Market Size, Productivity and Entrepreneurship in a Model a'la Melitz

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02/17/12

Outline

1. Motivation
   - Survey
   - Review of research
2. Our Results
   - Main Assumptions
   - Comparative Statics
3. Special cases of utilities
   - Linear upper utility function
   - Lower-tier utility function is CES
   - Upper-tier utility function is logarithm

Outline

Actuality

Recently the actual direction of researching in modern IO (especially in modeling of Monopolistic Competition) and International Trade is modeling of firms heterogeneity

The most popular approaches to this problem were developed in **Melitz (2003)** and **Melitz & Ottaviano (2008)**
But the fundamental defect in these works is exogeneity of hetergeneous productivity.

Our Aim is to develop model with endogenous heterogeneity and to investigate influence of market characteristics on profits and outputs of firms

Modern Approach

The first attempt to modeling of endogenous heterogeneity is proposed in **Oyama, D., Sato, Y., Tabuchi, T. and Thisse, J.-F. (2011)**

But two significant weakness in their approach:

1. using specific utility function (CES)
2. agents are homogenous in entrepreneurship ability and heterogeneous in unskilled work abilities.

Unspecific Utility Function Approach

This method has been proposed in **Krugman (1979)** and **Vives (1999)**
The most common form of that was developed in **Zhelobodko E., Kokovin S., Parenti M. and Thisse J.-F. (2011)**

In the last paper this approach was applied to the Melitz's model, but the firms were been as "black box" still

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Maintain Ideas

Our work is draft research of developing model with endogenous heterogeneity in Monopolistic Competition environment.

We will use approach proposed in **Lucas (1978)**:

- agents are homogenous in their preferences and unskilled work abilities, heterogeneous in entrepreneurship abilities
- agents choose their type of activity (to be an entrepreneur or to be a worker) comparing a reservation wage with a potential profit

Announce of Contributions

We have evaluated of elasticities the next characteristics of the Economy w.r.t. of market size changing and have found out their boundaries:

- share and total amount of entrepreneurs in population
- individual consumption of varieties
- profit and size of a firm
- prices and market statistics

and other interpretation

Outline

Economy

- 2-sector economy (diversified sector with increasing return in scale and traditional one with constant return in scale)
- only one factor for producing - a labor
- L agents in economy (continuously), mobile between sectors
- Agents are homogenous in their preferences (quasi-linear, linear in homogenous good)
- Agents are homogenous in their work abilities, each of one has a unit of labor
- Agents are differentiated in their entrepreneurship abilities, describing by the parameter c - marginal cost of production if organizing a firm (the smaller c the higher entrepreneurship ability). Each c-type has L copies in the population.
- Each agent knows her type and chooses between being an entrepreneur or a worker
- The parameter c is distributed on $[0;+\infty]$ (d.d.f. $\gamma(c)$, c.d.f $\Gamma(c)$)

Consumer problem

Each agent consumes infinite-dimensional vector of varieties
$\mathbf{x}(c) \equiv \{x(c) : [0;\bar{c}] \to R_+\}$ and scalar $A \geq 0$ of homogenous good
Consumer problem is

$$V\left(L\int_0^{\bar{c}} u(x_c)\gamma_c dc\right) + A \to \max_{x,A}, \text{ s.t. } L\int_0^{\bar{c}} p_c x_c \gamma_c dc + A = I$$

here:
I is agent's income, it equals the wage $w = 1$ for worker and the operating profit π_s for s-type entrepreneur
$\bar{c}$ (endogenously) is the "cutoff", type of "marginal" agent, who is indifferent between two types of activity: to be entrepreneur or to be a worker.

