How does the market size affect the firm's product line scope? The closed economy case

S.Kokovin, Ph.Ushchev, E.Zhelobodko

2012

Research agenda

Some issues on multiproduct firms frequently adressed in the empirical literature:

- How are the average outputs (intensive margins) correlated with the product line scopes (extensive margins)?
- What are the relative contributions of firms' entry and exit, changes in intensive margins and changes in extensive margins to total manufacturing output growth?
- What are the crucial factors of expansion/reduction of the product line scope?

Literature

- Ottaviano, G.I.P. and J.-F. Thisse. Monopolistic competition, multiproduct firms and product diversity (2011) \\ The Manchester School, Vol 79, No. 5, pp. 938–951.
- Zhelobodko, E., S.Kokovin, M. Parenti and J.-F. Thisse. Monopolistic competition in general equilibrium: beyond the CES (2011) \\ Working paper N°2011-08, Paris School of Economics.
- Bernard, A.B., S.J.Redding and P.K.Schott (2010). Multi-Product Firms and Product Switching \\ American Economic Review 100:70-97.

Plan

Firms (1)

- There is a continuum of firms of measure N.
- Each firm j, $j \in [0, N]$, chooses:
  - its product line scope n_j, $n_j \geq 0$;
  - its production plan $\mathbf{q}_j : [0, n_j] \to \mathbb{R}_+$
- Products are assumed to be horizontally differentiated across firms as well as within the product lines of the firms. Each firm is a monopolist on the market of each product it chooses to produce.

Firms (2)

- Each firm j is endowed with a production technology characterized fully by the fixed costs F_j and the variable costs function $\widetilde{C}_j(\mathbf{q}_j, n_j)$.
- Each firm maximizes its profit function $\widetilde{\Pi}_j(\mathbf{q}_j, n)$, defined as follows:

$$\widetilde{\Pi}_j(\mathbf{q}_j, n) = \int_0^{n_j} p_{ij} q_{ij} di - F_j - \widetilde{C}_j(\mathbf{q}_j, n_j).$$

- Here $q_{ij} = \mathbf{q}_j(i)$, p_{ij} is the price of the product which has number i in the product line of firm j.
- The profit of the firm which enters the market last equals zero (free entry condition): $\widetilde{\Pi}_N = 0$.

Consumers

- The economy is inhabited by L identical consumers, each of whom forms her individual demands x_{ij} in order to maximize her utility function:

$$\mathscr{U} = \int_0^N \int_0^{n_j} u(x_{ij})\, di\, dj,$$

subject to the budget constraint:

$$\int_0^N \int_0^{n_j} p_{ij}\, x_{ij}\, di\, dj \leq 1.$$

- The function u is the elementary utility function, assumed to be:
  - increasing and concave;
  - exhibiting the relative love for variety, i.e. $0 < r_u(x) < 1 \quad \forall x \geq 0$, where $r_u(x) = -\dfrac{x\, u''(x)}{u'(x)}$.

Inverse demand functions

- Solving the consumer's problem, we obtain the inverse demand functions:

$$p_{ij} = \frac{u'(x_{ij})}{\lambda}.$$

- λ is a Lagrange multiplier, which can be treated as some aggregate market statistics.
- NB!! As there is a continuum of firms, the individual ifluence of each firm on λ is negligible!

Basic assumptions about technologies

- Assume that all firms have identical technologies. So, in what follows, we drop the firm index j. Due to this, as well as the inverse demand functions, we can transform the profit function as follows:

$$\widetilde{\Pi}(\mathbf{q}, n) = \frac{1}{\lambda} \int_0^n u' \left(\frac{q_i}{L} \right) q_i \, di - F - \widetilde{C}(\mathbf{q}, n).$$

- Assume that the variable cost function $\widetilde{C}(\mathbf{q}, n)$ is convex in the production plan $\mathbf{q}$ by any value of the product line scope n.
- We would also like to assume that the variable cost function $\widetilde{C}(\mathbf{q}, n)$ is symmetric in the production plan $\mathbf{q}$, i.e. invariant to any "renumbering" of the components of the production plan.
- But what is the proper notion of a renumbering for the elements of a continuum?

Simmetry

Consider an operator $\mathscr{S}_j$, $j \in [0, n]$, mapping the set of all production plans into itself and defined as follows:

$$(\mathscr{S}_j \mathbf{q})_i = \begin{cases} q_{i+j}, & i+j \leq n, \\ q_{i+j-n}, & i+j > n. \end{cases}$$

Definition 1. Call $\mathscr{S}_j$ *the operator of cyclical shift by* j.

Definition 2. Let $\varphi(\mathbf{q})$ be some function defined on production plans. Call the function φ *symmetric*, if it is invariant to cyclical shift operators:

$$\varphi(\mathbf{q}) = \varphi(\mathscr{S}_j \mathbf{q}) \ \forall j \in [0, n].$$

Assume now that the ariable cost function $\widetilde{C}(\mathbf{q}, n)$ is symmetric in the production plan $\mathbf{q}$ by any product line scope n.

