj,$$

subject to the budget constraint:

$$\int_0^N \int_0^{n_j} p_{ij} x_{ij}\, di\, dj \leq 1.$$

- The function u is the elementary utility function, assumed to be:
  - increasing and concave;
  - exhibiting the relative love for variety, i.e. $0 < r_u(x) < 1 \quad \forall x \geq 0$, where $r_u(x) = -\dfrac{x\, u''(x)}{u'(x)}$.

Inverse demand functions

- Solving the consumer's problem, we obtain the inverse demand functions:

$$p_{ij} = \frac{u'(x_{ij})}{\lambda}.$$

- λ is a Lagrange multiplier, which can be treated as some aggregate market statistics.
- NB!! As there is a continuum of firms, the individual influence of each firm on λ is negligible.

Firms

- Each firm incurs costs of three types:
  - fixed costs F;
  - R&D costs (or control costs) $\psi(n)$, where n is the scope;
  - variable production costs $\varphi(y)$, where $y = \int_0^n q_i di$ is total output.
- The variable cost functions φ and ψ are assumed to be:
  - twice continously differentiable;
  - increasing;
  - convex, and at least one of them is strictly convex.
- Each firm maximizes its *profit function*:

$$\Pi = \int_0^n p_i q_i di - F - \varphi\left(\int_0^n q_i di\right) - \psi(n).$$

Symmetric equilibrium conditions

The "unit elasticity" condition:

$$\frac{\varphi'(y)\,y}{F+\varphi(y)+\psi(n)}+\frac{\psi'(n)\,n}{F+\varphi(y)+\psi(n)}=1.$$

The markup condition:

$$p=\frac{\varphi'(y)}{1-r_u}.$$

Free entry:

$$py=F+\varphi(y)+\psi(n).$$

Labour balance:

$$L=N\left(F+\varphi(y)+\psi(n)\right).$$

Existence and uniqueness of equilibrium

Definition

A *symmetric equilibrium* is a quadruple (y^*, n^*, p^*, N^*) which solves the system of equilibrium conditions.

Proposition

Assume that there exists some $\varepsilon > 0$ *such that* $\varepsilon < r_u(x) < 1 - \varepsilon \; \forall x \geq 0$. *Then a unique equilibrium* (y^*, n^*, p^*, N^*) *exists.*

The reactions of q^*, p^* and n^*N^*

Proposition

The average output q^*, the market price p^* and the total mass of varieties n^*N^* respond to an increase in market size according to the following three patterns, depending only on the RLV behavior:

RLV behavior	$r_u' > 0$	$r_u' = 0$	$r_u' < 0$
$\mathcal{E}_{p/L}$	$-r_u < \mathcal{E}_{p/L} < 0$	$\mathcal{E}_p = 0$	$\mathcal{E}_p > 0$
$\mathcal{E}_{q/L}$	$0 < \mathcal{E}_q < 1$	$\mathcal{E}_q = 0$	$\mathcal{E}_q < 0$
$\mathcal{E}_{nN/L}$	$0 < \mathcal{E}_{nN} < 1$	$\mathcal{E}_{nN} = 1$	$\mathcal{E}_{nN} > 1$

The reactions of n^*, y^* and Y^*

Proposition

The scope n^**, the total output of a firm* y^* *and the total output in the industry* $Y^* = N^* y^*$ *respond to an increase in market size according to the following three patterns:*

RLV behavior	$r'_u > 0$	$r'_u = 0$	$r'_u < 0$
$\mathcal{E}_{n/L}$	$-1 < \mathcal{E}_{n/L} < 0$	$\mathcal{E}_n = 0$	$\mathcal{E}_n > 0$
$\mathcal{E}_{y/L}$	$0 < \mathcal{E}_{y/L} < 1$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} < 0$
$\mathcal{E}_{Y/L}$	$\mathcal{E}_{Y/L} > 1$	$\mathcal{E}_{Y/L} = 1$	$\mathcal{E}_{Y/L} < 1$

Discussion

- If we “forget” for a while that some varieties may be produced by the same firm, we will see the same market outcome as in the single-product ZKPT model.
- The market outcome depends crucially on whether the elasticity of substitution is *decreasing* or *increasing* with respect to the individual consumption level. The case of CES preferences is *a borderline*.
- The supply side is *irrelevant* to the selection of a market outcome pattern.
- The average output q^* and the scope n^* always go in the opposite directions, i.e. *cannibalization effect* occurs.
- The function $Y^*(L)$ is the *aggregate production function* of the industry. The *increasing* (*decreasing*) marginal product of labour in the industry occurs under *decreasing* (*increasing*) elasticity of substitution.
- We compare the elasticity of total industrial output Y^* with unity but *not with zero*. Is it possible that the total industrial output *decreases* in response to a market size increase?