Low-tier Utility Function

Low-tier utility , $u(x_c)$, is the satisfaction from consuming x_c:

$$u(\cdot) \in C^3(\mathbb{R}_+ \mapsto \mathbb{R}_+),\, u'(\cdot) \geq 0,\;\; u''(\cdot) \leq 0$$

$$u(0) = 0,\;\; u'(0) = +\infty$$

The more convex is function u the higher is relative love for variety (that mirrors Arrow-Pratt theory of risk aversion)

$$r_{u_c} \equiv r_u(x_c) \equiv -\frac{u''_c x_c}{u'_c} \in (0;1)$$

Marginal utility function isn't too convex

$$r_{u'_c} \equiv r_{u'}(x_c) \equiv -\frac{u'''_c x_c}{u''_c} < 2$$

Upper-tier Utility Function

Interdependence of varieties with the numerarie is expressed by the upper-tier utility function $V(Y)$ of consumption a composite good:

$$V(\cdot) \in C^2(\mathbb{R}_+ \mapsto \mathbb{R}),\ V'(Y) \geq 0,\ \ V''(Y) \leq 0$$

The more convex is function V the higher is love for variety

$$r_V \equiv -\frac{YV''(Y)}{V'(Y)} \in [0;1]$$

Demand

First Order Condition for consumer problem is:

$$p(x_c) = \frac{u'(x_c)}{\lambda}, \; c \in [0;\bar{c}], \lambda \equiv 1/V'\left(L\int_0^{\bar{c}} u(x_c)\gamma_c dc\right)$$

here:
λ is market statistics, the higher its derivative the more income is spent on differentiated good whereas too concave V means quick satisfaction with varieties.
Assumptions on $u(\cdot)$ guarantee the neoclassic demand properties: $p(\cdot)$ decreases from infinity to zero.

Producer problem

A firm has only Variable Costs, proportionally to it's output

$$\pi_c = (p(x_c) - c)\, L x_c \to \max_{x_c},$$

The **First Order Condition** determines output for c-type agent:

$$\frac{(1 - r_u(x_c))\, p_c}{c} = 1$$

This condition allows to express outputs of firms with different marginal cost. Particular, it's possible to evaluate output of ordinal entrepreneur over marginal one.

Zero Profit Condition

The equilibrium is defined by formulating the "zero-profit condition"

$$\pi_{\overline{c}} = w = 1 \Leftrightarrow \frac{\overline{r}_u \overline{x}}{1 - \overline{r}_u} = \frac{1}{L\overline{c}}$$

If agent's type lets him to get profit more than wage, then that agent will chose entrepreneur activity.

Equilibrium

Definition

The equilibrium is the bundle $\left(\bar{c},\ \lambda,\ \{p_c;\ x_c\}_{c\in[0;\bar{c}]}\right)$ such that consumption x maximizes each consumer's utility under price vector $\{\mathbf{p} = p_c(x_c)\}_{c\in[0;\bar{c}]}$ and solves each producer's problem under $\bar{c},\ \lambda,\ \mathbf{p}(\cdot)$, the zero-profit condition holds and $\lambda = 1/V'\left(L\int_0^{\bar{c}} u(x_c)\gamma_c dc\right)$. For a given equilibrium, consumption of the numerarie A_c for each type follows from the budget constraint, that entails also the labor balance under our normalization $w = 1$

Outline

Marginal Productivity

Proposition 1. *The elasticity of the cutoff cost to the market size is determined with love to variety. It can't decrease/increase too fast:*

$$\mathcal{E}_L \overline{c} = \frac{\overline{r}_u}{1-\overline{r}_u} \frac{\frac{1}{r_V} - \frac{1}{\overline{r}_u} + \frac{J_0}{J}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\overline{r}_u} \frac{\overline{u}}{\tilde{u}} \frac{\overline{\gamma c}}{\overline{\Gamma}}} \in \left(-1; \frac{\overline{r}_u}{1-\overline{r}_u}\right)$$

here: bar signs the value expressed at the point $x_{\overline{c}}$, tilde singes an average under condition $c < \overline{c}$,

$$J_0 = \int_0^{\overline{c}} \frac{u'(x_c)x_c}{r_{u_c}} \cdot \frac{1-r_{u_c}}{2-r_{u'_c}} \gamma_c dc, \quad J = \int_0^{\overline{c}} u(x_c)\gamma_c dc$$

$$\overline{\Gamma} = \int_0^{\overline{c}} \gamma_c dc, \quad \tilde{u} = \frac{J}{\overline{\Gamma}}$$

Marginal Productivity, sufficient condition

Proposition 1. *For cutoff cost increase in response to the growing market size, any of the following two conditions are sufficient, either*

$$\bar{r}_u > r_V \;\Rightarrow\; \mathcal{E}_L \bar{c} > 0$$

or concavity of u decrease and concavity of $\ln u$ is bounded from below:

$$r_u' \leqq 0,\; r_{\ln u} \geq 1 \;\Rightarrow\; \mathcal{E}_L \bar{c} \geqq 0$$

