Endogenous Structure of Cities

City without External Trade

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Endogenous Market Structure and Spatial Economics

Terminology Agreement

- ***Monocentric City***: there exists unique Central Business District (CBD);
- ***Polycentric City***: along with CBD there exist additional Secondary Business District(s), connected to CBD;
- Extension: ***Hierarchic Polycentricity*** – Tertiary BDs, Quaternary BDs etc.

Stylized Facts

1. Monocentric Cities and Polycentric Cities co-exist in economy space;
2. "Large" cities are Polycentric, while "small" ones are Monocentric;
3. City pattern may change with the lapse of time (i.e. pattern is not predefined);
4. Population growth is disproportionally larger in suburbia (i.e., in SBDs);
5. Moreover, sometimes Central city may "freezes", while suburbia population still increases;

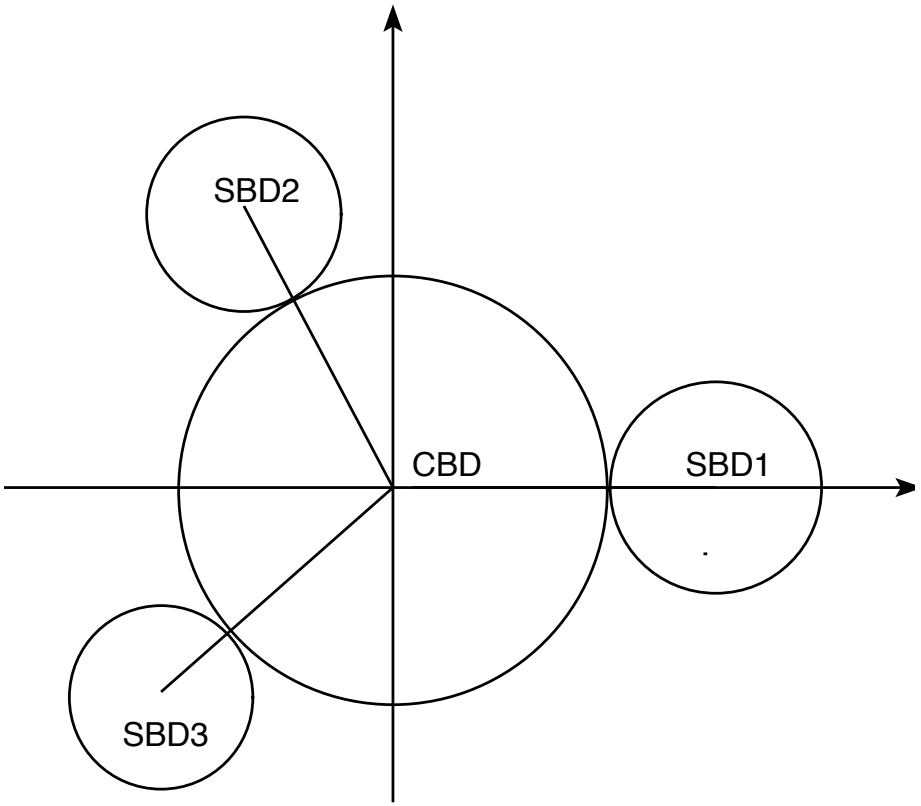
What Should be Explained?

- Monocentric/pre-Polycentric paradigms can't explain *WHY and HOW City pattern forms and changes;*
- *ENDOGENOUS nature* as outcome of *interplay of FORCES* ;
- ***Agglomeration force***: Economy of Scale, i.e. decreasing average production costs force firms to gather;
- ***Dispersion force***: Urban costs (e.g., commuting) force to form additional (secondary) job centers;

Similar Approaches

- Main insights are based on
  - J. Cavailhès, C. Gaigné, T. Tabuchi, J.-F. Thisse (2007), *Trade and the structure of cities*, Journal of Urban Economics
- ***Differences***: drop down "long narrow city" with exactly TWO secondary centers, 2D pattern with arbitrary number of subcenters: I—SBD_1—I———CBD———I—SBD_2—I
- Insight of job sub-centers' hierarchy (secondary, tertiary, etc) borrowed from
  - T. Tabuchi (2009), *Self-organizing marketplaces*, Journal of Urban Economics

Polycentric City



CBD – Central Business District, (Axis origin placed here)
SBDi – Secondary Business Districts
$i \in \{1, 2, \ldots, m\}$,
$m = 0$ – monocentric city

Why Circles?