Reactions of the number of firms N^*

Proposition

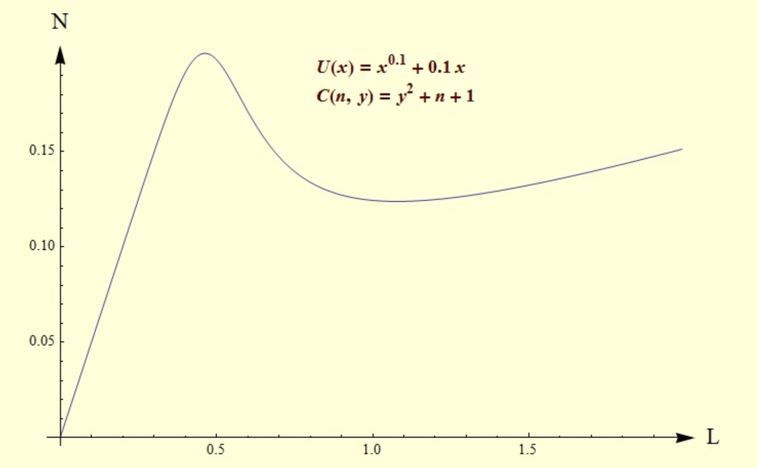
The reactions of the number of firms N^* to the changes in the market size are as follows:

Costs behavior	RLV behavior		
	$r_u' > 0$	$r_u' = 0$	$r_u' < 0$
$r_\varphi > r_\psi$	$0 < \mathcal{E}_{N/L} < 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} > 1$
$r_\varphi = r_\psi$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$
$r_\varphi < r_\psi$	$\mathcal{E}_{N/L} > 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} < 1$

Discussion

- The supply side is crucial for the behavior of the mass of firms.
- If, for example, φ is linear while ψ is strictly convex, economies of scale are *stronger* than economies of scope. The behavior of the mass of firms is then the same as in the single-product ZKPT model.
- If economies of scope are stronger than economies of scale, the reverse takes place.

$\mathcal{E}_{N/L} < 0$: an example



Scale-scope spillovers

- Empiricists find positive correlation between the firm's size and the efficiency its of R&D projects.
- The additively separable variable cost function is unable to catch this regularity.
- So, consider a variable cost function $C(y, n)$ of general type.
- Call C_y the *production marginal costs* (or *y-marginal costs*) and C_n the *scope marginal costs* (or *n-marginal costs*).

Definition

We say that the technology exhibits *scale-scope spillovers* (or *positive scope externality*) if y-marginal costs decrease with respect to scope, or, equivalently, if n-marginal costs decrease with respect to total output y. Formally:

$$C_{yn} < 0.$$

Elasticities of marginal costs

- The key-factor of the market outcome is the behavior of $r_u(x)$, which is the inverse demand elasticity.
- By analogy, we introduce marginal costs elasticities.
- But complications arise, for we have *two different marginal cost functions*: the y-marginal costs and the n-marginal costs.
- So, we have *four* marginal costs elasticities:
    - the y-elasticity of y-marginal costs $\dfrac{y\, C_{yy}}{C_y}$;
    - the n-elasticity of y-marginal costs $\dfrac{n\, C_{yn}}{C_y}$;
    - the y-elasticity of n-marginal costs $\dfrac{y\, C_{ny}}{C_n}$;
    - the n-elasticity of n-marginal costs $\dfrac{n\, C_{nn}}{C_n}$.