Market Statistics

Proposition 2. *If the market expands then the market statistics increases but it doesn't too fast*

$$\mathscr{E}_L \lambda = \frac{1 + \frac{\bar{r}_u}{1-\bar{r}_u} \frac{\bar{u}}{\tilde{u}} \frac{\overline{\gamma c}}{\bar{\Gamma}}}{\frac{1}{r_V} + \frac{J_0}{J} + \frac{1}{1-\bar{r}_u} \frac{\bar{u}}{\tilde{u}} \frac{\overline{\gamma c}}{\bar{\Gamma}}} > 0$$

$$\mathscr{E}_L \lambda \leq \max\{\bar{r}_u; r_V\}$$

This elasticity increases w.r.t. both r_V and $\bar{r}_u$

Average Productivity

Proposition 3. *Average productivity, $\tilde{c}$, goes in the same direction of the marginal productivity*

$$\mathcal{E}_L \tilde{c} = \left(\frac{\overline{c}}{\tilde{c}} - 1 \right) \frac{\overline{\gamma}\overline{c}}{\overline{\Gamma}} \mathcal{E}_L \overline{c}$$

here: $\tilde{c} = \frac{\int_0^{\overline{c}} c \gamma_c dc}{\overline{\Gamma}}$ - conditional average of entrepreneurship ability (conditional average of marginal cost)

Mass of Entrepreneurs

Total amount of entrepreneurs is $E = L\overline{\Gamma} = L\int_0^{\overline{c}} \gamma_c dc$
The fraction of entrepreneurs in population is $e = \frac{E}{L} = \overline{\Gamma} = \int_0^{\overline{c}} \gamma_c dc$
Proposition 4. *The fraction of entrepreneurs changes like as the marginal productivity*

$$\mathcal{E}_L e = \frac{\overline{\gamma_{\overline{c}}}\,\overline{c}}{\overline{\Gamma}} \mathcal{E}_L \overline{c}$$

Remark. *If the marginal productivity decreases the total amount of entrepreneurs might decreases too*

$$\mathcal{E}_L E = 1 + \frac{\overline{\gamma_{\overline{c}}}\,\overline{c}}{\overline{\Gamma}} \mathcal{E}_L \overline{c} \in \left(1 - \frac{\overline{\gamma_{\overline{c}}}\,\overline{c}}{\overline{\Gamma}};\ 1 + \frac{\overline{\gamma_{\overline{c}}}\,\overline{c}}{\overline{\Gamma}} \frac{\overline{r}_u}{1 - \overline{r}_u}\right)$$

Output and Consumption

Proposition 5. *Higher entrepreneurial ability implies bigger output and vice verse:*

$$Lx_{c_1} \gtreqless Lx_{c_2} \Leftrightarrow c_1 \lesseqgtr c_2$$

Theorem 2. *Elasticity of individual consumption of each variety w.r.t. the market size is negative:*

$$\mathcal{E}_L \overline{x} = -\frac{1-\overline{r}_u}{2-r_{u'_{\overline{c}}}} \cdot (1+\mathcal{E}_L \overline{c}) \leq 0, \ \mathcal{E}_L x_c = \frac{1-\overline{r}_u}{r_{u_c}} \cdot \frac{1-r_{u_c}}{2-r_{u'_c}} \cdot \left(\mathcal{E}_L \overline{c} - \frac{\overline{r}_u}{1-\overline{r}_u} \right) \leq 0$$

Remark. This effect is consist from direct and indirect ones. The last is different for marginal and non-marginal entrepreneur.