There is no reason to be something else

- Job centers (Central or Secondary) are dimensionless;
- Plane is initially featureless;
- Residents (workers) take a unite of area for residence and commuting costs depend on *Euclidean distance;*
- Each worker lives in residence zone around her job center;
- Thus Central Zone and Secondary Zones are circles and;
- All Secondary Zones are identical;

Main Feature of CBD

- SBDs lack some non-tradable goods/services presented in CBD: specific local public goods and business-to-business services such as marketing, banking, insurance;
- It reflects in additional *communication* costs $\mathscr{K}(x^S) = K + k \cdot ||x^S||$, $K > 0$, $k > 0$

Parameters: Exogenous vs Endogenous

- *Size of population* L – exogenous in separated city and endogenous in economy with migration;
- *Pattern (mono/polycentric)* – $m > 0$ or $m = 0$, endogenous;
- *Number* of SBDs m – ambiguously, will be discussed later;
- *Distance* $||x^S||$ of SBD x^S from CBD – endogenous;
- *Population shares* of residence zones (Central $\theta \in (0, 1]$ and Secondary $(1 - \theta)/m$) – endogenous;

Workers

Workers' welfare depends on the three goods:

- q_0 – numéraire good, homogenous and unproduced with initial endowment $\bar{q}_0$;
- $q(i)$, $i \in [0, n]$ a continuum n of varieties of a horizontally differentiated good under monopolistic competition and increasing returns, using labor as the only input. Traded costlessly for price $p(i)$ within the city of origin;
- one lot of land area for residence, price (rent) $R(x)$ depend on location. Normalization: one lot of the land $= \pi \approx 3.14159\ldots$

Consumer's Problem

For CBD worker:

$$U(q_0; q(i), i \in [0, n]) \to \max$$

$$s.t. \int_0^n p(i)q(i)\mathrm{d}i + q_0 = w^C + \bar{q}_0 - R^C(x) - t||x||$$

$R^C(x)$ – land rent at location x (in fact, depends on distance $||x||$ from the CBD only).
The budget constraint of SBD worker

$$\int_0^n p(i)q(i)\mathrm{d}i + q_0 = w^S + \bar{q}_0 - R^S(x) - t||x\text{-}x^S||.$$

Firm's Problem

Producing of variety i requires a given number f of labor units (fixed costs) and costs c units of numeraire.

- $c = 0$ without loss of generality;
- For firm producing i in CBD:

$$\Pi^C(i) = I(i) - f \cdot w^C \to \max$$

$I(i)$ – firm's revenue ($I(i) = p(i) \cdot Q(i)$ for local sales)

- For firm in SBDs:

$$\Pi^S(i) = I(i) - f \cdot w^S - \mathscr{K}(x^S) \to \max$$

City Equilibrium

- None of workers want to change her choice of job/residence (CBD or one of SBD zone and distance from job center);
- None of firms want change her choice of location (CBD or one of SBD);
- Firm's Cut-Off: profits $\Pi^C = \Pi^S = 0$;
- Mass of firms (=varieties of differentiated good) $n = \frac{L}{f}$;
- Consumer's Cut-Off:
  $w - C_u = w^C - (R^C(x) + t \cdot ||x|| - \bar{q}_0) \geq 0$;

Endogenous Central Share θ

- Costs $K > 0$, $k > 0$, $f > 0$ and $L \geq 0$, $m > 0$ are given;
- If $f \cdot t \leq k$ all firms stay in CBD, i.e. $\theta \equiv 1$;
- Otherwise, let $\delta = \dfrac{f \cdot t - k}{f \cdot t + k} \in (0,1)$ – *relative difference* of firm's commuting and communication costs;
- $L^M = \left(\dfrac{K}{k} \cdot \dfrac{(1-\delta)}{2\delta} \right)^2$

Theorem

If $L \leq L^M$, then polycentric city could not exists. i.e. $\theta \equiv 1$. Otherwise, for any $m > 0$ unique equilibrium $\theta^* \in \left(\frac{1}{1+m\delta^2}, 1 \right)$ exists.