Symmetrized cost function

In what follows, production plans where outputs of all products are equal play an essential role.

Definition. Call a production plan $\mathbf{q}$ *symmetric*, if the output of each product equals the average output by the whole product line:

$$\mathbf{q} \equiv q.$$

Now consider a function:

$$C(q, n) = \widetilde{C}(\mathbf{q}, n)|_{\mathbf{q} \equiv q}.$$

The function $C(q, n)$ is just a restriction of the variable cost function on the set of symmetric production plans.

Definition. Call the function $C(q, n)$ *the symmetrized cost function*.

Further assumptions

- Assume that the symmetrized cost function $C(q, n)$ increases in both arguments and is convex in the product line scope$n$ for any average output q.

- Assume that as the product line scope is zero, the variable costs are also zero, while as the average output is zero, the variable costs are positive at any positive product line scope:

$$C(q, 0) = 0 \, \forall \, q \geq 0, \quad C(0, n) > 0 \, \forall \, q > 0.$$

The symmetrized profit function

Define the symmetrized profit function:

$$\Pi(q, n) = \widetilde{\Pi}(\mathbf{q}, n)|_{\mathbf{q} \equiv q}.$$

Lemma. *The maxima of $\widetilde{\Pi}$ and Π exist or don't exist simultaneously, and if they do, the maximum points are the same.*

Due to lemma 2, we can state the producer will always choose a symmetric plan and a product line scope which solve the symmetrized producer's problem:

$$\Pi = \frac{1}{\lambda} n\, q\, u' \left(\frac{q}{L}\right) - C(q, n) - F \to \max_{(q,n)}$$

NB! To state the symmetrized producer's problem, we don't need to know precisely the variable cost function $\widetilde{C}(q_i, n)$, it suffices to know the symmetrized cost function $C(q, n)$!!!

The equivalence of cost functions

Definition. Call the variable cost functions $\widetilde{C}_1$ and $\widetilde{C}_2$ *equivalent*, if their symmetrized cost functions are the same.

Lemma. *Let $\widetilde{C}$ be some variable cost function. Then there exists an equivalent function $\widehat{C}$, which depends only on the product line scope and the average output.*

Proof. Let $C(q, n)$ be the symmetrized cost function of $\widetilde{C}$. Define $\widehat{C}$ as follows:

$$\widehat{C}(\mathbf{q}, n) = C\left(\frac{1}{n}\int_0^n q_i di,\ n\right).$$

QED.

The equivalence of cost functions: an example

Consider a variable cost function which is additive in production costs of different varieties:

$$\widetilde{C}(\mathbf{q}, n) = \int_0^n \varphi(q_i)di + \psi(n),$$

where φ, ψ are increasing and convex. The symmetrized cost function is:

$$C(q, n) = n\varphi(q) + \psi(n).$$

Consider now the following cost function $\widehat{C}$:

$$\widehat{C}(\mathbf{q}, n) = n\varphi\left(\frac{1}{n}\int_0^n q_i di\right) + \psi(n).$$

The functions $\widetilde{C}$ and $\widehat{C}$ are obviously equivalent.

Equilibrium conditions

From the producer's FOC and the free entry condition, we obtain the equilibrium conditions:

$$C_n n = C(q, n) + F,$$

$$1 - r_u\left(\frac{q}{L}\right) = \frac{q\,C_q}{n\,C_n}.$$

From the first equilibrium condition we immediately obtain the following result.

Proposition 2. *The firm always chooses the product line scope which minimizes the average total cost under optimum average output.*

Equilibrium conditions: an illustrative example

Take the following cost function:

$$C(q, n) = (qn)^2 + n^2.$$

The equilibrium conditions are:

$$n^2 = \frac{F}{1 + q^2},$$

$$r_u\left(\frac{q}{L}\right) = \frac{1}{1 + q^2}.$$

The equilibrium is a point of intersection of a falling curve with a vertical line on the (q, n) plane.

Correlation between q and n: possible outcomes

Proposition 2. *The correlation of the average output* q *and the product line scope* n *is positive (zero, negative) if and only if the elasticity of the marginal production costs* C_q *in the product line scope is less than (equal to, greater than) unity:*

$$\frac{dn}{dq} \gtreqless 0 \Leftrightarrow \frac{n\,C_{qn}}{C_q} \lesseqgtr 1.$$

Corollary 1. *If the marginal production costs* C_q *are concave (convex) in the product line scope, then the* q *correlation of the average output* q *and the product line scope* n *is positive (negative).*

Corollary 2. *The product line scope doesn't change with the changes of the market size* L *if the cost function is of the form:*

$$C(q, n) = n\,\varphi(q) + \psi(n),$$

where φ, ψ *are increasing and concave.*

Correlation between q and n: example 1

Consider the cost function which is separable in the total output (firm size) qn and the product line scope n:

$$C(q,n) = C_1(qn) + C_2(n),$$

where $C_{1,2}$ are increasing and convex functions. Then:

$$\frac{n\,C_{nq}}{C_q} = 1 + \frac{qn\,C_1''(qn)}{C_1'(qn)}.$$

As C_1 is convex, $\frac{n\,C_{qn}}{C_q} > 1$, and hence, due to proposition 2, the correlation between q and n is negative.