Direct and Indirect Effects

$$\mathcal{E}_L^d \overline{x} = -\frac{1-\overline{r}_u}{2-r_{u'_{\overline{c}}}} \cdot \leq 0, \ \mathcal{E}_L^d x_c = -\frac{\overline{r}_u}{r_{u_c}} \cdot \frac{1-r_{u_c}}{2-r_{u'_c}} \leq 0$$

$$\mathcal{E}_L^i \overline{x} = -\frac{1-\overline{r}_u}{2-r_{u'_{\overline{c}}}} \cdot \mathcal{E}_L \overline{c}, \ \mathcal{E}_L^i x_c = \frac{1-\overline{r}_u}{r_{u_c}} \cdot \frac{1-r_{u_c}}{2-r_{u'_c}} \mathcal{E}_L \overline{c}$$

Corollary. The individual consumption is changed with direct influence more than proportial the population if and only if concavity measure r_u is decreasing:

$$\left| \mathcal{E}_L^d x_c \right| \gtreqless 1 \Leftrightarrow r'_u \lesseqgtr 0$$

$$\mathcal{E}_L^d (L x_c) \begin{cases} \leq 0 & , \ r'_u \leq 0 \\ \in [0;1] & , \ r'_u \geq 0 \end{cases}$$

Ordinal and Marginal Entrepreneurs, outputs

Proposition 6. *The ratio* $\frac{Lx_c}{L\overline{x}}$ *of outputs increases if and only if new entrepreneurs enter to the market:*

$$\lim_{c \to \overline{c}} \mathcal{E}_L \frac{x_c}{\overline{x}} = \frac{1}{\overline{r}_u} \cdot \frac{1 - \overline{r}_u}{2 - \overline{r}_{u'}} \cdot \mathcal{E}_L \overline{c} \gtreqless 0 \Leftrightarrow \mathcal{E}_L \overline{c} \gtreqless 0,$$

and in the case $\mathcal{E}_L \overline{c} \geq 0$ *we have also an upper bound:*

$$\lim_{c \to \overline{c}} \mathcal{E}_L \frac{x_c}{\overline{x}} = \frac{1}{\overline{r}_u} \cdot \frac{1 - \overline{r}_u}{2 - \overline{r}_{u'}} \cdot \mathcal{E}_L \overline{c} \leqq \frac{1}{2 - \overline{r}_{u'}}$$

Prices

Proposition 7. *The price for variety produced by entrepreneur increases when the population grow*

$$\mathcal{E}_L p_c = -(1-\bar{r}_u)\frac{1-r_{u_c}}{2-r_{u'_c}}\left(\mathcal{E}_L \bar{c} - \frac{\bar{r}_u}{1-\bar{r}_u}\right) - \mathcal{E}_L \lambda$$

$$\mathcal{E}_L \bar{p} = \bar{r}_u \cdot \frac{1-\bar{r}_u}{2-\bar{r}_{u'}}(1+\mathcal{E}_L \bar{c}) - \mathcal{E}_L \lambda$$

Remark. There're both effects (direct and indirect) for prices as for outputs.

Ordinal and Marginal Entrepreneurs, prices

Proposition 7. *The limit of elasticity of price ratio* $\frac{\overline{p}}{p_c}$ *is positive:*

$$\lim_{c \to \overline{c}} \mathcal{E}_L \frac{\overline{p}}{p_c} = \frac{1 - \overline{r}_u}{2 - \overline{r}_{u'}} (1 + \mathcal{E}_L \overline{c}) \in \left(0; \frac{1}{2 - \overline{r}_{u'}}\right)$$

and it can be bigger or smaller than unit conditional upon increasing/decreasing concavity measure $\overline{r}_u = r_u(x_{\overline{c}})$ *and expanding/shrinking entrepreneurs fraction, as follows*

$\lim_{c \to \overline{c}} \mathcal{E}_L \frac{\overline{p}}{p_c}$	$\mathcal{E}_L \overline{c} < 0$	$\mathcal{E}_L \overline{c} = 0$	$\mathcal{E}_L \overline{c} > 0$
$\overline{r}'_u < 0$	< 1	< 1	?
$\overline{r}'_u = 0$	< 1	$= 1$	> 1
$\overline{r}'_u > 0$	?	> 1	> 1

Profit

Proposition 9. *The profit which is received by the entrepreneur from an agent decreases if the market size expands*

$$\mathcal{E}_L \frac{\pi_c}{L} = \left(2 - r_{u'_c}\right) \mathcal{E}_L x_c \leq 0$$

Remark *The total profit can grows when the market expands*

$$\mathcal{E}_L \pi_c = 1 + \left(2 - r_{u'_c}\right) \mathcal{E}_L x_c \overset{?}{\sim} 0$$