Comparative Statics

Theorem

$$\frac{\partial \theta^*}{\partial L} < 0, \ \frac{\partial \theta^*}{\partial m} < 0 \ \textit{for} \ L > L^M,$$

$$\lim_{L \to \infty} \theta^* = \frac{1}{1 + m\delta^2}, \ \lim_{m \to \infty} \theta^* = \frac{1}{L}\left(\frac{K}{k} \cdot \frac{(1-\delta)}{2\delta}\right)^2 = \frac{L^M}{L}$$

Corollary

$$\lim_{m \to \infty} \theta^* \cdot L \equiv L^M$$

Redistribution of Total Rent

Initial endowment

$$\bar{q}_0 = \frac{1}{L}\int_X R(x)\mathrm{d}x$$

In this case Consumer's Cut-Off:

$$w - C_u = w - t\sqrt{\theta^* L} + \frac{t}{3}\cdot\sqrt{L}\left[(\theta^*)^{3/2} + \frac{(1-\theta^*)^{3/2}}{\sqrt{m}}\right] \geq 0$$

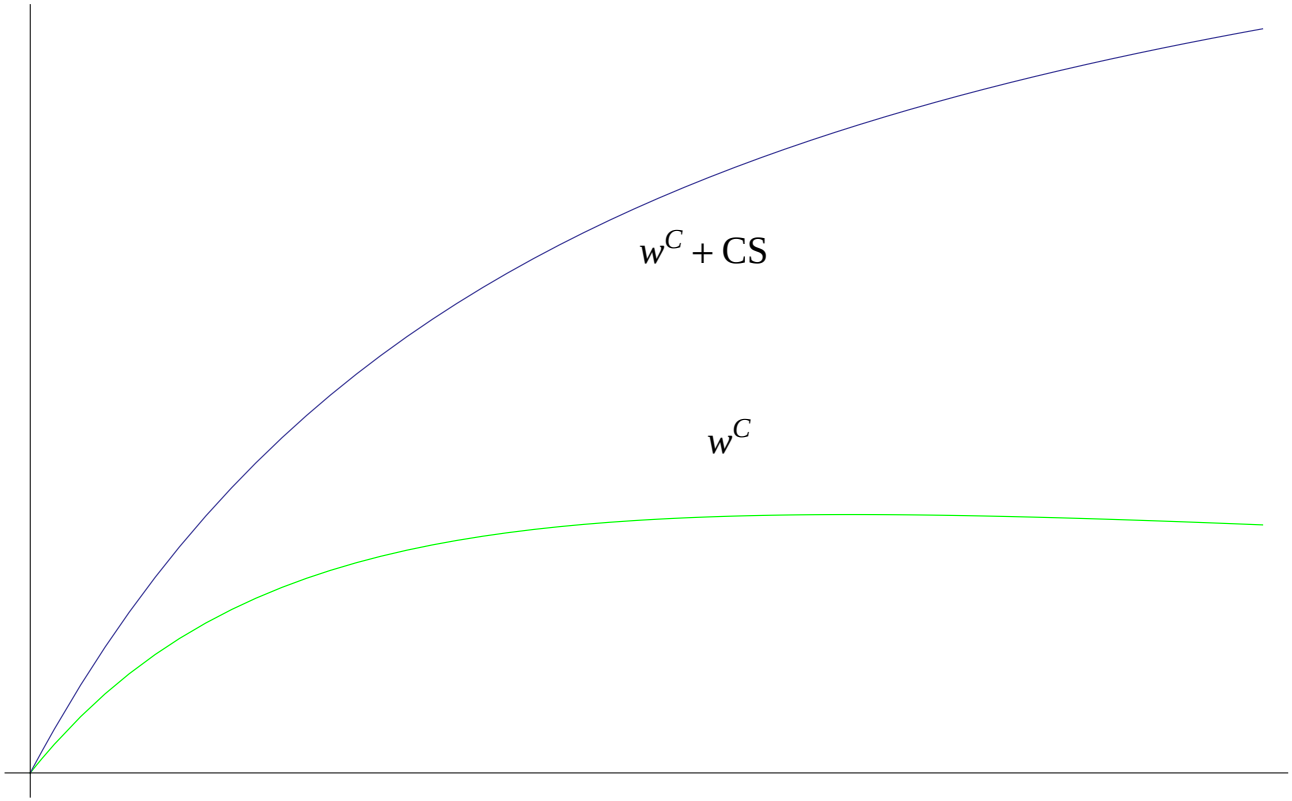
Consumer's Utility

- Ottaviano's quasi-linear utility for $\alpha > 0$, $\beta > 0$, $\gamma > 0$

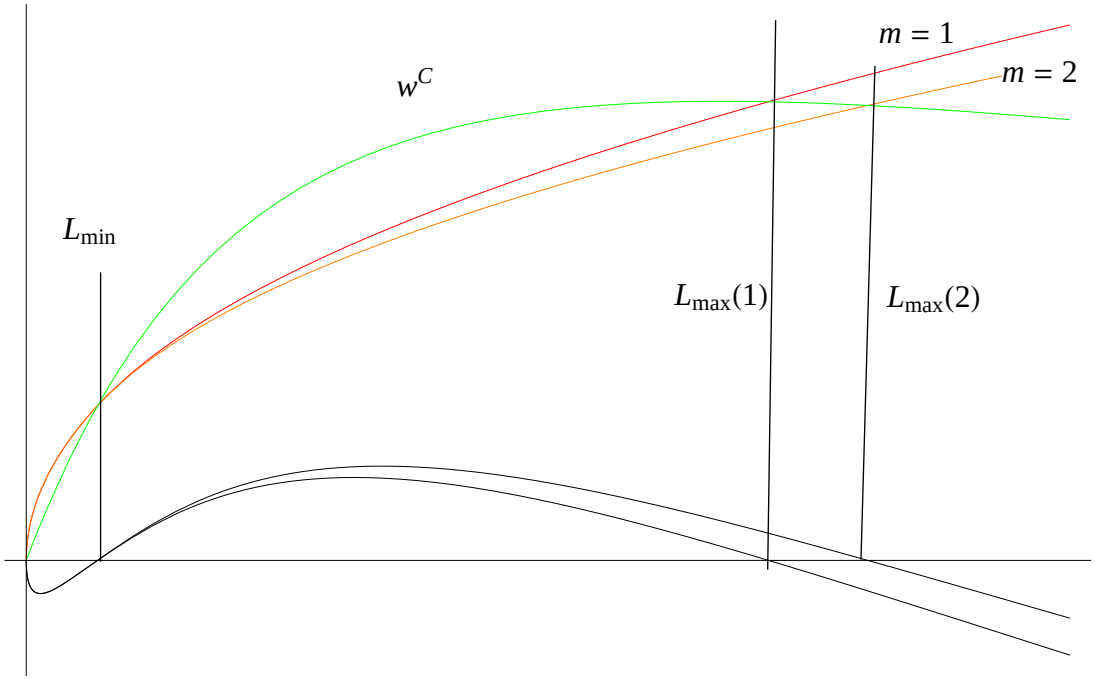
$$U(q_0; q(i), i \in [0, n]) = q_0 + \alpha \int_0^n q(i)\mathrm{d}i - \frac{\beta}{2} \int_0^n [q(i)]^2 \mathrm{d}i - \frac{\gamma}{2} \left[\int_0^n q(i)\mathrm{d}i \right]$$

  (studied for *linear cities* in J. Cavailhès et al. (2007))
- Generates *linear demand curve*