NB! In fact, in this case even qn and n are negatively correlated!

Correlation between q and n: example 2

Consider the following cost function:

$$C(q, n) = q\,n + e^{-q}(n - \log(1+n)).$$

It is increasing and concave in both arguments. The marginal production costs are:

$$C_q = \left(1 - e^{-q}\right)\, n + e^{-q} \log(1+n).$$

So, C_q is concave in n. Thus, by corollary 2, the correlation between q and n is positive.

Comparative statics of equilibria with respect to the market size L (1)

We study comparative statics with respect to the market size only for the case when the cost function is separable in the total output (firm size) qn and the product line scope n:

$$C(q, n) = C_1(qn) + C_2(n).$$

Such symmetrized function is generated, for example, by the following variable cost function:

$$\widetilde{C}(\mathbf{q}, n) = C_1\left(\int_0^n q_i di\right) + C_2(n).$$

Comparative statics of equilibria with respect to the market size L (2)

Proposition 3.
The comparative statics of equilibria in the model with the costs separability in the firm size and the product line scope is as follows:

RLV behavior	$r_u' > 0$	$r_u' = 0$	$r_u' < 0$
$\frac{\partial p}{\partial L}\frac{L}{p} \equiv \mathscr{E}_p$	$-r_u < \mathscr{E}_p < 0$	$\mathscr{E}_p = 0$	$\mathscr{E}_p > 0$
$\frac{\partial q}{\partial L}\frac{L}{q} \equiv \mathscr{E}_q$	$0 < \mathscr{E}_q < 1$	$\mathscr{E}_q = 0$	$\mathscr{E}_q < 0$
$\frac{\partial n}{\partial L}\frac{L}{n} \equiv \mathscr{E}_n$	$-1 < \mathscr{E}_n < 0$	$\mathscr{E}_n = 0$	$\mathscr{E}_n > 0$
$\frac{\partial y}{\partial L}\frac{L}{y} \equiv \mathscr{E}_y$	$0 < \mathscr{E}_y < 1$	$\mathscr{E}_y = 0$	$\mathscr{E}_y < 0$
$\frac{\partial (nN)}{\partial L}\frac{L}{nN} \equiv \mathscr{E}_{nN}$	$0 < \mathscr{E}_{nN} < 1$	$\mathscr{E}_{nN} = 1$	$\mathscr{E}_{nN} > 1$
$\frac{\partial Y}{\partial L}\frac{L}{Y} \equiv \mathscr{E}_Y$	$\mathscr{E}_Y > 1$	$\mathscr{E}_Y = 1$	$\mathscr{E}_Y < 1$

Comparative statics of equilibria with respect to the market size L (3)

Proposition 4.

Let $r_1(y) = -\frac{y\,C_1''(y)}{C_1'(y)}$, $r_2(n) = -\frac{n\,C_2''(n)}{C_2'(n)}$. *The reactions of the number of firms* N *to the changes in the market size are as follows:*

Costs\RLV	$r_u' > 0$	$r_u' = 0$	$r_u' < 0$
$-r_1 < -r_2$	$0 < \mathcal{E}_N < 1$	$\mathcal{E}_N = 1$	$\mathcal{E}_N > 1$
$-r_1 = -r_2$	$\mathcal{E}_N = 1$	$\mathcal{E}_N = 1$	$\mathcal{E}_N = 1$
$-r_1 > -r_2$	$\mathcal{E}_N > 1$	$\mathcal{E}_N = 1$	$\mathcal{E}_N < 1$

The reactions of the firm's revenue R *to the changes in the market size are as follows:*

Costs\RLV	$r_u' > 0$	$r_u' = 0$	$r_u' < 0$
$-r_1 < -r_2$	$0 < \mathcal{E}_R < 1$	$\mathcal{E}_R = 0$	$\mathcal{E}_R < 0$
$-r_1 = -r_2$	$\mathcal{E}_R = 0$	$\mathcal{E}_R = 0$	$\mathcal{E}_R = 0$
$-r_1 > -r_2$	$-1 < \mathcal{E}_R < 0$	$\mathcal{E}_R = 0$	$\mathcal{E}_R > 0$

Some questions unresolved yet

- Is it possible that the number of firms N and the total manufacturing output Y fall when the market size increases?
- Bernard, Redding and Schott find the following pattern for the US economy in 1972 - 1997: $\frac{\triangle N}{N} < 0 < \frac{\triangle n}{n} < \frac{\triangle q}{q}$. Within our setting, this pattern, in principle, cannot be explained by the market size variations by proposition 3. But can it be explained by technical progress?

Thank you for your attention!