If share of entrepreurs grows and RLV increases then $\mathcal{E}_L \pi_c > 0$
If share of entrepreurs decreases and $r_{u_c} < 0.5$ then $\mathcal{E}_L \pi_c < 0$

Outline

Specification

Under this utility specification the consumer's program is:

$$L\int_0^{\bar{c}} u_c \gamma_c dc + A \quad \to \quad \max_x,$$

$$\text{s.t. } L\int_0^{\bar{c}} p_c x_c \gamma_c dc + A \quad = \quad I$$

The markets are not related in this specification. The results are trivial in this case

Consumption and Output

z	$\mathcal{E}_L z$	bounds	behavior	comments
$\bar{c}$	$\frac{\bar{r}}{1-\bar{r}}$	$(0;+\infty)$	$\nearrow$in $\bar{r}$	under $\bar{r} > 0.5$ cutoff grows too fast
$\bar{x}$	$-\frac{1}{2-r_{u'_c}}$	$(-\infty;-1)$	$\searrow$in $r_{u'_c}$	consumption is reduced
$\bar{y} = L\bar{x}$	$\frac{1-r_{u'_c}}{2-r_{u'_c}}$	$(-\infty;0)$	$\searrow$in $r_{u'_c}$	under $r_{u'_c} > 1.5$ output is fast reduced with market size
x_c	0		*const*	consumption remains unchanged
$y_c = Lx_c$	1		*const*	output is proportional to the market size

Results (2), market statistic, prices and profits

z	$\mathcal{E}_L z$	bounds	behavior	comments
λ	0		*const*	market statistic is unchanged
$\overline{p}$	$\frac{\overline{r}}{2-r_{u_c'}}$	$(0;+\infty)$	$\nearrow$in $\overline{r}$ $\nearrow$in $r_{u_c'}$	by the absolute value the price is changing slower than consumption; under $\overline{r}>0.5$ price is changed faster than output;
p_c	0			price remains unchanged
$\frac{\pi_c}{L}$	-1		*const*	profit is unchanged
$\frac{\pi_c}{L}$	0		*const*	profit per capita is unchanged

Outline

Specification

Under this utility specification the consumer's program is:

$$V\left(L\int_0^{\bar{c}} x_c^{\rho}\gamma_c dc\right) + A \quad \to \quad \max_{x_c}, \;\; c \in [0;\overline{c}]$$

$$s.t.\; L\int_0^{\bar{c}} p_c x_c \gamma_c dc + A \quad = \quad I$$

In this specification with condition condition $r_V < 1$ CES isn't borderline due two-sector economy

Results (1), productivity, consumption and output

z	$\mathcal{E}_L z$	bounds	behavior	comments
$\bar{c}$	$\frac{\left(\frac{1}{r_V}-1\right)\frac{1-\rho}{\rho}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$\left(0;\frac{1-\rho}{\rho}\right)$	$\searrow$in r_V	under $\rho\longrightarrow 0$, upper boundary expands; under $\rho>0.5$ growth is slow
$\bar{x}$	$-\frac{\frac{1}{\rho}\left(1+\frac{1}{r_V}-\frac{\rho}{1-\rho}+\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}\right)}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$\left(-\frac{1}{\rho};-1\right)$	$\nearrow$in r_V	consumption declines speedily; lower boundary expands respect to the ρ
$\bar{y}=L\bar{x}$	$-\frac{\left(\frac{1}{r_V}-1\right)\frac{1-\rho}{\rho}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$\left(-\frac{1-\rho}{\rho};0\right)$	$\nearrow$in r_V	output is reciprocal to the marginal cost of producing;
x_c	$-\frac{\frac{1}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$(-1;0)$	$\searrow$in r_V	consumption is reduced slowly; it is changed slower than market size expands
$y_c=Lx_c$	$\frac{\frac{1}{r_V}-1}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\gamma c}}{\bar{\Gamma}}}$	$(0;1)$	$\searrow$in r_V	output is increased slowly, but it is changed slower than market size expands