Wage and Consumer's Surplus



Cut-Off Interval

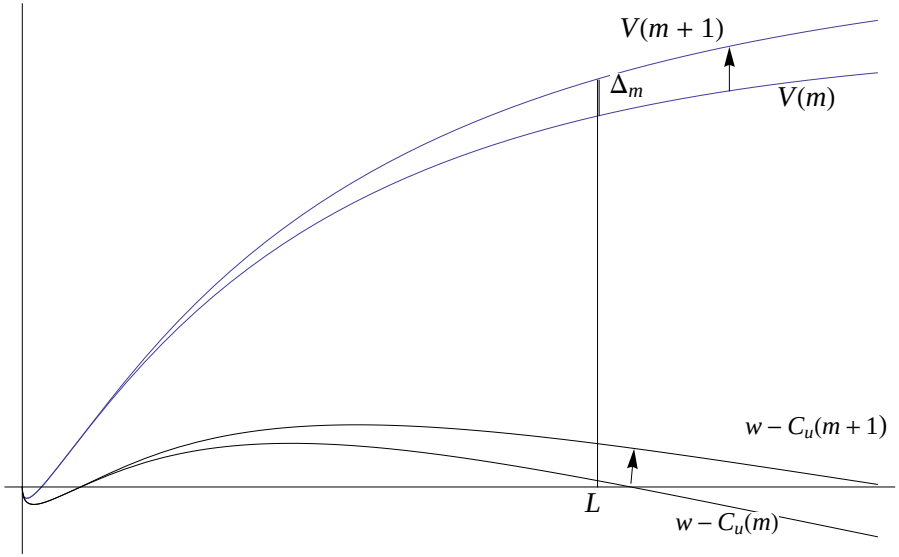


Cut-Off: $L \in [L_{\min}, L_{\max}] \subset (0, +\infty)$, non-emptiness condition:

$$t < \frac{3\alpha^2}{16\sqrt{2\beta\gamma f}}$$

Endogenous SBD Number m?

Assumption. SBD's number m is determined by "City Developers".



Welfare (or Real Wage) $V = w + CS - C_u$.

$\Delta_m = V(m+1) - V(m) > 0$, while $\sum_{m=0}^{\infty} \Delta_m < \overline{V}$, thus *marginal gains tent to zero* with increasing m.

Developing $m \to m+1$ is cost-efficient if marginal developing costs $M \leq L \cdot \Delta_m$

Endogenous Population L?

What are incentives for "immigrants":

- Disposable Income $w - C_u$;
- or Welfare (Real Wage) $w + CS - C_u$;
- Anyways, answer (endogenous L) requires extension to multi-regional setting with inter-city trade;

Correlation of L and m

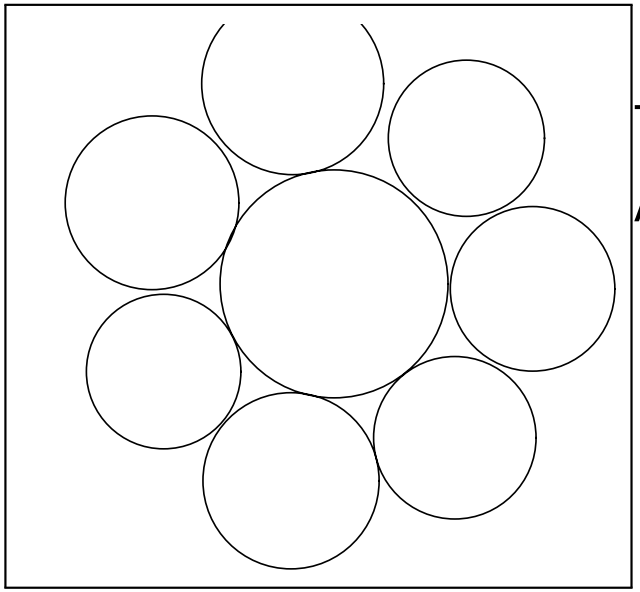
- Empiric evidences: *L and m positively correlate* (MacMillen and Smith (2003), *The number of subcenters in large urban areas*, JUE);
- Increasing in m strengthens incentives for immigrants, thus L will increase;
- Increasing in L requires to increase m, until marginal gains match marginal developing costs;

Conclusions

- Existence Threshold: $L_{\min} > 0$, upper limit $L_{\max} < \infty$
- Policentricity Threshold: $L^M = \left(\frac{K}{k} \cdot \frac{(1-\delta)}{2\delta} \right)^2$
- Developing of Suburbia ($m \uparrow$) improve Disposable Income and Welfare

What If City Reach Its Maximum?