The reactions of q^*, p^* and n^*N^*

Proposition

The output of a specific variety q^**, the market price* p^* *and the total mass of varieties* n^*N^* *respond to an increase in market size according to three patterns, depending only on the RLV behavior:*

RLV behavior	case $r_u' > 0$	case $r_u' = 0$	case $r_u' < 0$
$\mathcal{E}_{q/L}$	$0 < \mathcal{E}_{q/L} < 1$	$\mathcal{E}_{q/L} = 0$	$\mathcal{E}_{q/L} < 0$
$\mathcal{E}_{p/L}$	$-r_u < \mathcal{E}_{p/L} < 0$	$\mathcal{E}_{p/L} = 0$	$\mathcal{E}_{p/L} > 0$
$\mathcal{E}_{nN/L}$	$0 < \mathcal{E}_{nN/L} < 1$	$\mathcal{E}_{nN/L} = 1$	$\mathcal{E}_{nN/L} > 1$

The reactions of firm's size y^*

Proposition

The firm's size y^* responds to a market size increase according to the following nine patterns:

Costs behavior	RLV behavior		
	case $r_u' > 0$	case $r_u' = 0$	case $r_u' < 0$
case $\frac{C_{nn}n}{C_n} + \frac{C_{ny}y}{C_n} > 0$	$\mathcal{E}_{y/L} > 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} < 0$
case $\frac{C_{nn}n}{C_n} + \frac{C_{ny}y}{C_n} = 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} = 0$
case $\frac{C_{nn}n}{C_n} + \frac{C_{ny}y}{C_n} < 0$	$\mathcal{E}_{y/L} < 0$	$\mathcal{E}_{y/L} = 0$	$\mathcal{E}_{y/L} > 0$

The reactions of firm's scope n^*

Proposition

The firm's scope n^* responds to a market size increase according to the following nine patterns:

Costs behavior	RLV behavior		
	case $r_u' > 0$	case $r_u' = 0$	case $r_u' < 0$
case $\frac{C_{yy}y}{C_y} + \frac{C_{yn}n}{C_y} > 0$	$\mathcal{E}_{n/L} < 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} > 0$
case $\frac{C_{yy}y}{C_y} + \frac{C_{yn}n}{C_y} = 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} = 0$
case $\frac{C_{yy}y}{C_y} + \frac{C_{yn}n}{C_y} < 0$	$\mathcal{E}_{n/L} > 0$	$\mathcal{E}_{n/L} = 0$	$\mathcal{E}_{n/L} < 0$

The reactions of the mass of firms N^*

Proposition

The mass of firms N^* responds to a market size increase according to the following nine patterns:

Costs behavior	RLV behavior		
	case $r_u' > 0$	case $r_u' = 0$	case $r_u' < 0$
case $\frac{C_{yy}y}{C_y} + \frac{C_{yn}n}{C_y} > \frac{C_{nn}n}{C_n} + \frac{C_{ny}y}{C_n}$	$\mathcal{E}_{N/L} > 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} < 1$
case $\frac{C_{yy}y}{C_y} + \frac{C_{yn}n}{C_y} = \frac{C_{nn}n}{C_n} + \frac{C_{ny}y}{C_n}$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} = 1$
case $\frac{C_{yy}y}{C_y} + \frac{C_{yn}n}{C_y} < \frac{C_{nn}n}{C_n} + \frac{C_{ny}y}{C_n}$	$\mathcal{E}_{N/L} < 1$	$\mathcal{E}_{N/L} = 1$	$\mathcal{E}_{N/L} > 1$

Discussion

- The ordering of marginal costs elasticities is crucial for the market outcome under scale-scope spillovers.
- The impacts of the market size on the average output q^* and the scope n^* have opposite signs if and only if $\frac{C_{yy}y}{C_y} + \frac{C_{yn}n}{C_y} > 0$. Otherwise, the impacts have the same sign determined by the RLV behavior.
- The condition $\frac{C_{yy}y}{C_y} + \frac{C_{yn}n}{C_y} > 0$ is thus a *cannibalization condition.* Intuitively, cannibalization arises under *relatively low positive scope externalities*.

Plans for further work

- Heterogeneity of firms;
- The open economy case;
- Endogenous choice between producing a single product and multiple products.

Thank you for your attention!