Results (2), market statistic, prices and profits

z	$\mathcal{E}_L z$	bounds	behavior	comments
λ	$\frac{1+\frac{1-\rho}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\bar{\gamma}\bar{c}}}{\bar{\bar{\Gamma}}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\bar{\gamma}\bar{c}}}{\bar{\bar{\Gamma}}}}$	$(0;1$-$\rho)$	$\searrow$in r_V $\nearrow$in ρ	level of market statistic increases slowly, lower boundary expands with ρ
$\bar{p}$	$\frac{\frac{1-\rho}{\rho}\frac{1}{r_V}-1}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\bar{\gamma}\bar{c}}}{\bar{\bar{\Gamma}}}}$	$\left(0;\frac{1}{\rho}-1\right)$	$\searrow$in r_V	upper boundary shrinks with ρ
p_c	0		*const*	price is unchanged
$\frac{\pi_{\bar{c}}}{L}$	$-\frac{1+\frac{1}{r_V}-\frac{\rho}{1-\rho}+\frac{\bar{u}}{\tilde{u}}\frac{\overline{\bar{\gamma}\bar{c}}}{\bar{\bar{\Gamma}}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\bar{\gamma}\bar{c}}}{\bar{\bar{\Gamma}}}}$	$(-1;-\rho)$	*const*	the value is exactly -1
$\frac{\pi_c}{L}$	$-\frac{\frac{\rho}{1-\rho}+\frac{\bar{u}}{\tilde{u}}\frac{\overline{\bar{\gamma}\bar{c}}}{\bar{\bar{\Gamma}}}}{\frac{1}{r_V}+\frac{\rho}{1-\rho}+\frac{1}{\rho}\frac{\bar{u}}{\tilde{u}}\frac{\overline{\bar{\gamma}\bar{c}}}{\bar{\bar{\Gamma}}}}$	$(-\rho;0)$	$\searrow$in r_V	profit slowly increases

Outline

Specification

Under this utility specification the consumer's program is:

$$\ln\left(L\int_0^{\bar{c}} u_c\gamma_c dc\right) + A \quad \to \quad \max_{x_c},\ c\in[0;\bar{c}]$$

$$\text{s.t. } L\int_0^{\bar{c}} p_c x_c\gamma_c dc + A \quad = \quad I$$

This specification is is pretty border case, because measure of concavity for logarithm is maximum: $r_V = 1$. In this case CES is borderline.

Results (1), productivity

The result about expand/shrinking entrepreneur sector when population grows is displayed in the table:

<table>
<tr><th>$\mathcal{E}_L \overline{c}$</th><th>$r'_{u_c} < 0$</th><th>$r'_{u_c} = 0$</th><th>$r'_{u_c} > 0$</th></tr>
<tr><td>$r_{\ln(u_c)} < 1$</td><td>?</td><td>< 0</td><td rowspan="2">< 0</td></tr>
<tr><td>$r_{\ln(u_c)} = 1$</td><td rowspan="2">> 0</td><td>$= 0$</td></tr>
<tr><td>$r_{\ln(u_c)} > 1$</td><td>> 0</td><td>?</td></tr>
</table>

The market statistics is bounded from below: $\mathcal{E}_L \lambda \leq r_u$

Results (2), consumption, output and profit

The estimates of elasticities of consumption, output and profit of the commodity produced by an entrepreneur are displayed in the following table:

$\mathcal{E}_L \frac{\pi_c}{L}$ $\mathcal{E}_L x_c$ $\mathcal{E}_L y_c$	$r'_{u_c} < 0$	$r'_{u_c} = 0$	$r'_{u_c} > 0$
$r_{\ln(u_c)} < 1$	?	$> r'_{u_c} - 2$ > -1 > 0	$> r'_{u_c} - 2$ > -1 > 0
$r_{\ln(u_c)} = 1$	$< r'_{u_c} - 2$ < -1 < 0	$= r'_{u_c} - 2$ $= -1$ $= 0$	
$r_{\ln(u_c)} > 1$		$< r'_{u_c} - 2$ < -1 < 0	?

Outcome

- Structure of employment/entrepreneurship can be influenced by the economy size
- Population growth changes the equilibrium
- *CES* isn't borderline in two-sector model

Further Directions

- Comparison of equilibrium with social optimum
- Modeling of trade
- Studying similar model, that allow for natural income effects
  1. one sector model (it is proved that *CES* is borderline there)
  2. two sector model without quasi-linearity of preferences

THANK FOR YOUR ATTENTION!!!