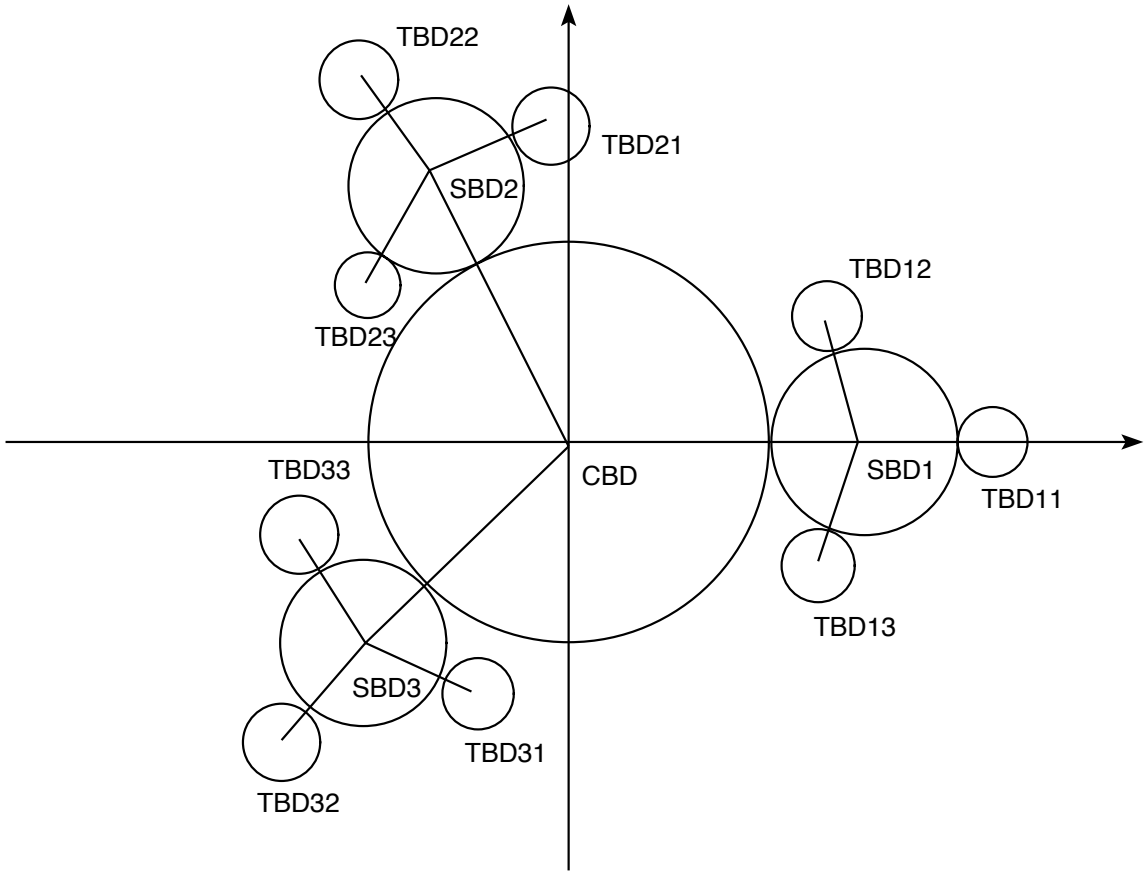
Does It Mean that Moscow Is NON-Rubber?



There exists theoretical maximum of SBDs.
Asymptotically (for large L):

$$M^* \approx \frac{\pi}{\arcsin\left(\frac{\delta}{1+\delta}\right)} \gtrapprox \frac{\pi \cdot (1+\delta)}{\delta}$$

Hierarchy of Business Districts



$$
\begin{aligned}
&m_0 \equiv 1 \text{ CBD},\\
&m_1 \geq 1 \text{ SBDs},\\
&m_2 \geq 1 \text{ TBDs},\\
&\quad\vdots
\end{aligned}
$$

Do We Need to Extend the Model?

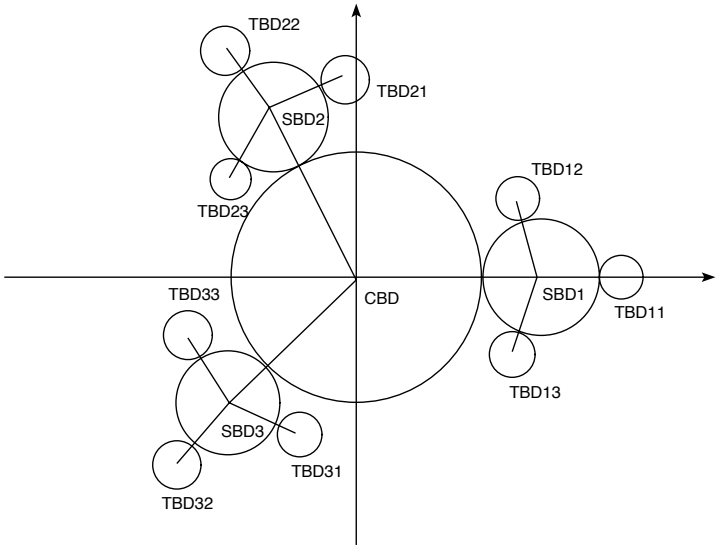
Theorem

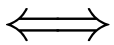
If $L > L^M$, then for any given hierarchy $(m_1, m_2, \ldots, m_n)$ there exists unique hierarchic equilibrium.
It is technically equivalent to two-tier city equilibrium with an "effective" number of SBDs
$m_{eff} = m_1(m_2 \cdot \delta^2 + 1)(m_3 \cdot \delta^2 + 1)\ldots(m_n \cdot \delta^2 + 1)$, *where*
$\delta = \dfrac{f \cdot t - k}{f \cdot t + k} \in (0,1)$ – *relative difference between commuting and communication costs. Moreover,* $m_{eff} \geq (1 + \delta^2)^n \to \infty$ *for* $n \to \infty$.

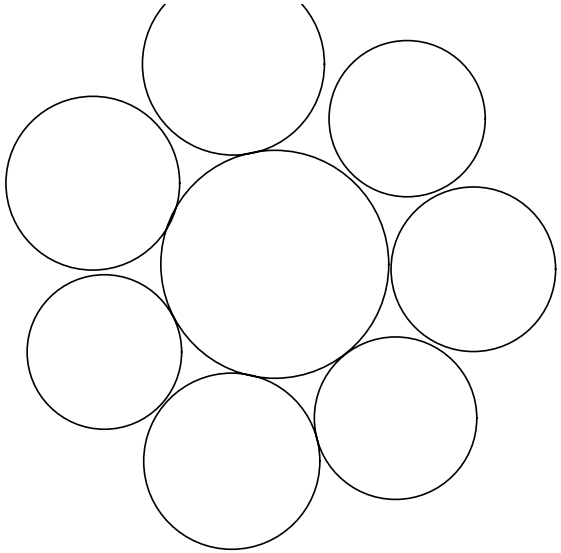
Corollary

We can do not care about theoretical maximum M^*!

"Equivalence"







Let $m_1 = m_2 = 3,\ k = 1, f = 5, t = 1,$ then $\delta = 2/3$ and
$m_{eff} = 3 \cdot \left(3 \cdot (2/3)^2 + 1\right) = 7,$
while theoretic maximum: $M^* \gtrapprox 7.85.$

That's All, Folks!

Thank You for Attention!

Any questions?